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Strategies of linear feedback control and its classification

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Abstract

This paper is concerned with the control problem for a class of nonlinear dynamical (hyperchaotic) systems based on linear feedback control strategies. Since the obtaining positive feedback coefficients are required for these strategies. From this point of view, the available ordinary/dislocated/enhancing and speed feedback control strategies can be classified into two main aspects: control the dynamical systems or can't be control although it own *a positive feedback coefficients. So, we focused on* these cases, and *suggest a new method to recognize which system can be controller it or not. In this method, we divided the positive feedback coefficient which obtain from these strategies in to four categories according to possibility of suppression and show the reason for each case. Finally, numerical simulations are given to illustrate and verify the results.*

Keywords: *dislocated feedback control, enhancing feedback control, ordinary feedback control, routh-hurwitz criterion, speed feedback control*

1. Introduction

Chaos theory deals with nonlinear dynamical systems that are very sensitive to even small changes in the initial conditions, known as chaotic/hyperchaotic systems. Chaos theory has applications in several areas of science and engineering such as plasma systems, chemical reactor, control theory, biological networks, artificial neural networks, telecommunications and secure communication [1-3]. Chaos control, in a broader sense, can be divided into two categories: one is to suppress the chaotic dynamical behavior and the other is to generate or enhance chaos in nonlinear systems [4-8]. Recently, after the pioneering work of Ott et al. [9-13], many different techniques and methods have been proposed to achieve chaos control, such as OGY method, impulsive control method, sliding method control, adaptive control method, chaos suppression method [14-19], and so on. Among them, the linear feedback control is especially attractive and has been commonly applied to practical implementation due to its simplicity in configuration and implementation [2-4]. Generally speaking, there are two main approaches for controlling chaos: linear feedback control and nonlinear feedback control. The linear feedback control approach offers many advantages such as robustness and computational complexity over the nonlinrar feedback control method [9-15].

However, in a linear feedback control strategies the necessary condition for suppressing the chaotic/hyperchaotic system is a feedback coefficient must be a positive. Most of the previous works suppressed chaotic/hyperchaotic systems which satisfy this condition. But in Ref [12] founded some of cases can't be control although it's contains a positive feedback coefficient. In this paper we focused on these cases and found a new method for classified which system can control it or not. Based on above discussion, we know that the existing results on the stabilization (controller) of nonlinear dynamical systems are aimed at some special systems. In these results, motivated by the stabilization method implemented by rendering the nonlinear systems be passive with designed feedback controller, the stabilization of nonlinear systems based on linear feedback control with positive feedback coefficient will be studied in this paper, which, to the author's knowledge, is an open problem at present. The contributions of this paper include following two aspects: Firstly, a suitable controller is designed to transform the chaotic/hyperchaotic systems into stable systems and achieve

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the required disturbance attenuation performance. Secondly, a coefficient computation based new method is presented to solve the feasible interval value of the coefficients. Comparing with the existing results, with the method of this paper, it is deals with all possible casas to find the active coefficient. In addition, with the feasible interval solution of the coefficients contained in the feedback controller, the disturbance attenuation capability of the system is adjustable. The remainder of this paper is organized as follows. Section 2 is the problem formulation. The main contribution of this paper is then given in section 3, in which the adaptive classification and disturbance attenuation control problem of nonlinear dynamical system is studied with the positive feedback coefficient. In section 4, a numerical example to support the results is given, which is followed by the conclusion in section 5.

2. Research Method

Consider a class of nonlinear dynamical system in the form of [20-25]:

$$\dot{X} = AX + f(X) \tag{1}$$

where $(t) = [x_1, x_2, \dots\dots, x_n]^T \in R^{n\times 1}$, $i = 1, 2, \dots, n$ is the state variables of the system, $A = (a_{ij})_{n\times n}$ is the matrix of the system parameters, and $f: R^n \to R^n$ is the nonlinear part of the dynamical system.

The controller system is given by the following form:

$$\dot{X} = AX + f(X) + U \tag{2}$$

where $U = [u_i]^T = [u_1, u_2, \dots\dots, u_n]^T \in R^{n\times 1}$ represent the control, the purpose of the control problem is to choose suitable a feedback controller U to make that $\lim_{t\to\infty} \|X(t)\| = 0$.

This control U based on linear feedback control strategies which consist of three sub-strategies such as ordinary, dislocated and enhancing as well as to speed feedback control, can be written in a succinct mathematical form as:

$$U = u_i = \begin{cases} -kx_i & ; \; if \quad i = j \quad (\, ordinary) \\ -kx_j & ; \; if \quad i \neq j \quad (dislocated) \\ -k\dot{x}_j & ; \; if \quad i \neq j \quad (\; speed \;) \\ -k[x_i]_{i=1}^{n} & ; \; if \quad i = j \quad (enhancing) \end{cases} \tag{3}$$

where k is called a feedback coefficient, and $k > 0$.

Some time, more than one positive feedback coefficients are obtain based on Routh-Hurwitz criterion. So, we used the following formulation in order to select active feedback coefficient [15].

$$k = \cap_{i=1}^{n} k_i \; = k_1 \cap k_2 \cap \dots\dots \cap k_n \tag{4}$$

Based on above these sub-strategies, the nonlinear dynamical system can control if it has a positive feedback coefficient. This condition is necessary and sufficient for controlled nonlinear dynamical systems. But in some time, its failure to control although satisfied this condition. In order to overcome this problem and this weakness, we extend and classified these sub-strategies by the following theorem (Theorem 1).

3. Main Results

The following theorem explain how can recognize the chaotic and hyperchaotic systems to suppress.

Theorem

The control problem of dynamical (chaotic/ hyperchaotic) systems by using linear feedback control strategies (ordinary, dislocated, speed, enhancing) with a positive feedback coefficient k have the following cases.

a. control it if has only one positive feedback gain k, and $k >$ constant,
b. not control if has only one positive feedback gain k, and $k <$ constant,

c. control it if more than one positive feedback coefficient and $\bigcap_{i=1}^{n} k_i \neq \emptyset$,
d. not control if more than one positive feedback coefficient and $\bigcap_{i=1}^{n} k_i = \emptyset$.

Proof

The proof of this theorem depended on Routh-Hurwitz, suppose that cubic equation form:

$$\lambda^3 + A\lambda^2 + B\lambda + C = 0 \tag{5}$$

where A, B and C are functions with respect to feedback coefficient or real numbers. Now,

a. If A and C are real numbers, then the feedback coefficient (k) is not exist, and if A and C are line equation i.e., have the form $c_1 k + c_2$ (c_1 and c_2 are constants), then we get $k > -\frac{c_2}{c_1} = -c_3$ (condition $A, C > 0$), therefore k is neglects (since k is negative), and from the third condition $AB - C > 0$, we get only one positive feedback coefficient and greater then constant such that ($k >$ constant) so, the system can be control, the proof is complete.
b. If A and C are the line equation i.e., have the form $-c_1 k + c_2$, then we get $k < \frac{c_2}{c_1} = c_3$ from the first and second condition of Routh-Hurwitz criterion ($A, C > 0$), therefore we obtain a necessary condition for suppress ($k > 0$). But, the third condition of Routh-Hurwitz method is not satisfied $AB - C > 0$. So, omitted k or become negative, therefore we get only one positive feedback coefficient and smaller then constant such that ($k <$ constant), in this case the chaotic and hyperchaotic system can't be control, the proof is complete.
c. If A and C are linear or quadratic or cubic function then obtain positive feedback coefficients (k_1) from the condition ($A, C > 0$), and the feedback coefficient (k_1) has one of forms (k_1 constant) or ($k_1 <$ constant), and from the third condition, we get the second positive feedback coefficient (k_2). Now, if there intersection between k_1 and k_2, then the chaotic/hyperchaotic system can be control, but if no intersection between them, then the chaotic/hyperchaotic system can't be control the proof of 3.4 is complete.

4. Description System

In this section, we take 4D nonlinear dynamical systems which is called hyperchaotic Lorenz system and contains nine terms with two cross-product nonlinearities terms, for example to show how to use the results obtained in this paper to analyze the controlling a class of nonlinear dynamical systems. The hyperchaotic system which is described by the following mathematical form:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = cx_1 - x_2 - x_1 x_3 + x_4 \\ \dot{x}_3 = x_1 x_2 - bx_3 \\ \dot{x}_4 = -\, dx_1 \end{cases} \tag{6}$$

where a, b, c and d are constant, and it has only one equilibrium point $O(0,0,0,0)$, when parameters $a = 10, b = \frac{8}{3}$, $c = 28, d = 5$, system (6) has two positive Lyapunov exponents. So, this system is hyperchaotic system [12]. Figure 1 show the attractor of system (6).

4.1. Ordinary Feedback Control Strategies

According to the strategy of ordinary feedback control, four forms are obtain, but only three control with a positive feedback coefficient such as:

$$U = \begin{cases} [u_1, 0,0,0]^T; \;\; u_1 = -kx_1 \\ [0, u_2, 0,0]^T; \;\; u_2 = -kx_2 \\ [0,0,0, u_4]^T; \;\; u_4 = -kx_4 \end{cases} \tag{7}$$

and the characteristic equation of system (6) with above control are respectively as:

$$(\lambda + \tfrac{8}{3})[\lambda^3 + (k + 11)\lambda^2 + (k - 270)\lambda + 50] = 0 \tag{8}$$

$$(\lambda+\frac{8}{3})[\lambda^3+(k+11)\lambda^2+(10k-270)\lambda+50]=0 \quad (9)$$

$$(\lambda+\frac{8}{3})[\lambda^3+(k+11)\lambda^2+(11k-270)\lambda+50-270k]=0 \quad (10)$$

Now, based on Routh-Hurwitz criterion we get from the: In (8) has only one positive feedback coefficient and $k>270.1778$. According to Theorem1, first case, the system (6) with control $[-kx_1, 0,0,0]^T$ can be suppress, and (9) has only one positive feedback coefficient and $k>27.1311$. According to Theorem1, first case, the system (6) with control $[0,-kx_2,0,0]^T$ can be suppress, (10) has two positive feedback coefficients and $k_1<0.1851$ and $k_2>11.958$. According to Theorem1, four case, the system (6) with control $[0,0,0,-kx_4]^T$ can't be suppress since $k_1 \cap k_2=\emptyset$. For the ordinary feedback control, the three controls with positive feedback coefficient k obtained, their characteristic equations and corresponding of index of Routh-Hurwitz theorem, are listed in Table 1.

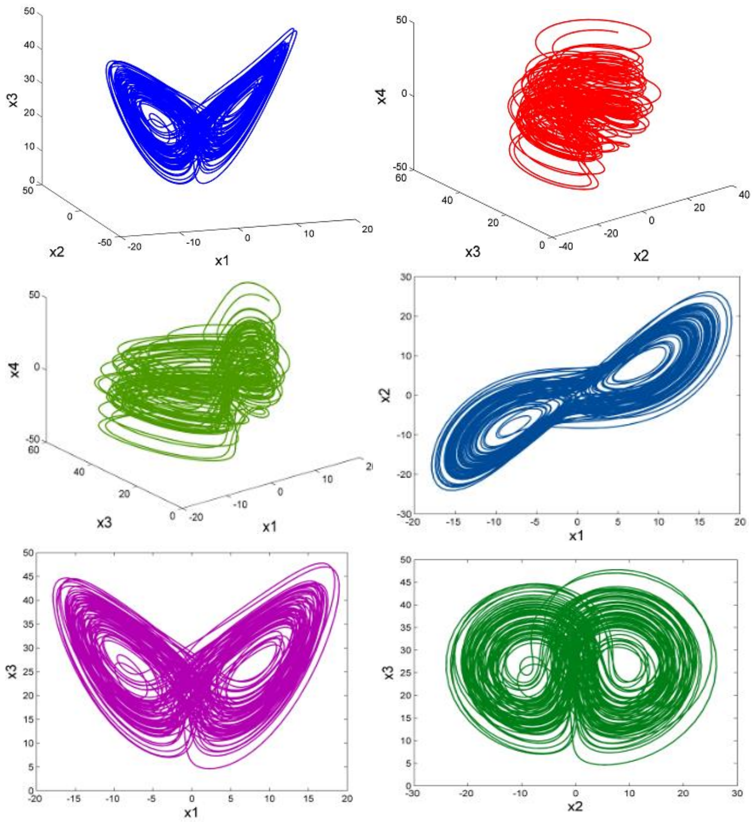


Figure 1. The attractor of system (6)

Table 1. Control, their characteristic equations and corresponding of index of Routh-Hurwitz criterion

No.	Control $U=[u_i]^T$	Characteristic equations	Routh-Hurwitz criterion
1	$[-kx_1, 0,0,0]^T$	$\lambda^3+(k+11)\lambda^2+(k-270)\lambda+50=0$	$A=k+11>0$ $C=50>0$ $AB-C>0 \Rightarrow k>270.1778$
2	$[0,-kx_2,0,0]^T$	$\lambda^3+(k+11)\lambda^2+(10k-270)\lambda+50=0$	$A=k+11>0$ $C=50>0$ $AB-C>0 \Rightarrow k>27.1311$
3	$[0,0,0,-kx_4]^T$	$\lambda^3+(k+11)\lambda^2+(11k-270)\lambda+50-270k=0$	$A=k+11>0$ $C=50-270k>0 \Rightarrow k_1<0.1851$ $AB-C>0 \Rightarrow k_2>11.9583$

Obviously, system (6) with the first and second control can be control only, and the infimum feedback coefficient for this strategy is $k > 27.1311$, Figures 2 (a) and (b) show divergence and convergent to origin point when $k > 27$, $k > 27.5$ respectively for system (6) with control $[0, -kx_2, 0,0]^T$, while Figure 3 show the attractor for system (6) with the same control.

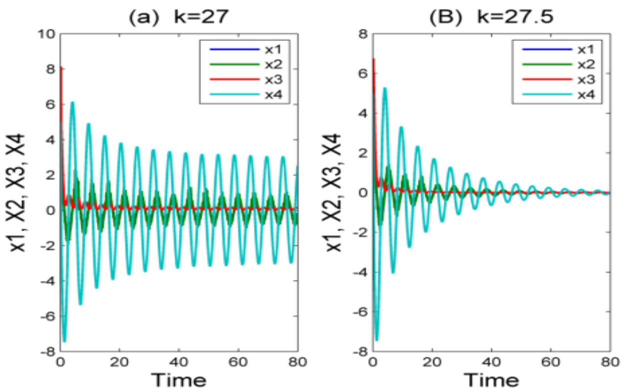


Figure 2. The divergence and convergent of the state system (6) at: (a) k=27, (b) k=27.5

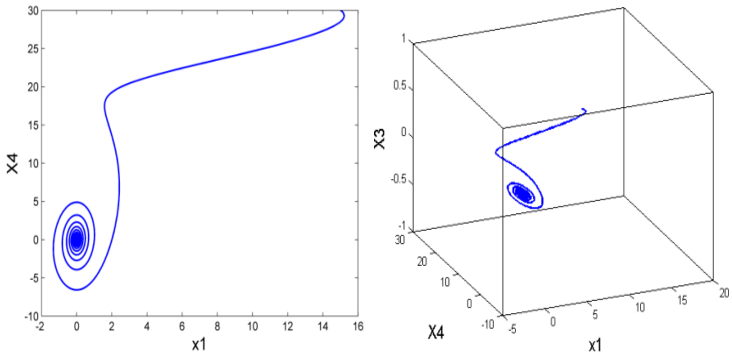


Figure 3. The attractor of system (6) with control $[0, -27.5x_2, 0,0]^T$

4.2. Dislocated Feedback Control Method

According to the startegy of dislocated feedback control, we have twelve forms, but only five control with a positive feedback coefficient for system (6). These five control k obtained and their characteristic equations and corresponding of index of Routh-Hurwitz criterion, are listed in Table 2.

The four root for these equations is $\lambda = -8/3$. In the first control, we have two positive feedback coefficients $k_1 < 10$ and $k_2 > 9.6485$. According to Theorem1, third case, then system (6) can be suppress after add the term $-\boldsymbol{kx_2}$ to first equation, since $k_1 \cap k_2 = (9.6485,10) \neq \emptyset$. While, only one positive feedback coefficient $k_1 < 10$ is obtain in the second control. According to Theorem1, second case, then system (6) can't be suppress after add the term $-\boldsymbol{kx_4}$ to first equation. Also, only one positive feedback coefficient $k > 27.4545$ is obtain in the third control. According to Theorem1, first case, then system (6) can be suppress after add the term $-\boldsymbol{kx_1}$ to second equation.

In fourth and fifth control, two positive feedback coefficients $k_1 < 1$, $k_2 > 60.4$ and $k_1 < 5$, $k_2 > 137.272$ are obtain respectively, but there no intersection between them. So, the system (6) can't be suppress according to Theorem1, fourth case, after add the terms $-\boldsymbol{kx_4}$,$-\boldsymbol{kx_2}$ to second and fourth equations respectively. Obviously, only the first and second control succeed to achieve control for system (6), and the infimum feedback coefficient for this strategy is $k \in (9.6485,10)$. Figure 4 (a) and (b) show divergence and convergent to origin point when $k = 9.6, k = 9.7$ respectively, while Figure 5 show the attractor for system (6) with the control $[-9.7x_2, 0,0,0]^T$.

Table 2. Control, their characteristic equations and corresponding of index of Routh-Hurwitz criterion

No.	Control $U = [u_i]^T$	Characteristic equations	Routh-Hurwitz criterion
1	$[-kx_2, 0,0,0]^T$	$\lambda^3 + 11\lambda^2 + (28k - 270)\lambda + 50 - 5k = 0$	$A = 11 > 0$ $C = 50 - 5k > 0 \Rightarrow k_1 < 10$ $AB - C > 0 \Rightarrow k_2 > 9.6485$
2	$[-kx_4, 0,0,0]^T$	$\lambda^3 + 11\lambda^2 - (270 + 5k)\lambda + 50 - 5k = 0$	$A = 11 > 0$ $C = 50 - 5k > 0 \Rightarrow k_1 < 10$ $AB - C > 0 \Rightarrow k_2 < -60.4$
3	$[0, -kx_1, 0,0]^T$	$\lambda^3 + 11\lambda^2 + (10k - 270)\lambda + 50 = 0$	$A = k + 11 > 0$ $C = 50 > 0$ $AB - C > 0 \Rightarrow k > 27.4545$
4	$[0, -kx_4, 0,0]^{TT}$	$\lambda^3 + 11\lambda^2 - 270\lambda + 50 - 50k = 0$	$A = 11 > 0$ $C = 50 - 50k > 0 \Rightarrow k_1 < 1$ $AB - C > 0 \Rightarrow k_2 > 60.4$
5	$[0,0,0, -kx_2]^T$	$\lambda^3 + 11\lambda^2 + (k - 270)\lambda + 50 - 10k = 0$	$A = 11 > 0$ $C = 50 - 10k > 0 \Rightarrow k_1 < 5$ $AB - C > 0 \Rightarrow k_2 > 137.272$

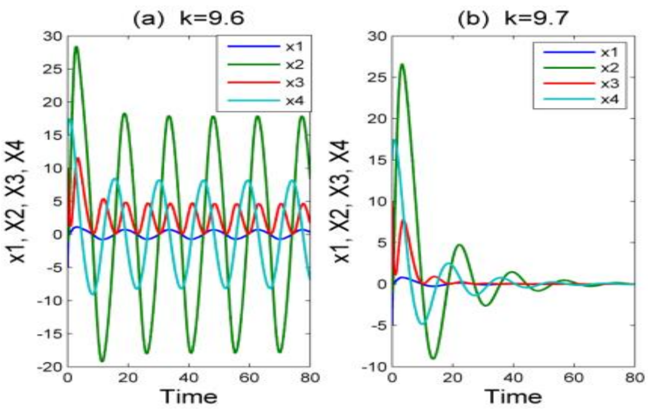


Figure 4. The divergence and convergent of the state system (6) at: (a) k=9.6, (b) k=9.7

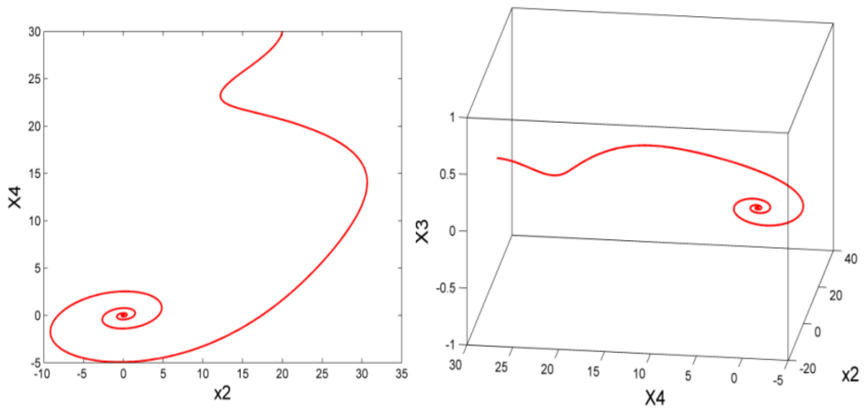


Figure 5. The attractor of system (6) with control $[-9.7x_2, 0, ,0,0]^T$

4.3. Speed Feedback Control Method

In this strategy, twelve forms are possible. But only three control obtain positive feedback coefficient for system (6). The three control k obtained and their characteristic equations and corresponding of index of Routh-Hurwitz theorem, are listed in Table 3.

Table 3. Control, their characteristic equations and corresponding of index of Routh-Hurwitz criterion

No.	Control $U = [u_i]^T$	Characteristic equations	Routh-Hurwitz criterion
1	$[-k\dot{x}_4, 0,0,0]^T$	$\lambda^3 + (11 - 5k)\lambda^2 - (270 + 5k)\lambda + 50 = 0$	$A = 11 + 5k > 0 \Rightarrow k_1 < 2.2$ $C = 50 > 0$ $AB - C > 0 \Rightarrow k_2 > 2.2355$
2	$[0,0,0, -k\dot{x}_1]^T$	$\lambda^3 + 11\lambda^2 + (10k - 270)\lambda + 50 = 0$	$A = 11 > 0$ $C = 50 > 0$ $AB - C > 0 \Rightarrow k > 27.4545$
3	$[0,0,0, -k\dot{x}_2]^T$	$\lambda^3 + (k + 11)\lambda^2 + (10k - 270)\lambda + 50 = 0$	$A = k + 11 > 0$ $C = 50 > 0$ $AB - C > 0 \Rightarrow k > 27.1311$

The four root for these equations is $\lambda = -8/3$. In first control, two positive feedback coefficients $k_1 < 2.2$, $k_2 > 2.2355$ are obtain, but there no intersection between them. So, the system (6) can't be suppress since $k_1 \cap k_2 = \emptyset$. according to Theorem1, fourth case, after add the term $-\boldsymbol{k\dot{x}_4}$ to first equation.

In second and third control, only one positive feedback coefficients, $k > 27.4545$, $k > 27.1311$ are obtain respectively. So, the system (6) can be suppress according to Theorem1, first case, after add the terms $-\boldsymbol{k\dot{x}_1}$, $-\boldsymbol{k\dot{x}_2}$ to four equation of system (6) respectively. Obviously, only second and third control succeed to achieve control for system (6), and the infimum feedback coefficient for this strategy is $k > 27.1311$. Figure 6 (a) and (b) show divergence and convergent to origin point when $k = 26.5$, $k = 27.5$ respectively.

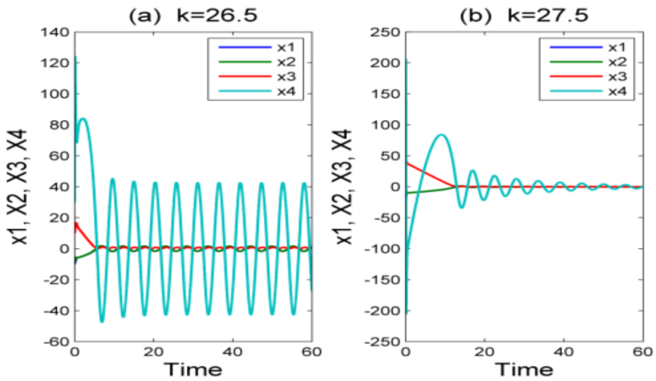


Figure 6. The divergence and convergent of the state system (6) at: (a) k=26.5, (b) k=27.5

4.4. Enhancing Feedback Control Method

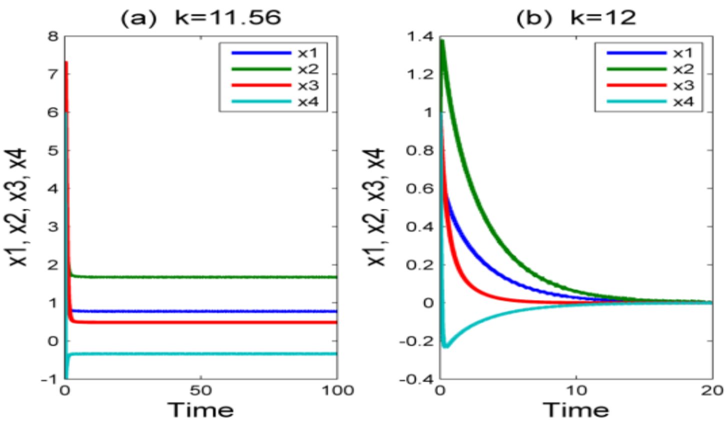
In this strategy, eleven forms are possible. All controller obtain positive feedback coefficients for system (6). The controller k obtained and their characteristic equations and corresponding of index of Routh-Hurwitz theorem, are listed in Table 4. The four root for these equations is $\lambda = -8/3$. In the first, second, fourth and seventh control, only one positive feedback coefficients are obtain and their coefficients are $k > 11.8692$, $k > 270.1778$, $k > 27.1311$ and $k > 11.8692$ respectively. So, satisfied first case of Theorem 1, therefore system (6) can be suppress. While two positive feedback coefficients are obtain in third, fifth, eighth, ninth, tenth and eleventh control and their coefficients are given respectively as:

$k_1 \cap k_2 = (269.81, \infty) \cap (11.5886, \infty) \neq \emptyset \Rightarrow k \in (269.81, \infty)$
$k_1 \cap k_2 = (26.8135, \infty) \cap (7.5527, \infty) \neq \emptyset \Rightarrow k \in (26.8135, \infty)$
$k_1 \cap k_2 \cap k_3 = (11.7040, \infty) \cap (0.1866, \infty) \cap (5.9453, \infty) \neq \emptyset \Rightarrow k \in (11.7040, \infty)$
$k_1 \cap k_2 \cap k_3 = (269.8147, \infty) \cap (0.1865, \infty) \cap (6.7091, \infty) \neq \emptyset \Rightarrow k \in (269.8147, \infty)$
$k_1 \cap k_2 \cap k_3 = (26.8135, \infty) \cap (0.1866, \infty) \cap (7.5527, \infty) \neq \emptyset \Rightarrow k \in (26.8135, \infty)$
$k_1 \cap k_2 \cap k_3 = (11.7040, \infty) \cap (0.1866, \infty) \cap (5.9453, \infty) \neq \emptyset \Rightarrow k \in (11.7040, \infty)$
there intersection between them, so system (6) can be suppress by these controller (satisfied third case for Theorem 1).

Table 4. Control, their characteristic equations and corresponding of index of Routh-Hurwitz criterion

No.	Control $U = [u_i]^T$	Characteristic equations	Routh-Hurwitz criterion
1	$-k[x_1, x_2, 0,0]^T$	$\lambda^3 + (2k + 11)\lambda^2 + (k^2 + 11k - 270)\lambda + 50 = 0$	$A = 2k + 11 > 0$ $C = 50 > 0$ $AB - C > 0 \Rightarrow k > 11.8692$
2	$-k[x_1, 0, x_3, 0]^T$	$\lambda^3 + (k + 11)\lambda^2 + (k - 270)\lambda + 50 = 0$	$A = k + 11 > 0$ $C = 50 > 0$ $AB - C > 0 \Rightarrow k > 270.1778$
3	$-k[x_1, 0,0, x_4]^T$	$\lambda^3 + (2k + 11)\lambda^2 + (k^2 + 12k - 270)\lambda + k^2 - 270k + 50 = 0$	$A = 2k + 11 > 0$ $C = k^2 - 270k + 50 > 0 \Rightarrow k_1 > 269.81$ $AB - C > 0 \Rightarrow k_2 > 11.5886$
4	$-k[0, x_2, x_3, 0]^T$	$\lambda^3 + (k + 11)\lambda^2 + (10k - 270)\lambda + 50 = 0$	$A = k + 11 > 0$ $C = 50 > 0$ $AB - C > 0 \Rightarrow k > 27.1311$
5	$-k[0, x_2, 0, x_4]^T$	$\lambda^3 + (2k + 11)\lambda^2 + (k^2 + 21k - 270)\lambda + 10k^2 - 270k + 50 = 0$	$A = 2k + 11 > 0$ $C = k^2 - 27k + 5 > 0 \Rightarrow k_1 > 26.8135$ $AB - C > 0 \Rightarrow k_2 > 7.5527$
6	$-k[0,0, x_3, x_4]^T$	$\lambda^3 + (k + 11)\lambda^2 + (11k - 270)\lambda - 270k + 50 = 0$	$A = k + 11 > 0$ $C = -270k + 50 > 0 \Rightarrow k_1 < 0.1851$ $AB - C > 0 \Rightarrow k_2 > 11.95$
7	$-k[x_1, x_2, x_3, 0]^T$	$\lambda^3 + (2k + 11)\lambda^2 + (k^2 + 11k - 270)\lambda + 50 = 0$	$A = 2k + 11 > 0$ $C = 50 > 0$ $AB - C > 0 \Rightarrow k > 11.8692$
8	$-k[x_1, x_2, 0, x_4]^T$	$\lambda^3 + (3k + 11)\lambda^2 + (3k^2 + 22k - 270)\lambda + k^3 + 11k^2 - 270k + 50 = 0$	$A = 3k + 11 > 0$ $C > 0 \Rightarrow k_1 > 0.1866,\ k_2 > 11.7040$ $AB - C > 0 \Rightarrow k_3 > 5.9453$
9	$-k[x_1, 0, x_3, x_4]^T$	$\lambda^3 + (2k + 11)\lambda^2 + (k^2 + 12k - 270)\lambda + k^2 - 270k + 50 = 0$	$A = 2k + 11 > 0$ $C > 0 \Rightarrow k_1 > 269.8147, k_2 > 0.1853$ $AB - C > 0 \Rightarrow k_3 > 6.7091$
10	$-k[0, x_2, x_3, x_4]^T$	$\lambda^3 + (2k + 11)\lambda^2 + (k^2 + 21k - 270)\lambda + 10k^2 - 270k + 50 = 0$	$A = 2k + 11 > 0$ $C > 0 \Rightarrow k_1 > 26.8135, k_2 > 0.1865,$ $AB - C > 0 \Rightarrow k_3 > 7.5527$
11	$-k[x_1, x_2, x_3, x_4]^T$	$\lambda^3 + (3k + 11)\lambda^2 + (3k^2 + 22k - 270)\lambda + k^3 + 11k^2 - 270k + 50 = 0$	$A = 2k + 11 > 0$ $C > 0 \Rightarrow k_1 > 0.1866,\ k_2 > 11.7040$

At last, the only control which failure to achieve suppress system (6) in this strategy is the sixth control, although contains two positive feedback coefficients, but on there intersection between them. Therefore, satisfied fourth case for Theorem 1. Obviously, most of above strategies can be control the system (6) except the sixth control can't be control. The infimum feedback coefficient for this strategy is $k > 11.7040$**.** Figure 7 a, b show divergence and convergent to origin point when is $k > 11.56$, $k > 12$ respectively. The controller U obtained and their active control and corresponding of index of Routh-Hurwitz theorem, are listed in Table 5.



Figuer 7. The divergence and convergent of the state system (6) at (a) k=11.56, (b) k=1

Table 5. Relationship between a Positive Feedback Controlled and Suppressed for System (6) According to the a Theorem 1

Type of feedback control	Control $U=[u_i]^T$	Number of a positive feedback	Feedback controller	Intersection $k=\cap_{i=1}^{n} k_i$	Satisfied all Routh-Hurwitz	Result
ordinary	$-[kx_1, 0,0,0]^T$	one	$k > 270.1778$	....	Yes	control
ordinary	$-[0, kx_2, 0,0]^T$	one	$k > 27.1311$	....	Yes	control
ordinary	$-[0,0,0, kx_4]^T$	two	$k_1 < 0.1851$ $k_2 > 11.9583$	$k = k_1 \cap k_2 = \emptyset$	Yes	not control
dislocated	$-[kx_2, 0,0,0]^T$	two	$k_1 < 10$ $k_2 > 9.6485$	$(9.6485,10)$ $\neq \emptyset$	Yes	control
dislocated	$-[kx_4, 0,0,0]^T$	one	$k < 10$	....	No	not control
dislocated	$-[0, kx_1, 0,0]^T$	one	$k > 27.4545$	....	Yes	control
dislocated	$-[0, kx_4, 0,0]^T$	two	$k_1 < 1$ $k_2 > 60.4$	$\emptyset$	Yes	not control
dislocated	$-[0,0,0, kx_2]^T$	two	$k_1 < 5$ $k_2 > 137.2727$	$\emptyset$	Yes	not control
speed	$-[k\dot{x}_4, 0,0,0]^T$	two	$k_1 > 2.2355$ $k_2 < 2.2$	$\emptyset$	Yes	not control
speed	$-[0,0,0, k\dot{x}_1]^T$	one	$k > 27.4545$	....	Yes	control
speed	$-[0,0,0, k\dot{x}_2]^T$	one	$k > 27.1311$	......	Yes	control
enhancing	$-k[x_1, x_2, 0,0]^T$	one	$k > 11.8692$	......	Yes	control
enhancing	$-k[x_1, 0, x_3, 0]^T$	one	$k > 270.1778$	......	Yes	control
enhancing	$-k[x_1, 0,0, x_4]^T$	two	$k_1 > 11.5886$ $k_2 > 269.8146$	$(269.8146, \infty)$ $\neq \emptyset$	Yes	control
enhancing	$-k[0, x_2, x_3, 0]^T$	one	$k > 27.1311$	......	Yes	control
enhancing	$-k[0, x_2, 0, x_4]^T$	two	$k_1 > 26.8135$ $k_2 > 7.5527$	$(26.8135, \infty)$ $\neq \emptyset$	Yes	control
enhancing	$-k[0,0, x_3, x_4]^T$	two	$k_1 > 11.95$ $k_2 < 0.1851$	$\emptyset$	Yes	not control
enhancing	$-k[x_1, x_2, x_3, 0]^T$	one	$k > 11.8692$	......	Yes	control
enhancing	$-k[x_1, x_2, 0, x_4]^T$	three	$k_1 > 5.9453$ $k_2 > 11.7040$ $k_3 > 0.1866$	$(11.7040, \infty)$ $\neq \emptyset$	Yes	control
enhancing	$-k[x_1, 0, x_3, x_4]^T$	three	$k_1 > 6.7091$ $k_2 > 269.8147$ $k_3 > 0.1853$	$(269.8147, \infty)$ $\neq \emptyset$	Yes	control
enhancing	$-k[0, x_2, x_3, x_4]^T$	three	$k_1 > 7.5527$ $k_2 > 26.8135$ $k_3 > 0.1865$	$(26.8135, \infty)$ $\neq \emptyset$	Yes	control
enhancing	$-k[x_1, x_2, x_3, x_4]^T$	three	$k_1 > 5.9453$ $k_2 > 11.7040$ $k_3 > 0.1866$	$(11.7040, \infty)$ $\neq \emptyset$	Yes	control

5. Conclusion

In this paper, we discussed the control problem of hyperchaotic systems by using linear feedback control strategies with a positive feedback coefficient based on Routh-Hurwitz method. We found a new method for recognize which systems can be control or not, and we show if get only one positive feedback coefficient and greater than constant, then system can be control, and if smaller than constant, then system can't be control. Also, if we get more than one positive feedback coefficient and there exist intersection between them, then system can be control, else if not exist intersection between them then system can't be control. Finally, we can use a new method for any chaotic and hyperchaotic systems.

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