

A Novel Technique for Fault-Tolerant Control of Single-Phase Induction Motor

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Abstract

This research discusses about vector control of single-phase Induction Motor (IM) with two main and auxiliary windings under stator winding open-phase fault based on Indirect Rotor Flux-Oriented Control (IRFOC). Unlike conventional controller which can only be used for single-phase IM with two windings, the proposed technique in this paper can also be used for single-phase IM under open-phase fault. The proposed fault-tolerant drive system in this paper is based on using transformation matrix. Simulations results confirm the validity of the theoretical analysis, shown that the performance of the proposed scheme is highly satisfactory for controlling both healthy and faulty.

Keywords: fault-tolerant control, single-phase induction motor, open-phase fault, indirect rotor flux-oriented control, transformation matrix

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1. Introduction

Single-phase Induction Motor (IM) is one the most commonly IMs which is used in washing machines, drills, compressors, refrigerators, pumps, dryers and many other applications. A single-phase IM is principally an unbalanced IM since it is constructed with two unequal main and auxiliary windings [1].

AC drives of single-phase IM demonstrate better performance, lower protection, improved reliability and are cost-benefit in comparison with their DC counterparts [2, 3]. In the literature, numerous studies have been conducted on Variable Frequency Control (VFC) techniques of single-phase IM drives such as scalar and vector control techniques. These control techniques assure energy saving, improved efficiency, decrease in torque response pulsation and etc [4-8]. Recent research trend on IM is toward vector control of single-phase IMs or unbalanced two-phase IMs [9-22]. Generally, in all proposed vector control methods for single-phase IMs, the start-up and running capacitors are disregarded and single-phase IM is considered as an asymmetric 2-phase IM. Use of hysteresis current controller for Field-Oriented Control (FOC) of unbalanced 2-phase IM has been suggested in [9]. In [10], FOC technique for a single-phase IM with current double sequence controller for decreasing the torque pulsation has been proposed. Using current double sequence controller is a complex controller due to using many PI controllers. To solve problems in [9, 10], in [11, 12], decoupling vector control of single-phase IM has been presented. In [11, 12], by introducing two new decoupling signals in addition to the decoupling signals like ones used in 3-phase IM, a novel technique for vector control of single-phase IM based on FOC has been proposed. However, decoupling vector control technique depends on variation of single-phase IM parameters. In [13-22], some methods for high performance FOC of single-phase IM or unbalanced 2-phase IM have been presented which can be listed as follows; In [13-16], speed sensorless Indirect FOC (IFOC) of 2-phase IM using Extended Kalman Filter (EKF), in [17], Model Reference Adaptive System (MRAS) observer for rotor speed estimation, in [18], sensorless FOC of single-phase IM with on line stator resistance estimation, in [19, 20], two techniques for speed sensorless IFOC of unbalanced 2-phase IM based on motor model, in [21], FOC of 2-phase IM using Genetic Algorithm (GA) for speed PI controller tuning and in [22], Virtual High Frequency Injection Method (VHFIM) to determine the speed and position in IFOC of 2-phase IM have been presented.

Reliability of an electrical drive has been always a most important concern in many industrial and critical applications. In these applications, an appropriate control strategy must be capable of managing the drive system during the fault conditions. In spite of good performance of conventional vector control methods for single-phase IM drives, its capability in controlling faulty single-phase IMs is unsatisfactory. To overcome this problem, a novel vector control method for single-phase IM is proposed in this paper. The new vector control technique which is based on FOC, is suitable for both healthy single-phase IM drives and single-phase IMs when works with just main winding. MATLAB simulation results show that the performance of the proposed scheme is highly satisfactory for controlling healthy and faulty single-phase IM.

2. d-q Model of Single-Phase IM with Two Main and Auxiliary Windings

The single-phase IM d-q equations with two different main and auxiliary windings in the stationary reference frame can be shown as following equations [9] (in this paper superscript "s" indicates that the variables are in the stationary reference frame):

Stator and rotor voltage equations:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{ds} + L_{ds} \frac{d}{dt} & 0 & M_{ds} \frac{d}{dt} & 0 \\ 0 & R_{qs} + L_{qs} \frac{d}{dt} & 0 & M_{qs} \frac{d}{dt} \\ M_{ds} \frac{d}{dt} & \omega_r M_{qs} & R_r + L_r \frac{d}{dt} & \omega_r L_r \\ -\omega_r M_{ds} & M_{qs} \frac{d}{dt} & -\omega_r L_r & R_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (1)$$

Stator and rotor flux equations:

$$\begin{bmatrix} \lambda_{ds}^s \\ \lambda_{qs}^s \\ \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} = \begin{bmatrix} L_{ds} & 0 & M_{ds} & 0 \\ 0 & L_{qs} & 0 & M_{qs} \\ M_{ds} & 0 & L_r & 0 \\ 0 & M_{qs} & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (2)$$

Electromagnetic torque equations:

$$\tau_e = \frac{Pole}{2} (M_{qs} i_{qs}^s i_{dr}^s - M_{ds} i_{ds}^s i_{qr}^s)$$

$$\frac{Pole}{2} (\tau_e - \tau_l) = J \frac{d\omega_r}{dt} + F \omega_r \quad (3)$$

Where, v_{ds}^s , v_{qs}^s , i_{ds}^s , i_{qs}^s , i_{dr}^s , i_{qr}^s , λ_{ds}^s , λ_{qs}^s , λ_{dr}^s and λ_{qr}^s are the d-q axes voltages, currents, and fluxes of the stator and rotor. R_{ds} , R_{qs} and R_r denote the stator and rotor resistances. L_{ds} , L_{qs} , L_r , M_{ds} and M_{qs} denote the stator, and the rotor self and mutual inductances. ω_r is the machine speed. τ_e , τ_l , J and F are electromagnetic torque, load torque, inertia and viscous friction coefficient.

3. d-q Model of Single-Phase IM with One Winding (Faulty Single-Phase IM)

Modeling of engineering systems has been subject of attention of many researchers within the last decades (e.g. [23, 24]). In this paper, the single-phase IM model with only main winding based on d-q model is presented (it is assumed that a phase cut-off fault is occurred in the auxiliary winding of single-phase IM). Stator a-axis (main winding) and d-q axes can be shown as Figure 1 (in this Figure, " f_{as} " can be flux, voltage or current).

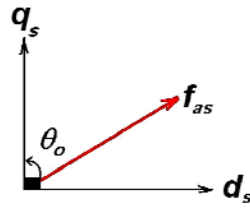


Figure 1. Stator a-axis and d-q axes

In this Figure θ_o is the angle between stator a-axis and stator q-axis. Based on Figure 1, d-q variables for stator can be shown as:

$$d_s = f_{as} \sin \theta_o, \quad q_s = f_{as} \cos \theta_o \quad (4)$$

Since the d and q axes are orthogonal, therefore their inner product should be equal to zero. So,

$$\sin \theta_o \cos \theta_o = 0 \Rightarrow \theta_o = 0 \text{ or } \frac{\pi}{2} \quad (5)$$

If we consider $\theta_o=0$, the transformation matrix for stator variables is obtained as:

$$\begin{bmatrix} d_s \\ q_s \end{bmatrix} = [T_s][f_a] = \begin{bmatrix} 0 \\ 1 \end{bmatrix} [f_a] \quad (6)$$

Therefore, the d-q model of single-phase IM under open-phase fault is obtained as follows:

Stator and rotor voltage equations:

$$\begin{bmatrix} 0 \\ v_{qs}^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & R_{qs} + L_{qs} \frac{d}{dt} & 0 & M_{qs} \frac{d}{dt} \\ 0 & \omega_r M_{qs} & R_r + L_r \frac{d}{dt} & \omega_r L_r \\ 0 & M_{qs} \frac{d}{dt} & -\omega_r L_r & R_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} 0 \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (7)$$

Stator and rotor flux equations:

$$\begin{bmatrix} 0 \\ \lambda_{qs}^s \\ \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & L_{qs} & 0 & M_{qs} \\ 0 & 0 & L_r & 0 \\ 0 & M_{qs} & 0 & L_r \end{bmatrix} \begin{bmatrix} 0 \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (8)$$

Electromagnetic torque equations:

$$\tau_e = \frac{Pole}{2} M_{qs} i_{qs}^s i_{dr}^s$$

$$\frac{Pole}{2} (\tau_e - \tau_l) = J \frac{d\omega_r}{dt} + F \omega_r \quad (9)$$

As can be seen from Equation (7)-(9), the structure of the single-phase IM under open-phase fault is similar to the structure of the single-phase IM with two main and auxiliary windings. The only difference between these two types of motors is the motor parameters. In the single-phase IM under open-phase fault it is obtained: $L_{ds}=M_{ds}=R_{ds}=0$ but in the healthy single-phase IM: $L_{ds}\neq L_{qs}$, $M_{ds}\neq M_{qs}$ and $R_{ds}\neq R_{qs}$.

In the next section, a vector control system based on Rotor FOC (RFOC) for healthy single-phase IM is presented. It is obvious by substituting $L_{ds}=M_{ds}=R_{ds}=0$, this control system can be used for faulty single-phase IM.

4. Proposed Method for Vector Control of Single-Phase IM Based on RFOC

In this section, using transformation matrix, a method for vector control of single-phase IM is presented. The reason of using this transformation matrix is changing the unbalanced equations of the single-phase IM (due to the $L_{ds}\neq L_{qs}$, $M_{ds}\neq M_{qs}$ and $R_{ds}\neq R_{qs}$) to the balanced equations.

4.1. Transformation Matrix for Stator Current Variables

The idea of using this transformation matrix is obtained from equivalent circuit of the single-phase IM. Figure 2 shows the equivalent circuit of single-phase IM.

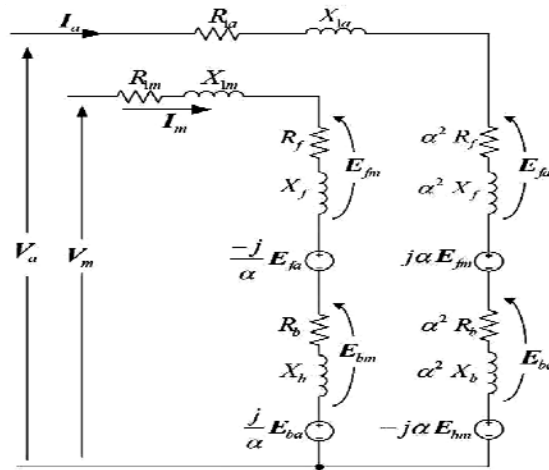


Figure 2. Equivalent circuit of single-phase IM

All the parameters in Figure 2, have been defined in Appendix. It can be shown that using following definitions,

$$\begin{aligned}
 I_1 &= -j\alpha I_a + I_m \\
 V_1 &= Z_3 V_m + jZ_4 V_a
 \end{aligned}
 \tag{10}$$

Figure 2 can be simplified as Figure 3 (see Appendix).

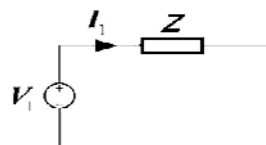


Figure 3. Simplified equivalent circuit of single-phase IM

In Figure 3,

$$Z = \left(\frac{(Z_3 + Z_4)(Z_{lm} + 2Z_f)}{2} + \frac{(Z_3 + Z_4)(-\alpha Z_{lm} + jZ_{la} + 2\alpha Z_f(\alpha j - 1))(Z_{lm} + 2Z_b)}{2(\alpha Z_{lm} + jZ_{la} + 2\alpha Z_b(\alpha j + 1))} \right) \quad (11)$$

In (11), Z_3 and Z_4 are the parameters in terms of inductances (L_{ds} and/or L_{qs} and/or M_{ds} and/or M_{qs}). As can be seen using (10), the equivalent circuit of single-phase IM (Figure 2) changed into a balanced circuit (Figure 3). Equation (10) can be re-written as follows:

$$\begin{cases} jI_1 = \frac{N_a}{N_m} I_a + jI_m \\ I_1 = -j \frac{N_a}{N_m} I_a + I_m \end{cases} \quad (12)$$

And,

$$\begin{cases} jV_1 = -Z_4 V_a + jZ_3 V_m \\ V_1 = jZ_4 V_a + Z_3 V_m \end{cases} \quad (13)$$

Equation (12) and (13) are transformation matrices for transformation of variables from unbalanced set (eg., Figure 2) to the balanced set (eg., Figure 3). Based on (12) and (13), following transformation matrices for stator voltage and current variables can be derived:

Transformation matrix for stator voltage variables:

$$\begin{bmatrix} v_{ds}^e \\ v_{qs}^e \end{bmatrix} = [T_{vs}^e] \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} -Z_4 \cos \theta_e & Z_3 \sin \theta_e \\ Z_4 \sin \theta_e & Z_3 \cos \theta_e \end{bmatrix} \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} \quad (14)$$

Transformation matrix for stator current variables:

$$\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} = [T_{is}^e] \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} = \begin{bmatrix} \frac{M_{ds}}{M_{qs}} \cos \theta_e & \sin \theta_e \\ -\frac{M_{ds}}{M_{qs}} \sin \theta_e & \cos \theta_e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (15)$$

To obtain (14) and (15), following substituting have been considered (in (14) and (15) superscript "e" indicates that the variables are in the rotating reference frame. Moreover, " θ_e " is the angle between the stationary reference frame and the rotor flux oriented reference frame).

$$\begin{aligned} j &\rightarrow \sin \theta_e, 1 \rightarrow \cos \theta_e, \frac{N_m}{N_a} = \frac{M_{qs}}{M_{ds}}, jV_1 \rightarrow v_{ds}^e, V_1 \rightarrow v_{qs}^e \\ V_a &\rightarrow v_{ds}^s, V_m \rightarrow v_{qs}^s, jI_1 \rightarrow i_{ds}^e, I_1 \rightarrow i_{qs}^e, I_a \rightarrow i_{ds}^s, I_m \rightarrow i_{qs}^s \end{aligned} \quad (16)$$

It is expected by using (14) and (15) the unbalanced equations of single-phase IM become similar to the balanced equations.

4.2. Equations of RFOC for Single-Phase IM Based on Proposed Method

In this section vector control equations of single-phase IM based RFOC is presented. Using (15) and after simplifying, the equations of single-phase IM are obtained as follows:

Rotor voltage equations:

$$\begin{aligned} \begin{bmatrix} T_s^e \\ 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} M_{ds} \frac{d}{dt} & \omega_r M_{qs} \\ -\omega_r M_{ds} & M_{qs} \frac{d}{dt} \end{bmatrix} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix}^{-1} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \\ &+ \begin{bmatrix} T_s^e \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} R_r + L_r \frac{d}{dt} & \omega_r L_r \\ -\omega_r L_r & R_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix}^{-1} \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix} \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \end{aligned} \quad (17)$$

After simplifying Equation (17) can be written as:

$$\begin{aligned} \begin{bmatrix} 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} M_{qs} \frac{d}{dt} & (\omega_r - \omega_e) M_{qs} \\ -(\omega_r - \omega_e) M_{qs} & M_{qs} \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} \\ &+ \begin{bmatrix} R_r + L_r \frac{d}{dt} & (\omega_r - \omega_e) L_r \\ -(\omega_r - \omega_e) L_r & R_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{dr}^e \\ i_{qr}^e \end{bmatrix} \end{aligned} \quad (18)$$

Electromagnetic torque equation:

$$\begin{aligned} \tau_e &= \frac{Pole}{2} (M_{qs} i_{qs}^s i_{dr}^s - M_{ds} i_{ds}^s i_{qr}^s) = \frac{Pole}{2} \begin{bmatrix} i_{dr}^s & i_{qr}^s \end{bmatrix} \begin{bmatrix} 0 & M_{qs} \\ -M_{ds} & 0 \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \\ &= \left(\frac{Pole}{2} \begin{bmatrix} i_{dr}^s & i_{qr}^s \end{bmatrix} \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix}^T \left(\begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix}^{-1} \right)^T \begin{bmatrix} 0 & M_{qs} \\ -M_{ds} & 0 \end{bmatrix} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix}^{-1} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \right) \end{aligned} \quad (19)$$

After simplifying Equation (19) can be written as:

$$\tau_e = \frac{Pole}{2} M_{qs} (i_{qs}^e i_{dr}^e - i_{ds}^e i_{qr}^e) \quad (20)$$

In summary based on Equation (17)-(20), equations of RFOC for single-Phase IM are obtained as following equations. In the process of obtaining these equations the assumption $\lambda_{dr}^e = |\lambda_r|^e$ and $\lambda_{qr}^e = 0$ is considered (in RFOC method, the rotor flux vector is aligned with d-axis; $\lambda_{dr}^e = |\lambda_r|^e$ and $\lambda_{qr}^e = 0$):

$$|\lambda_r|^e = \frac{M_{qs} i_{ds}^e}{1 + T_r \frac{d}{dt}} \quad (21)$$

$$\tau_e = \frac{Pole}{2} |\lambda_r|^e \frac{M_{qs}}{L_r} i_{qs}^e \quad (22)$$

$$\omega_e = \omega_r + \frac{M_{qs} i_{qs}^e}{T_r |\lambda_r|^e} \quad (23)$$

In Equation (21), T_r is rotor time constant ($T_r=L_r/R_r$). As can be seen from equations (21)-(23) the structure of RFOC equations of single-phase IM using proposed transformation matrix for stator current variables, become like balanced equations.

Consequently, Figure 4 is proposed for IRFOC of single-phase IM. As mentioned before, by substituting $L_{ds}=M_{ds}=R_{ds}=0$, this control system can be used for faulty single-phase IM. In This Figure, 2 to 2 transformation matrix for stator currents is as follows:

$$\begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (24)$$

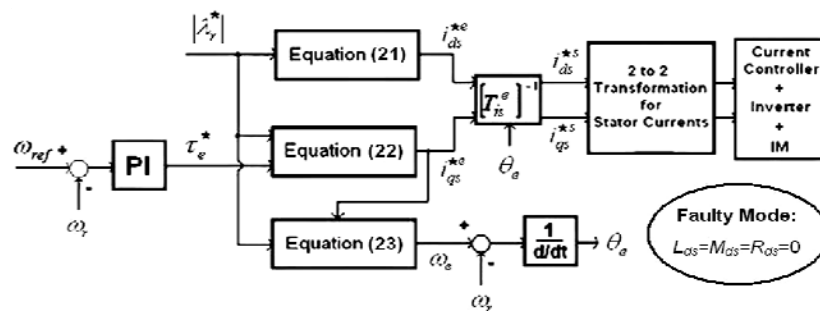


Figure 4. Block diagram of proposed IRFOC for both healthy and faulty single-phase IM

5. Simulation Results

To evaluate the effectiveness of the proposed controller for vector control of healthy and faulty single-phase IM, simulation is conducted using MATLAB simulation package. The simulated single-phase IM is fed by two-leg Voltage Source Inverter (VSI) as used in [9]. The d-q model based on Equation (1)-(3) and (7)-(9) has been used for healthy and faulty single-phase IM respectively. The Runge-Kutta algorithm is used to solve dynamic equations of single-phase IM under healthy and faulty conditions. Moreover, a controller system based on Figure 4 is considered for vector control of both single-phase IM when work with one (faulty single-phase IM) and two windings (healthy single-phase IM). In all simulations a phase cut-off is occurred in auxiliary winding. The proposed drive system is tested under various operating conditions namely, healthy and faulty conditions and variations of load torque and reference speed. In this paper, immediate fault detection is assumed as considered in [25]. The parameters of the simulated machine are as follows:

Voltage: 110 V, $f = 50$ Hz, No. of poles = 4, Power = 475 Watt, $J = 0.0038$ kg.m², $R_{ds} = 20.6$ Ω , $R_{qs} = 6.2$ Ω , $L_{lr} = L_{ls} = 0.0814$ H, $R_r = 19.15$ Ω , $L_{ms} = 0.851$ H, $L_{ds} = 1.28$ H $L_{qs} = 0.43$ H

Figure 5 shows the simulation results of the proposed controller for single-phase IM under healthy and faulty conditions. In this Figure, from $t=0$ s to $t=2$ s, the single-phase IM works with two main and auxiliary windings and from $t=2$ s to $t=7$ s, the single-phase IM works with only main winding. Moreover, at $t=2.5$ s, a step load torque equal to 0.1N.m is applied. Figure 5(c) and 5(d) show the electromagnetic torque of the single-phase IM. It can be seen that the electromagnetic torque has a quick response with no pulsations in both healthy and faulty conditions. Figure 5(e), 5(f) and 5(g) show the reference and real motor speed, when the speed reference varied from 500rpm to 450rpm. It is obvious from these Figures that the single-phase IM can follow the reference speed without any overshoot and steady-state error even under load. Based on Figure 5, the maximum error between reference and real speed in the healthy condition and at steady state is ~ 0.2 rpm and the maximum error in the faulty condition after applying load and at steady state is ~ 5 rpm. From the presented simulation results of Figure 5 it can be seen that the dynamic performance of the proposed IRFOC drive system for vector control of single-phase IM under healthy and faulty conditions are acceptable.

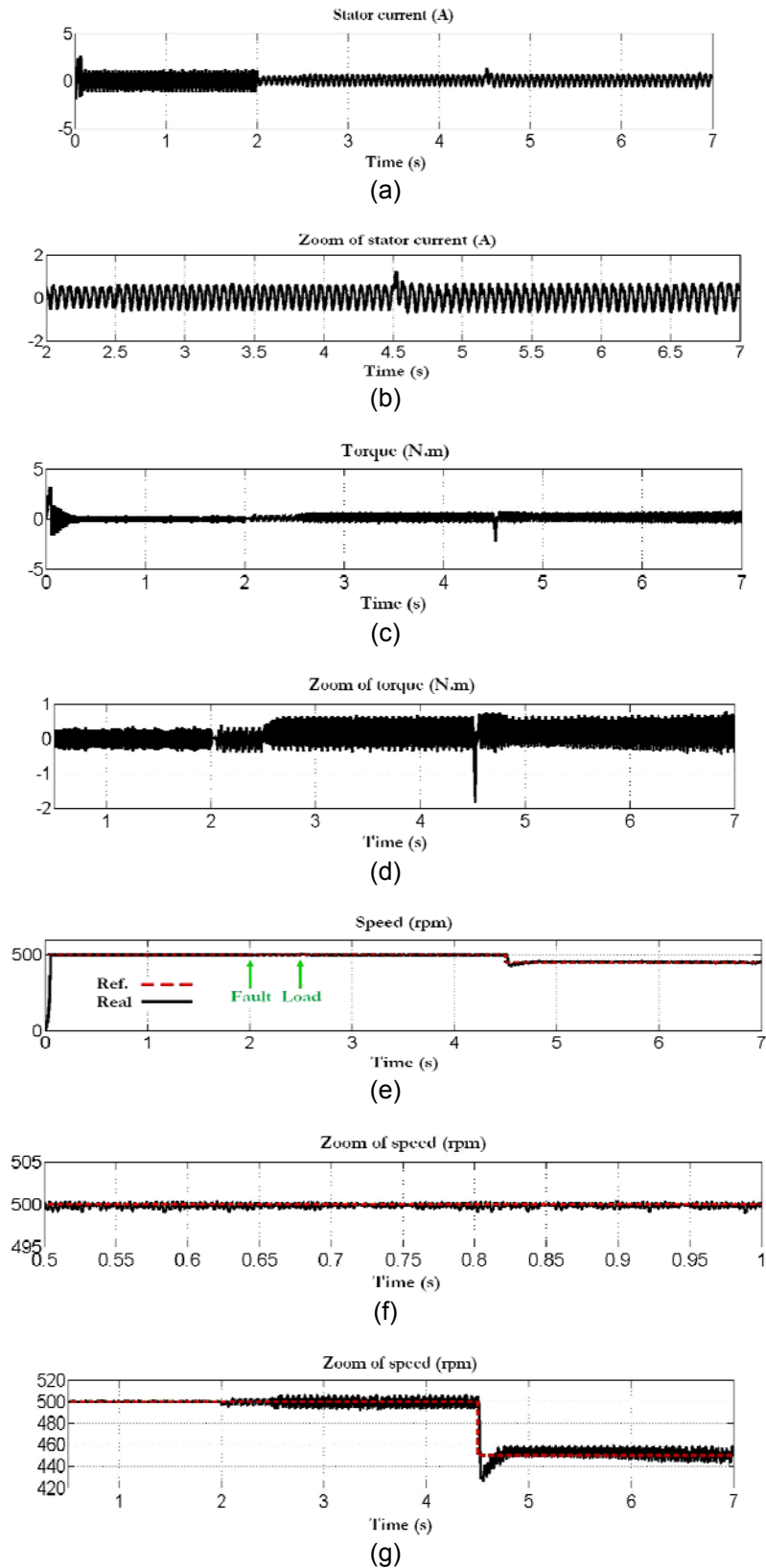


Figure 5. Simulation results of the proposed controller; (a) Stator current, (b) Zoom of stator current, (c) Torque, (d) Zoom of torque, (e) Speed, (f) Zoom of speed in the normal condition, (g) Zoom of speed in the faulty condition

6. Conclusion

In this paper, a new scheme for vector control of healthy and faulty single-phase IM (open-phase fault) has been presented. A novel single-phase IM model with one and two windings based on d-q model has been developed to provide a useful solution to apply FOC technique to control the single-phase IM under normal and open-phase fault conditions. Simulation results have validated the proposed methodology. The performance of the proposed scheme is highly satisfactory for controlling both healthy and faulty single-phase IM. In future research, the other fault conditions will be addressed. Experimental results will be carried out to emphasize the simulation results, which are so far acceptable.

Appendix

Based on Figure 2, the stator main and auxiliary voltages can be written as:

$$V_m = Z_{lm} I_m + E_{fm} - \frac{j}{\alpha} E_{fa} + E_{bm} + \frac{j}{\alpha} E_{ba}$$

$$V_a = Z_{la} I_a + E_{fa} + j\alpha E_{fm} + E_{ba} - j\alpha E_{bm}$$

Where:

$$E_{fm} = Z_f I_m, \quad E_{bm} = Z_b I_m, \quad E_{fa} = \alpha^2 Z_f I_a$$

$$E_{ba} = \alpha^2 Z_b I_a, \quad Z_f = R_f + jX_f, \quad Z_b = R_b + jX_b$$

$$Z_{lm} = R_{lm} + jX_{lm}, \quad Z_{la} = R_{la} + jX_{la}$$

In Figure 2, V_m , V_a , I_m and I_a are the main and auxiliary voltages and currents, $a=N_a/N_m$ and j is the turn ratio and square root of "-1". E_{bm} , E_{ba} , E_{fm} and E_{fa} are the backward and forward voltage of magnetizing branch of the main and auxiliary windings. R_{lm} , R_{la} , X_{lm} and X_{la} are the leakage resistance and inductance of the main and auxiliary winding. Moreover, R_f , R_b , X_f and X_b are the forward and backward stator resistance and inductance. Using following change of variables,

$$I_m = \frac{1}{2}(I_1 + I_2), I_a = \frac{j}{2\alpha}(I_1 - I_2)$$

Ratio of current is obtained as following equations:

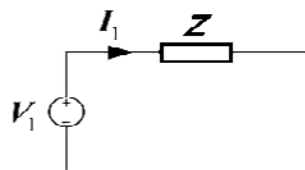
$$\frac{I_m}{I_a} = \frac{Z_{la} + \alpha^2(Z_f + Z_b) + j\alpha(Z_f - Z_b)}{Z_{lm} + Z_f + Z_b - j\alpha(Z_f - Z_b)}$$

$$\frac{I_1}{I_2} = \frac{\alpha Z_{lm} + jZ_{la} + 2\alpha Z_b(\alpha j + 1)}{-\alpha Z_{lm} + jZ_{la} + 2\alpha Z_f(\alpha j - 1)}$$

Let us assumed:

$$V_1 = Z_3 V_m + jZ_4 V_a$$

Consequently, Figure 2 can be simplified as Figure 3.



where:

$$Z = \left(\frac{(Z_3 + Z_4)(Z_{lm} + 2Z_f)}{2} + \frac{(Z_3 + Z_4)(-\alpha Z_{lm} + jZ_{la} + 2\alpha Z_f(\alpha j - 1))(Z_{lm} + 2Z_b)}{2(\alpha Z_{lm} + jZ_{la} + 2\alpha Z_b(\alpha j + 1))} \right)$$

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