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Real interpolation method for transfer function approximation of distributed parameter system

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Abstract

Distributed parameter system (DPS) presents one of the most complex systems in the control theory. The transfer function of a DPS possibly contents: rational, nonlinear and irrational components. This thing leads that studies of the transfer function of a DPS are difficult in the time domain and frequency domain. In this paper, a systematic approach is proposed for linearizing DPS. This approach is based on the real interpolation method (RIM) to approximate the transfer function of DPS by rational-order transfer function. The results of the numerical examples show that the method is simple, computationally efficient, and flexible.

Keywords*: a distributed parameter system, approximation, real interpolation method*

1. Introduction

All real industrial processes are characterized by variables which can vary both temporally and spatially. Mathematical models of these processes are commonly known as distributed parameter systems (DPS). A mathematical description of this class of systems is represented by partial differential equations (PDEs), which lead to the infinite-dimensional model as well as to irrational transfer function representations [1, 2]. Therefore, due to the mathematical complexity, analysis of DPS is much more complicated than in the case of lumped parameter systems (LPS), where spatial effects are averaged. Consequently, infinite-dimensional DPS models are often approximated by finite-dimensional ones. In general, modeling of DPS can be classified into two groups, modeling of unknown DPS and modeling of known DPS.

For unknown DPS, widely, the model is determined by the input-output relationship. This technique is so-called black-box identification. The dynamic identification models can be estimated using traditional system identification techniques such as nonlinear autoregressive model [3], a method in the real domain [4], spectral method [5] and artificial methods: neural networks [6]; genetic method [7].

For the known DPS, the linearization method can approximate the infinite dimensional system into a finite order of ordinary differential equations. Many studies have been done in order to simulate DPSs to LPSs over the last decade [8-15]. The proposed methods in these studies are presented more complex and requiring high computation resources. In this paper, we propose a real interpolation method for linearizing DPS to the LPS. The method has an advantage in the computation, the simplicity of the algorithm.

2. Real interpolation method

RIM is one of the methods, which works on mathematical descriptions of the imaginary domain [15-17]. The method is based on a real integral transform:

$$F(\delta) = \int_0^\infty f(t)e^{-\delta \cdot t}dt, \delta \in [C, \infty), C \geq 0 \quad (1)$$

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which assigns the image function $F(\delta)$ in accordance to the original function $f(t)$ as a function of the real variable δ. Formula of direct transform can be considered as a special case of the direct Laplace transform by replacing the complex variable s for real δ variable. Another step towards the development of the instrumentation method is the transition from continuous functions $F(\delta)$ to their discrete form, using the computing resources and numerical methods. For these purposes, RIM is represented by the numerical characteristics $\{F(\delta_i)\}_N$. They are obtained as a set of values of function $F(\delta)$ in the nodes δ_i $where$ $i \in 1,2,\dots N$, where N is the number of elements of numerical characteristics, called its dimension [18-21].

Selecting of interpolations δ_i is a primary step in the transition to a discrete form, which has a significant impact on the numerical computing and accuracy of problem solutions. Distribution of nodes in the simplest variant is uniform. Another important advantage of the RIM is cross-conversion property. It dues to the fact that the behavior of the function $F(\delta)$ for large values of the argument δ is determined mainly by the behavior of the original $f(t)$ for small values of the variable t. In the opposite case, the result is the same: the behavior of the function $F(\delta)$ for small values of the argument δ is determined mainly by the behavior of the original $f(t)$ for large values of the variable t.

3. Approximation Algorithm

In this paper, we consider the following approximation task of distributed parameter systems. The transfer function of DPS is complex, containing fractional and/or transcendental components. The transfer functions of these systems typically have the form such as formula (1). Because of the presence of fractional and/or transcendental components in the transfer function, which not allow using method and means of lumped parameters systems.

$$G(s) = G\left(e^{-\sqrt{T_1 s}}, \frac{1}{\sqrt{T_2 s}}, \sqrt{s}, sh(s), ch(s) \dots.\right) \quad (2)$$

Let us consider rational transfer function:

$$\mathrm{W(s)} = \frac{\mathrm{B(s)}}{\mathrm{A(s)}} = \frac{\mathrm{b_m s^m + \dots + b_1 s + b_0}}{\mathrm{a_n s^n + \dots + a_1 s + a_0}} \quad (3)$$

where $m \le n$; m, n are the integer, which should be used to approximate transfer function $G(s)$ of linear the fractional order system. For $(G(0) \neq 0, b_0 = 1)$ or $(G(0) = 0, a_0 = 1)$ there are $N = n + m + 1$ real coefficients which should be determined from N equations obtained from the condition of overlapping the numerical characteristics in the corresponding discrete points:

$$G(\delta_i) - \frac{B(\delta_i)}{A(\delta_i)} = 0, i = \overline{1, N}, \quad (4)$$

$$G(\delta_i)A(\delta_i) - B(\delta_i) = 0,\ i = \overline{1, N}. \quad (5)$$

or for $\mathrm{G(0) = 0, a_0 = 1}$ one obtained

$$a_n \delta_i{}^n G(\delta_i) + \dots + a_1 \delta_i G(\delta_i) - b_m \delta_i{}^m - \dots - b_0 = -G(\delta_i),\ i = \overline{1, N}. \quad (6)$$

For fixed δ_i both numerator and denominator polynomials are linear combinations of the unknown process parameters. Thus, the set of (6) represents a linear system of equations having N linear equations, one obtains N coefficients of the rational approximation (4). The obtained (6) are conveniently rewritten in the following matrix form, which is easily solved using some of the modern computer algebra packages, in particular, introducing

$$\mathrm{M} = \begin{bmatrix} \delta_{N,1}{}^n G(\delta_{N,1}) \dots \delta_{N-n,1} G(\delta_{N-n,1}) - \delta_{N-n-1,1}{}^m \dots -1 \\ \delta_{N,2}{}^n G(\delta_{N,2}) \dots \delta_{N-n,2} G(\delta_{N-n,2}) - \delta_{N-n-1,2}{}^m \dots -1 \\ \dots \\ \delta_{N,N}{}^n G(\delta_{N,N}) \dots \delta_{N-n,N} G(\delta_{N-n,N}) - \delta_{N-n-1,N}{}^m \dots -1 \end{bmatrix} \quad (7)$$

$$B = \begin{bmatrix} -G(\delta_1) \\ -G(\delta_2) \\ \dots \\ -G(\delta_N) \end{bmatrix} \quad (8)$$

one easily obtains the desired system of linear equations in matrix form

$$M \cdot X = B \quad (9)$$

where X is the vector of unknown parameters:

$$X = \begin{bmatrix} a_n \\ a_{n-1} \\ \dots \\ a_1 \\ b_m \\ \dots \\ b_0 \end{bmatrix}. \quad (10)$$

It is important to mention that the selected set of points $\delta \in [\delta_1, \delta_2, .., \delta_N]$ can produce a singular matrix from the set of equations. In such a case, another, more appropriate set of points should be used. It is also significant to note that it is also possible to use more than *n* incident points in the selected set. The exact solution cannot be found in such a case.

4. Numerical examples and discussion

In the first example, we consider the system with the distributed parameter transfer function, in which exact solution is known to estimate the result of approximation task, by comparing between the Heaviside responses, Bode characteristics of the approximation models and the exact model [22-26]. The given transfer function has a form:

$$G_1(s) = e^{-\sqrt{s}} \quad (11)$$

The transient function of Heaviside excitation is a function: $h_1(t) = erfc(\frac{1}{2\sqrt{t}})$.

According to the considered model (10): $G_1(\infty) \to 0$ and $G_1(0) = 1$, we have chosen the approximation model (4) with $b_0 = 1, a_0 = 1$ and the order of denominator of transfer function must be higher than the order of numerator. Simply, the orders of the approximated rational transfer function being considered are $m = 6$ in numeratthe or, and $n = 7$ in denominathe tor. It means that number of a unknown coefficients is $N = n + m = 13$. Corresponding to 7th order of the ration approximated transfer function, the number of unknown coefficient is $N = 13$. In the RIM we choose values of the nodes δ_i in range the $[0.001 \div 1]$ with *N*=13 nodes equally spaced. The results of approximation process will be analysed in the time and frequency domains and compared to other methods with same order of approximation model, where $\Delta h_1(t)$ is the error of time response by RIM method.

Figure 1 and Figure 2 show that the approximation model is fitted with a real model. The high accuracy presents in the range [0-100] sec and approximation error reaches a peak at around 300 s. The approximation errors of time responses are illustrated in Figure 2, where $h_1(t)$ is the exact time response; $h_{1A}(t)$ - time response by RIM method.

The Bode plots of the approximation models are shown in Figures 3 and 4. The Bode plots illustrate the logarithmic magnitude, phase responses, and errors plots, respectively. Figures 3 and 4 show that the errors in the magnitude and phase responses of the considered approximation methods present the lowest value in low frequency ranged [10^{-3}, 10] Hz. In the higher frequency regime, results of the RIM introduce less accuracy.

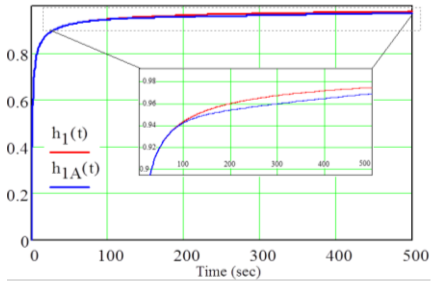


Figure 1. Time responses

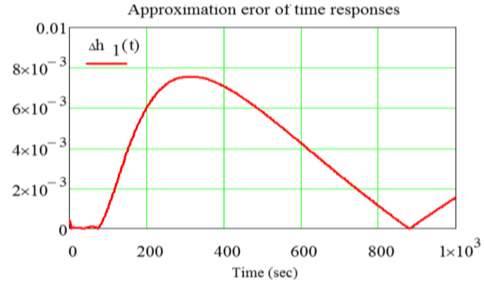
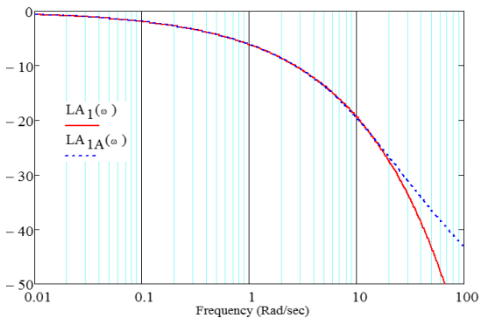
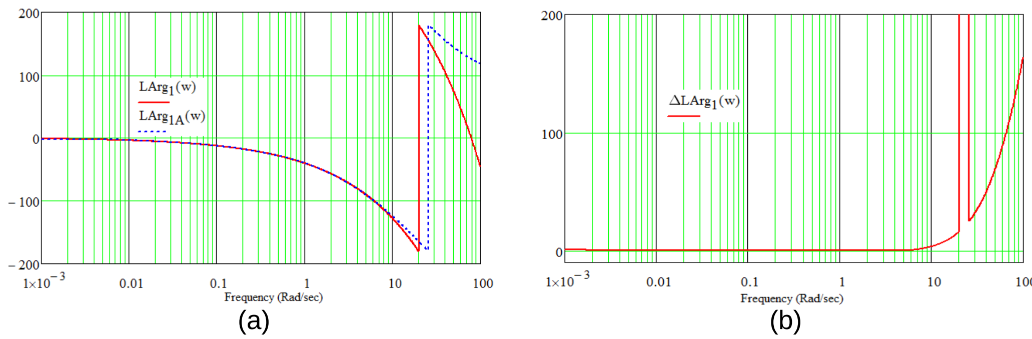


Figure 2. Approximation error of time responses



$LA_1(\omega)$ - exact magnitude response;
$LA_{1A}(\omega)$ - magnitude response by RIM method

Figure 3. Magnitude responses



$LArg_1(\omega)$: Phase response of real model; $LArg_{1A}(\omega)$: approximation model by RIM;
$\Delta Arg_1(\omega)$: error of phase response by RIM

Figure 4. Phase responses and error of the phase response

The second part of examples we considered the transfer function of a distributed parameter system, having the form as (12) [12]:

$$G_2(s) = \frac{\dfrac{\sinh\left(\frac{z_0\sqrt{s}}{v}\right)}{\sinh\left(\frac{l\sqrt{s}}{v}\right)}}{1-\frac{1}{2}e^{-z_0 s}+\dfrac{\sinh\left(\frac{z_0\sqrt{s}}{v}\right)}{\sinh\left(\frac{l\sqrt{s}}{v}\right)}} \qquad (12)$$

The parameters of the model $G_2(s)$ in the formula are given (12), for $\ell = 1, z_0 = \ell/2, \vartheta = 1$. From this transfer function is impossible to find the exact time response. For this cause we will find the approximation model and estimate the fitness of the model in the frequency and time domains.

According to the model (11): $\lim_{\delta\to 0} G_2(\delta) = 0.5$ and $\lim_{\delta\to\infty} G_2(\delta) = 0$. we have chosen approximation model (3) with $b_0 = 0.5, a_0 = 1$ and the order of denominator of the transfer function must be higher than the order of numerator. Simply, the orders of the approximated rational transfer function being considered are $m = 6$ in numerator, and $n = 7$ in denominator. It means that number of unknown coefficients is $N = n + m = 13$. In the RIM we choose values of the nodes δ_i in range $[0.001 \div 15]$ with *N*=13 nodes equally spaced. The results of approximation process will be considered in the time and frequency domains. The time response of the linearization model is presented in Figure 5. Magnitude responses and error of the magnitude responses as shown in Figure 6.

The Bode plots of the approximation models are shown in Figures 7 and 8. The approximation model presents high accuracy to first resonant. In the higher frequency range, the approximation is less accurate. The diagrams show that it leads to the same above conclusion, the fitness of RIM model in Bode domain is high accuracy in the frequency range [0.001-10] Hz. However, in the higher frequency range [10-100] RIM model is inaccurate.

To estimate the RIM for the approximation of distributed parameter systems, we carry out numerical examples with different systems. In the examples, there was conducted an in-depth analysis of the results of the RIM in the time domain and the frequency domain. The above results show that the accuracy of the RIM in the time domain is significantly in the first example. In the frequency domain as the Bode characteristics, the RIM models show highly fitting in the low-frequency range [0.001-10] Hz. However, in the higher considered frequency range, the RIM is less accurate. Generally, the characteristic of RIM model in the Bode diagrams fit the exact model in the wide range comparing to the other methods.

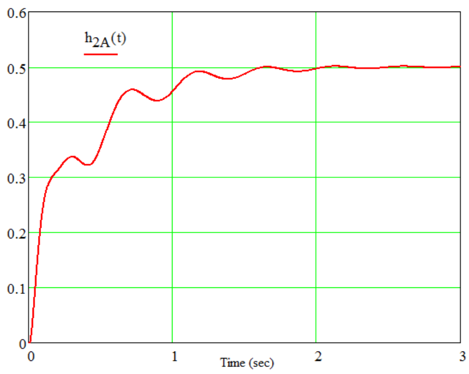
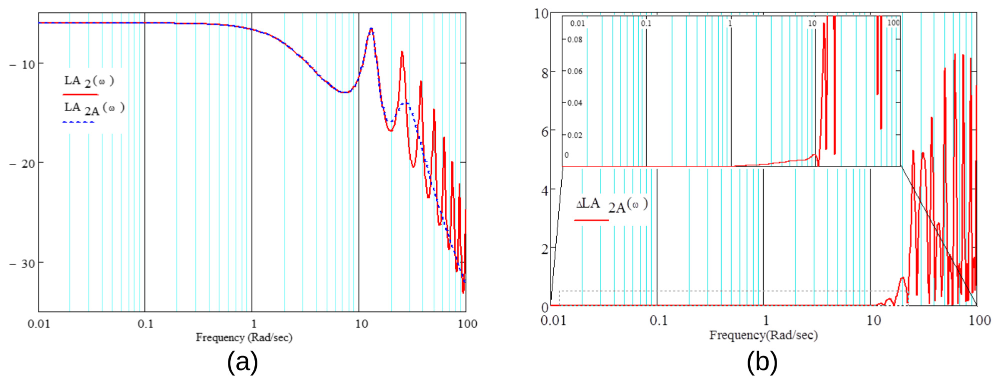
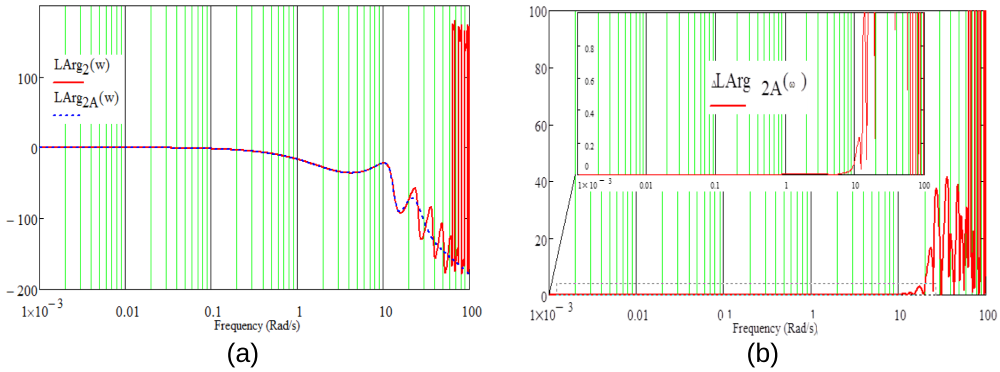


Figure 5. Time responses of the approximation model



LA2(ω): exact magnitude response; LA2A(ω): magnitude response by RIM method;
ΔL2A(ω): magnitude response error by RIM method

Figure 6. Magnitude responses and error of the magnitude responses



$Arg_2(\omega)$: exact phase response; $Arg_{2A}(\omega)$ phase response by RIM method;
$\Delta Arg_{1R}(\omega)$: phase response error by RIM method.

Figure 7. Phase responses and error of the phase responses

4. Conclusion

In this paper, a new approximation method for distributed parameter system is presented. The most significant feature of the proposed method is its computational efficiency. The computation is carried out in the real domain. This method is straightforward both conceptually and computationally. The obtained results from the previous examples are entirely satisfactory. The main drawback of the proposed method is that it is uncertain of the approximation model in the frequency domain.

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