# Binary LDPC Codes Decoding Algorithm Based on MRF and FPGA Implementation

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## Abstract

The improved LDPC code decoding algorithm mainly refereed to improving decoding performance or reducing the decoding computation complexity. No matter hard decision or soft decision LDPC code decoding algorithm, we can get all ring number by one test, instead of testing each long ring number, after optimizing ring detection algorithm. We putted forward the application of Gaussian Markov Random Field model to realize the source parameter estimation, and make logarithmic likelihood ratio correction of bit sequence received by the channel decoding end. Joining source residual redundancy information is to increase the decoder error correction ability. Source estimation adaptive variable can correct coefficient, and it was regulated by error rate. Under the condition that computational complexity increasedlittlely, the LDPC code decoding algorithm based on MRF effectively improved the decoding performance and implemented the improvement of LDPC code decoding algorithm .In the end, we realized the decoding algorithm by using FPGA.

Keywords: LDPC code, MRF, parameters estimation, decoding algorithm, FPGA

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## 1. Introduction

LDPC code, low density parity check code [1-3], belongs to the linear block code. Its check matrix is sparse matrix of 0 and 1. Generalized LDPC codes have shown relatively good performance. Scholars Gallage brought forward the related theory basis of LDPC codes in 1962 [4]. Because the LDPC code decoding complexity is not high, and its description is relatively simple, it can be realized by hardware, so it immediately draw attention and became academic research hot [5]: increasing the transmission distance, proposing energy-saving scheme and improving anti interference ability, etc [6]. In fact, the research steps of further enhancing the performance of LDPC codes never stop. LDPC codes have been extensively studied in the literature using code modifying techniques such as information nulling or shortening, extending [7, 8], puncturing [9], and combining [10]. We focused on studying the structure of LDPC code and LDPC code decoding algorithm, improved LDPC code decoding algorithm became the most crucial part [11].

Due to the progress of computer computing capacity, the calculation of stochastic model is no longer the bottleneck, we give more and more attention to stochastic model, and Markov [12] Random Field is one of the important branches. MRF introduced structure information through the neighborhood system, so that it can express interaction model between spatial relative random variables, and find the solution of the problem according to the statistical decision and estimation theory optimal criterion. No matter hard decision or soft decision LDPC code decoding algorithm, after optimizing ring detection algorithm, we put forward the application of Gaussian Markov Random Field model to realize the source parameter estimation, and realize the LDPC code decoding algorithm based on MRF.

The best way of testing whether an algorithm is effective is to apply the algorithm to practice it. So we use FPGA [13] model to realize the algorithm, and to verify the effectiveness, so that it can provide guidance when we use this algorithm in bad communication environment.

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# 2. LDPC Code Decoding Algorithm Based on MRF

Currently, in the process of research on LDPC codes, the improved decoding algorithm became a very important and a key part, and it mainly include improving the decoding performance or reducing computing complexity, or making a compromise.

# 2.1. Research Method

# 2.1.1. Ring Analysis Detection Algorithm Optimization

Both hard decision and soft decision belong to the iterative decoding algorithm, the key lies in the independence of the message transfer. For any given LDPC code, rapid detecting ring-number of Tanner graph which check matrix corresponding to in very important. Optimization algorithm, pointed at the characteristics of Tanner graph which LDPC code check matrix corresponding to, was constructed according to Dijkstra algorithm. The key gist is that we can get all ring number by one testing, instead of testing ring-number in turn, which ring length is 4, 6, 8, 10, etc, so as to realize the rapid detection of ring-number. So it can reduce the computational complexity and improve detection speed.

# 2.1.2. Parameter Estimation Method

For parameter model, parameter estimation is very important. But parameter estimation need to identify accurate results, so we need to alternately iterate parameter estimation and identification, that is to say we will use previous parameter estimation for the next pattern recognition, to update parameter estimation after getting recognition results. In the end, recycling these steps until convergence. This article uses the Dynamic Monte Carlo method estimation.

Gibbs sampling, which will be used in Dynamic Monte Carlo method, has a fast convergence speed, so the parameter estimation usually uses Gibbs sampling.

We assume that  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, x_2, \dots, x_n)$  are two realities which have differences only in the first component in the Markov Field, then define the transition probability:

$$P_{1}(y \mid x) = z_{1}^{-1} \exp(-U(y_{1}x_{s \setminus 1}))$$
(1)

Among them  $y_1 x_{s \setminus 1} = y = (y_1, x_2, \dots, x_n)$ 

Thus extending, two realities in any Markov Field,  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$ , after *n* steps,  $(x_1, x_2, \dots, x_n) \rightarrow (y_1, x_2, \dots, x_n) \dots (y_1, y_2, \dots, y_n)$ , then transfer of x to y is realized. Transfer matrix is:

$$P(y | x) = P_1 P_2 \cdots P_n(x, y)$$
 (2)

Limiting distribution of the transition matrix probability is Markov Field true distribution P(x).

## 2.1.3. Markov Random Field Model

The choice of distance measure has an important relationship with pattern recognition, because the directly affect the iterative convergence effect in the end. Frobenius norm can effectively measure matrix and vector, it also can be regarded an extension of two or three-dimensional vector length.

For changes of calculating amount, this area usually relatively concentrated, if we can find the closed curve which distinguishes between change and non-change regions, then it can be separated from the curve inside and outside and even residual won't affect it.  $PL(\gamma)$  represent closed curve boundary probability density function of  $\gamma$ , then closed curve internal probability density <br/>shoundary probability density, closed curve external probability density <br/>shoundary probability density of crossing the closed curve. In other words, we only need to find the largest one of the boundary probability density.

$$PL(\gamma) = Pin(\gamma)Pout(\gamma)$$
(3)

Among them,  $Pin(\gamma)$  represents probability within the area,  $Pout(\gamma)$  represents probability outside the area. For Markov chain, the boundary curve of the next moment is only related to the boundary curve of present moment, and it must satisfy aperiodicity and ergodicity. The best way to construct Markov chain is to use the dynamic Monte Carlo method to estimate parameters. More than 50 times, we should stop operation, and the 50<sup>th</sup> state will be optimal boundary closed curve.

Gauss-Markov random field belongs to the stationary autoregressive process, and it is a linear model. So its covariance matrix is positive definite and its neighborhood system is symmetric.

 $V_c(x)$  is a potential function of potential group C, the object's prior model is:

$$P(X = x) = \frac{1}{Z} \exp[-\sum_{c \in C} V_c(x)]$$
(4)

Z is a regular constant, a priori energy will be defined as the following:

$$U(x) = \sum_{r \in R} \sum_{r' \in N(r)} V(x(r), x(r'))$$
(5)

Using the maximum posteriori probability estimation:

$$X^* = \arg\max_{X} P(X|Y) \tag{6}$$

According to the Bayesian formula:

$$P(X|Y) \propto P(Y|X)P(X) \tag{7}$$

So there are:

$$X^* = \arg\max_{X} P(Y|X)P(X)$$
(8)

Tagging the voxel with the way of minimum energy, so that we can deal with posterior energy problem.

$$X^* = \arg\min\left\{ U(D|x;\theta) + U(x) \right\}$$
(9)

Among them, the posterior for energy is:

$$U(D|x;\theta) = \sum_{r \in \mathbb{R}} U(D(r)|x(r)) = \sum_{r \in \mathbb{R}} \left[ \frac{\|D(r) - \mu_k\|_F^2}{2\sigma_k^2} + \frac{1}{2}\log(\sigma_k^2) \right]$$
(10)

The Gaussian parameters  $\theta = \{\mu_k, \sigma_k | k \in K \}$ 

In the class K, defining the center of the class:

$$\mu_k = \frac{1}{L_k} \sum_{X(r)=k} D(r) \tag{11}$$

Defining the variance of the class:

$$\sigma_{k} = \frac{1}{L_{k}} \sum_{X(r)=k} \left\| D(r) - \mu_{k} \right\|_{F}^{2}$$
(12)

And  $L_k$  is the voxel number of, which belongs to the class K,  $\|D(r) - \mu_k\|_F$  is the Frobenius norm. It also meets the characters of positive definiteness, homogeneity, triangle inequality and compatibility.

## 2.1.4. LDPC Code Decoding Algorithm Based on MRF

The main steps of LDPC code decoding algorithm based on MRF are as follows:

First, we will initialization and set the allowed maximum iteration number, use Gauss-Markov random field model to realize the source parameter estimation, and make logarithmic likelihood ratio correction of bit sequence received by the channel decoding end. Source estimation adaptive variable can correct coefficient, and it will be regulated by error rate.

The second step, it will calculate respectively from the horizontal direction and vertical direction. In decoding iteration for the first time, check nodes will not get any information on input code word from variable node. So the external information of information nodes from check nodes is zero.

The third step, it will make judgment information for all variable nodes, and decide whether it should end the iterative decoding process according to that judgment information. If it gets a zero vector, then showing that it is a legal code word, and it has decoded successfully. Otherwise, it will begin the iterative process of check nodes and variable nodes. It will repeat the above steps until all errors pattern is zero or the maximum iteration number has been reached, and output the decoding results.

#### 2.2. Results and Analysis

Simulation test in this paper mainly includes Performance simulation results and average iteration(we set all maximum iteration number to 100) number of LDPC code soft decision decoding algorithm and LDPC codes decoding algorithm based on MRF. The result is shown in Figure 1.

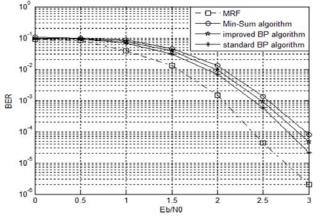


Figure 1. Decoding performance comparisons

We can know from Figure 1: The decoding performance difference of LDPC code decoding algorithms difference will begin to emerge when the SNR increases. Especially when the SNR is more than 0.5, the decoding performance of LDPC codes decoding algorithm based on MRF is obviously better than the standard BP decoding algorithm, improved BP decoding algorithm and Min-Sum decoding algorithm. Specifically when the bit error rate is 10<sup>-3</sup>, compared with standard BP decoding algorithm, improved BP decoding algorithm and Min-Sum decoding performance of LDPC code decoding algorithm and Min-Sum decoding performance of LDPC code decoding algorithm and Min-Sum decoding performance of LDPC code decoding based on MRF increased 0.3dB, 0.38dB, 0.5dB.

# 3. The FPGA Implementation of LDPC Decoding Algorithm Based on MRF

Now we will implement the above algorithm through FPGA and verify the effectiveness of this algorithm. So that it will provide guidance in bad communication environment. In order to verify the efficient performance of the algorithm, code length and code rate of binary LDPC code will be set to 512 and 0.5 respectively.

# 3.1. Research Method

The tools adopted by this part: the Verilog language; Altera Quartusll14.0 development tools; DE1-SOC platform developed by Taiwan friend Crystal Company, and its main chip; partial parallel decoder.

# 3.1.1. The Overall Architecture Diagram of Decoder

The overall architecture diagram is shown in Figure 2.

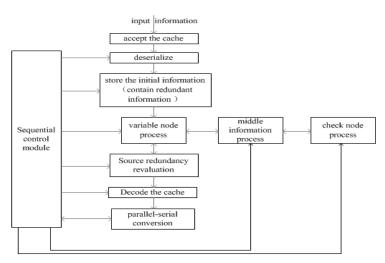


Figure 2. Overall architecture diagrams

The above figure is the overall architecture diagram of FPGA decoder. Simply, the whole process is that: After receiving information, it will be converted through buffer, and it will get initial information which includes redundant information. The will start decoding.

The next step is to update information of check node and variable node, then to make a reassessment of source redundant information, and to iterate again. After reaching the maximum iteration number, it will start decoding cache and outputting the results through the String and conversion module.

# 3.1.2. Check Node Module

This algorithm adopts the binary LDPC codes logarithmic domain algorithm. In this process, it will use tanh(x) function. It can be divided into two parts. One part represents the sign bit, and another part represents numerical bit. We can implement the numerical bit through look-up table. The structure diagram is shown in Figure 3.

## 3.1.3. Variable Node Module

Variable node updating module mainly absorbs the information from other check node processing, then it will merger these information and pass them to check nodes. Variable nodes decoding part mainly absorbs the information from all check nodes and the corresponding initial information, and then it will start decoding. If the code word is not appropriate, it will estimate source redundant information when the number of iteration didn't reach it maximum point. Then it will merge this information into the initial information, and begin the next process. The structure diagramis shown in Figure 4.

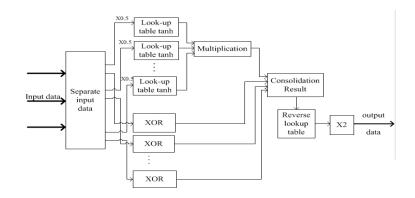


Figure 3. check node structure diagram

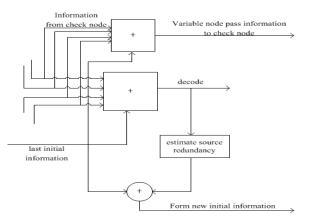
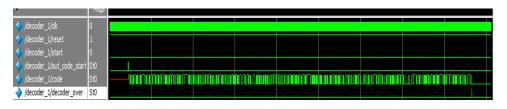


Figure 4. Variable node structure diagrams

## 3.2. Results and Analysis

The parameter of binary LDPC code, adopted in this part, is(512,3,6), and K=5. The end result is shown in Figure 5. From the picture we can see that it began to decode when the reset is invalid and start decode=1. After check nodes and variable nodes update iteration, decode over=1, and it outputted the code words when the number of iteration reached the maximum point. It can be verified that the code word is consistent with that outputted by MATLAB.





This part uses Cyclone V series 5CESMA5F31C6 developed by Altera company. We make code compilation and testing through the QuartusII 14.0 software. After comprehensive report, we can see that consumed resources LE was 50%, the RAM was 60%, the maximum clock frequency was 200 MHZ. From the result we can see that the logical resources can meet the requirement, and it can achieve the compromise between resources and efficiency.

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#### 4. Conclusion

For Ring analysis detection, algorithm we can get all ring number through one test, and it no longer requires testing ring number of each ring length. From unique perspective, this paper first put forward that we can combine with Markov random field to implement the source parameter estimation. At the same time, we can improve the ability of error correction through the using of residual redundancy information when decoding. In the end, we use FPG A to implement the above algorithm.

Simulation result showed that the performance of LDPC code decoding algorithm based on MRF increased 0.4dB. So it is better than the mainstream decoding algorithm, and it Achieve a high performance target. But the complexity of the algorithm is in the same order of magnitude, so we need to simplify the computational complexity of the algorithm in the follow-up study.

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