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Integral Backstepping Approach for Mobile Robot Control

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Abstract

This paper presents the trajectory tracking problem of a unicycle-type mobile robots. A robust output tracking controller for nonlinear systems in the presence of disturbances is proposed, the approach is based on the combination of integral action and Backstepping technique to compensate for the dynamic disturbances. For desired trajectory, the values of the linear and angular velocities of the robot are assured by the kinematic controller. The control law guarantees stability of the robot by using the lyapunov theorem. The simulation and experimental results are presented to verify the designed trajectory tracking control.

Keywords: *Robot Mobile, Backstepping, Trajectory tracking, nonlinear systems*

1. Introduction

In the recent years, the mobile robots have been used and exploited in dangerous and difficult works. They can also be found in: industry, science, education, medical, domestic machines, entertainment and military applications [1].They are used in a range of applications [2, 3]. Many research articles have been proposed for the trajectory tracking problem in the literature. Studies and approaches have been developed in this field [4, 5]. In order to design control laws for mobile robots, many models with different methods have been proposed and has been studied by researchers. In [6], an Adaptive PID control law is developed for the control of mobile robot in order to follow a path planning. In [7], a controller based on distance measures using the sonar sensors is proposed to follow a wall, and to command the robot in the workspace while avoiding obstacles this algorithm is tested in the real implementation cases.

Many proposed methods in literature are based on the kinematics design, but in the real experimental, the dynamic models are must be used and included to perform at high speeds and heavy work. Thus, some controllers based on dynamic modeling have been proposed. As an example in [8], a fuzzy Logic Controller is proposed for tacking of mobile robot type unicycle using Mamdani model and backstepping technique, the dynamics parameters are included for the development of the control approach with the Lagrangian Method. Moreover, no experimental results were reported.

In [9], a controller fuzzy logic is proposed to control the system with unknown dynamics variables for the development of the design control, this approach is used on mobile robot with PIC 16f877 microcontroller, but the real implementation necessary to use a computer with high configuration processor and acquisition cards. An adaptive approach for trajectory applied to the model dynamic equation of mobile robot type unicycle is designed in [10], a stability analysis using Lyapunov theory is analyzed to ensure the robot stability, the parameters are updated online where the parameters of the system are not known or change the task, this developed method control is tested with mobile robot type Pioneer 3-DX. Fuzzy adaptive trajectory tracking control of mobile robot type nonholonomic is developed in [11], the objective

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of this method is to eliminate the perturbations in the dynamic and kinematic equations, the designed approach is tested only with simulation results and no experimental results were presented.

The robust controller is proposed in [12], to resolve the tracking problems and to compensate the unknown nonlinear dynamics of the robot. The control structure is based on the combination of the neural adaptation technique and the sliding mode control, real implementation is presented. In [13], the Backstepping approach is introduced and applied on robot type Arduino, the equations control laws developed for motors robot are studied and tested with real implementation, the actual and reference linear and angular velocities are calculated by the kinematics structure control, and for the stability stady, lyapunov theory is used to guarantee the stability of the system.

Some of parameters systems are not modeled or are prone to change in time, with is often impossible for the application. In this study, robust PI/Backstepping contro is applied to compensate for the dynamic disturbances, and its stability is analyzed using the Lyapunov theory, the objective is to ameliorate the Backstopping approach by adding nonlinear PI action to control law equation. Proportional integral action control is added to reduce the error of the trajectory tracking. A comparative study between the proposed approach and the backstepping control is presented by simulations to show the performance of the control structure for stability.

The remainin part of the paper is presented as follows, first we gives the dynamic model of unicycle type robot. Section 2 gives the dynamic unicycle-like robot model. Section 3, presents and explains the proposed PI or backstepping controller design. Respectively, in section 4, The findings of the paper, focusing on the designed trajectory tracking control method. Finally, in section 5 we conclude this paper.

2. Robot Model

In this section, the model of the mobile robot type unicycle proposed by La Cruz and Carelli in [14] is considered, the figure 1 show the mobile robot, where G is mass center, h is the targeted point used. ω is angular velocity. ψ is the angle of rotation, a, b, c and d are distances.

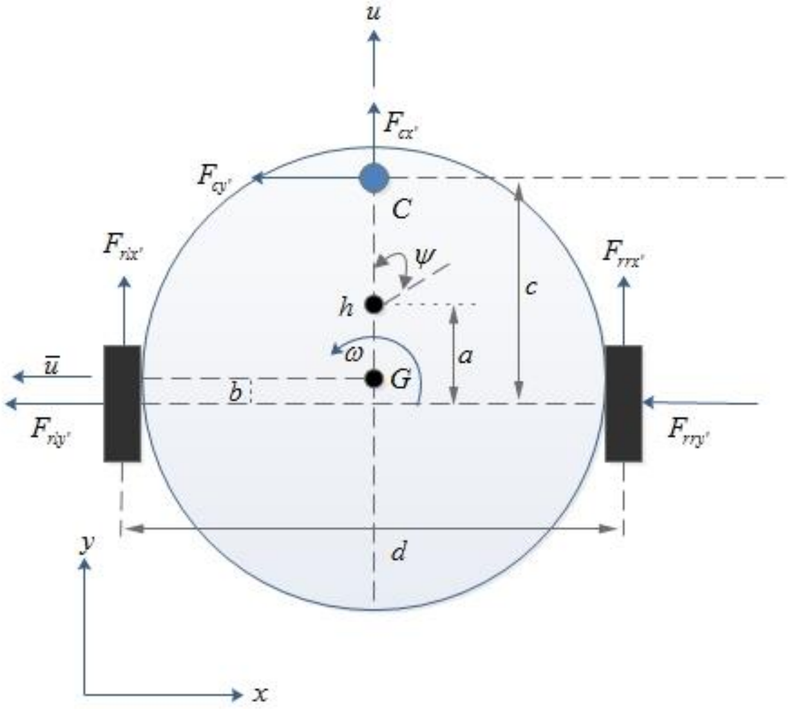


Figure 1. Mobile robot type unicycle

Forces and Moments equations for the system are written as [15]:

$$\sum F_{x'} = m\left(\dot{u} - \bar{u}\omega\right) = F_{rlx'} + F_{rrx'} + F_{cx'} \ , \sum F_{y'} = m(\dot{\bar{u}} + u\omega) = F_{rly'} + F_{rry'} + F_{cy'}$$

$$\sum M_z = I_z\dot{\omega} \qquad (1)$$

and $F_{rrx}, F_{rry}, F_{rlx'}, F_{rly'}, F_{cx'}$ and F_{cy} are the longitudinal and lateral tire forces applied the wheels the robot. The moment of inertia at mass center is Iz. The first part of model is written in [14] as :

$$\dot{x} = ucos\psi - \bar{u}sin\psi - (a-b)\omega sin\psi \ \ , \dot{y} = usin\psi - \bar{u}cos\psi + (a+b)\omega cos\psi \qquad (2)$$

where $\bar{u}$ is the lateral velocity of the mass center. The final model of mobile robot type unicycle is given in [14] by De La Cruz and Carelli as:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{u} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} u\cos(\psi) - aw\sin(\psi) \\ \mathrm{usin}(\psi) + \mathrm{awcos}(\psi) \\ \omega \\ \frac{\lambda_3}{\lambda_1}\omega^2 - \frac{\lambda_4}{\lambda_1}u \\ -\frac{\lambda_5}{\lambda_2}u\omega - \frac{\lambda_6}{\lambda_2}\omega \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\lambda_1} & 0 \\ \frac{1}{\lambda_2} & 0 \end{pmatrix} \begin{pmatrix} u_{ref} \\ \omega_{ref} \end{pmatrix} \qquad (3)$$

and the parameters of the system are defined in [14] as:

$$\lambda_1 = \frac{1}{2}\frac{2rk_{DT} + \frac{R_a}{k_a}(2I_e + mR_t r)}{(rk_{PT})} \ , \lambda_2 = \frac{1}{2}\frac{2rdk_{DR} + \frac{R_a}{k_a}(2R_t mb^2 + r(I_z) + I_e d^2)}{(rdk_{PR})} \ , \lambda_3 = \frac{R_a}{k_a}\frac{mbR_t}{2k_{PT}}$$

$$\lambda_4 = \frac{\frac{R_a}{k_a}(\frac{k_a k_b}{R_a} + B_e)}{(rk_{PT})} + 1 \ , \lambda_5 = \frac{R_a}{k_a}\frac{mbR_t}{2k_{PR}} \ , \lambda_6 = \frac{\frac{R_a}{k_a}(\frac{k_a k_b}{R_a} + B_e)d}{(rk_{PR})} + 1$$

where m is the Arduino robot mass, Ie is the moment of inertia, B_e is the viscous friction coefficient, r is the right and left wheel radius, and R_t is the tire radius, k_a is the torque coefficient, k_b is electromotive coefficient, R_a is the motors resistance. P_D controllers are implemented with PI k_{PT} and k_{PR}, and derivative gains k_{DT} and k_{DR} to control the velocities of the right and left motor.

3. Architecture Control

For the architecture control, we are used the kinematics controller for external loop and a PI/backstepping for the dynamics model as see in Figure 2.

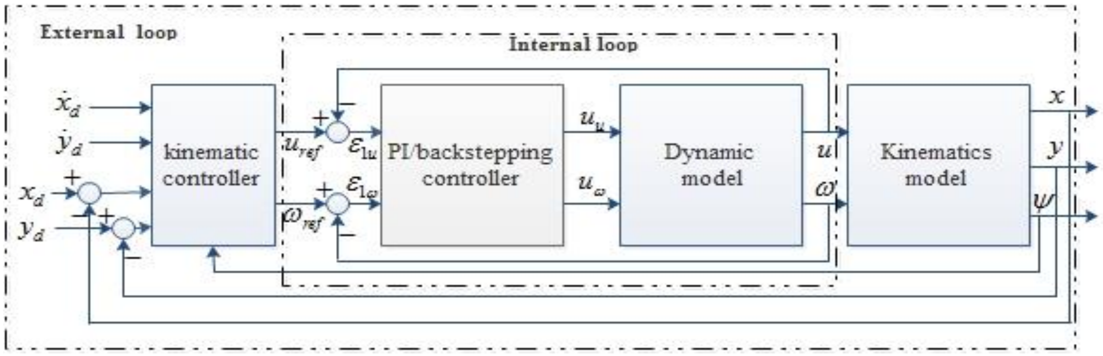


Figure 2. Structure control of PI/backstepping

3.1. Kinematic Control

This design controller is developed with the inverse kinematics equations of the robot mobile. The objective of this controller is to generate the reference values for the PI/Backstepping controller is designed in order the desired values of the linear and angular velocities. From (3), the kinematic model is given as:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} cos(\psi) & -asin(\psi) \\ sin(\psi) & acos(\psi) \end{pmatrix} \begin{pmatrix} u_{ref} \\ \omega_{ref} \end{pmatrix} \tag{4}$$

where $\mathrm{u_{ref}}$ the reference is value of the linear velocity, and ω_{ref} is angular velocity. The matrix inverse is (5).

$$A^{-1} = \begin{pmatrix} \cos(\psi) & \sin(\psi) \\ -\frac{1}{a}\sin(\psi) & \frac{1}{a}\cos(\psi) \end{pmatrix} \tag{5}$$

The control law can be chosen as:

$$\begin{pmatrix} u_{ref}^{k} \\ \omega_{ref}^{k} \end{pmatrix} = \begin{pmatrix} cos(\psi) & sin(\psi) \\ -\frac{1}{a}sin(\psi) & \frac{1}{a}cos(\psi) \end{pmatrix} \begin{pmatrix} \dot{x}_d+\varepsilon_x \\ \dot{y}_d+\varepsilon_y \end{pmatrix} \tag{6}$$

where $\varepsilon_{\mathrm{x}} = \mathrm{x_d} - \mathrm{x}$ and $\varepsilon_y = y_d - y$ the current are position errors, $h(x,y)$ and $\mathrm{h_d(x_d,y_d)}$ are the actual and reference points. The study of the stability for the kinematic controller is detailed in [13].

3.2. Nonlinear PI-based Backstepping Controller Design

The combination of integral action and backstepping equations is proposed to design the structure control that improves the robustness when the dynamics parameters of the robot are not well-known. The dynamic part of (3) is:

$$\dot{u} = \frac{\lambda_3}{\lambda_1}\omega^2 - \frac{\lambda_4}{\lambda_1}u + \frac{u_{ref}}{\lambda_1} \quad , \dot{\omega} = \frac{\lambda_5}{\lambda_2}u\omega - \frac{\lambda_6}{\lambda_2}\omega + \frac{\omega_{ref}}{\lambda_2} \tag{7}$$

first, we consider the errors control laws:

$$\varepsilon_{1u} = u_{ref} - u \Rightarrow \dot{\varepsilon}_{1u} = \dot{u}_{ref} - \dot{u}, \varepsilon_{1\omega} = \omega_{ref} - \omega \Rightarrow \dot{\varepsilon}_{1\omega} = \dot{\omega}_{ref} - \dot{\omega} \tag{8}$$

the lyapunov functions is derived as

$$F(\varepsilon_{1u}) = \frac{1}{2}\varepsilon_{1u}^2, F(\varepsilon_{1\omega}) = \frac{1}{2}\varepsilon_{1\omega}^2 \tag{9}$$

the time derivative of the Lyapunov candidate functions is calculated as

$$\dot{F}(\varepsilon_{1u}) = \varepsilon_{1u}\dot{\varepsilon}_{1u}, \dot{F}(\varepsilon_{1\omega}) = \varepsilon_{1\omega}\dot{\varepsilon}_{1\omega} \tag{10}$$

the stabilization of the dynamics errors system can be obtained by introducing a virtual controls input:

$$u_u^v = \dot{u}_{ref} + K_{1u}\varepsilon_{1u} + \alpha_u\chi_u \quad , u_\omega^v = \dot{\omega}_{ref} + K_{1\omega}\varepsilon_{1\omega} + \alpha_\omega\chi_\omega \tag{11}$$

Where the integral actions are:

$$\chi_u = \int \varepsilon_{1u}(\tau)\,\partial\tau \ , \ \chi_\omega = \int \varepsilon_{1\omega}(\tau)\,\partial\tau \tag{12}$$

with $K_{1u}, K_{1\omega}, \alpha_u$ and $\alpha_\omega$are design parameters. And now, we calculate the new errors as:

$$\varepsilon_{2u} = \dot{u}_{ref} - \dot{u} + K_{1u}\varepsilon_{1u} + \alpha_u \chi_u \ , \ \varepsilon_{2\omega} = \dot{\omega}_{ref} - \dot{\omega} + K_{1\omega}\varepsilon_{1\omega} + \alpha_\omega \chi_\omega \tag{13}$$

and

$$\dot{\varepsilon}_{2u} = \ddot{u}_{ref} - \ddot{u} + K_{1u}\dot{\varepsilon}_{1u} + \alpha_u \varepsilon_{1u} \ , \ \dot{\varepsilon}_{2\omega} = \ddot{\omega}_{ref} - \ddot{\omega} + K_{1\omega}\dot{\varepsilon}_{1\omega} + \alpha_\omega \varepsilon_{1\omega} \tag{14}$$

from (11) and (14), it follows that:

$$\dot{\varepsilon}_{1u} = -K_{1u}\varepsilon_{1u} - \alpha_u \chi_u + \varepsilon_{2u} \ , \ \ \dot{\varepsilon}_{1\omega} = -K_{1\omega}\varepsilon_{1\omega} - \alpha_\omega \chi_\omega + \varepsilon_{2\omega} \tag{15}$$

and

$$\dot{\varepsilon}_{2u} = -K_{2u}\varepsilon_{2u} - \varepsilon_{1u} \ , \ \dot{\varepsilon}_{2\omega} = -K_{2\omega}\varepsilon_{2\omega} - \varepsilon_{1\omega} \tag{16}$$

where $K_{2u}, K_{2\omega}$ are positive constant of stability. So, that is result, the integral backstepping control laws of elevation, linear and angular velocities are:

$$u_u = \begin{bmatrix} \lambda_1(\ddot{u}_{ref} + (K_{1u} + K_{2u})\varepsilon_{2u} \\ +(1 - K_{1u}^2 + \alpha_u)\varepsilon_{1u} - \alpha_u \chi_u) \\ +2\lambda_3 \omega \dot{\omega} - \lambda_4 \dot{u} \end{bmatrix} \tag{17}$$

and

$$u_\omega = \begin{bmatrix} \lambda_2(\ddot{\omega}_{ref} + (K_{1\omega} + K_{2\omega})\varepsilon_{2\omega} \\ +(1 - K_{1\omega}^2 + \alpha_\omega)\varepsilon_{1\omega} - \alpha_\omega \chi_\omega) \\ -\lambda_5(\dot{u}\omega + u\dot{\omega}) - \lambda_6 \dot{\omega} \end{bmatrix} \tag{18}$$

the stability of the controller can be studied with the Lyapunov theory. The lyapunov candidate functions is given as:

$$F(\varepsilon_{1u}, \varepsilon_{2u}) = \frac{\alpha_u \chi_u^2 + \varepsilon_{1u}^2 + \varepsilon_{2u}^2}{2} \ , \ F(\varepsilon_{1\omega}, \varepsilon_{2\omega}) = \frac{\alpha_\omega \chi_\omega^2 + \varepsilon_{1\omega}^2 + \varepsilon_{2\omega}^2}{2} \tag{19}$$

whose first time derivative

$$\dot{F}(\varepsilon_{1u}, \varepsilon_{2u}) = -K_{1u}\varepsilon_{1u}^2 - K_{2u}\varepsilon_{2u}^2 \leq 0 \ , \ \ \dot{F}(\varepsilon_{1\omega}, \varepsilon_{2\omega}) = -K_{1\omega}\varepsilon_{1\omega}^2 - K_{2\omega}\varepsilon_{2\omega}^2 \leq 0 \tag{20}$$

are negative defined, which means that, the tracking error is stable.

4. Experiment results

The fourth section present the implementation of the proposed control law on a Arduino Robot Mobile, with Radius: 185 mm, height: 85 and weight: 1.50 kilogram, it has also two processors based on the ATmega32u4, see Figure 3, the wheels are driven by DC motors having rated torque 30 mNm at 15000 rpm, encoders output of 500 ticks/revolution are integrated for this motors, the sample time of the robot is 0.1s. Arduino Yun is connected to the robot to send the control law from MATLAB to the robot using the wireless network connection. The odometric sensors are used to sensing the point h of robot position. The approach is programmed with the MATLAB using the windows system. In order to test the proposed PI/Backstepping control law of this paper, several experiments were carried out with circular track reference trajectory of 2 m. The robot start from the position P_0 = (x, y, ψ) = (0, 0, 0). The gains of the implemented controller are selected as:

$$[K_{1u} = 120, K_{2u} = 75]\ [K_{1\omega} = 14, K_{2\omega} = 2]\ \ [\alpha_u = 12.75, \alpha_\omega = 3.5]$$

The robot mobile tracks the circular way with smaller error. Thus, show good performance as see in Figure 4 (a), the evolutions of the desired and actual linear velocity and angular velocity are plotted in Figure 4 (b), in the figures results, we show that the robot tracks the reference trajectory correctly and the errors tend to zero. Figure 5 (a) illustrates the control the robot's wheels speed signal generated by the Motor board processor. The linear velocity and angular control laws are indicated in Figure 5 (b). The Yun Arduino card is used to receive the control laws from MATLAB to the robot through wireless card.

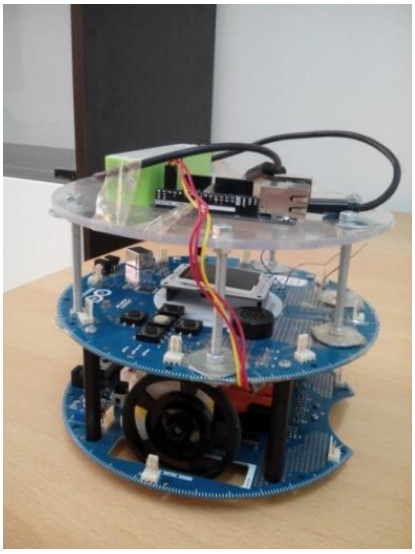
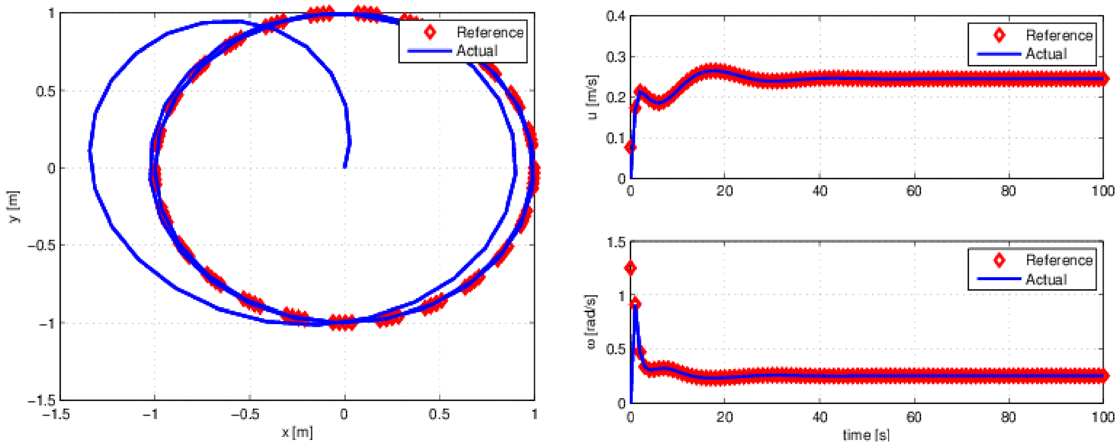


Figure 3. Arduino robot mobile

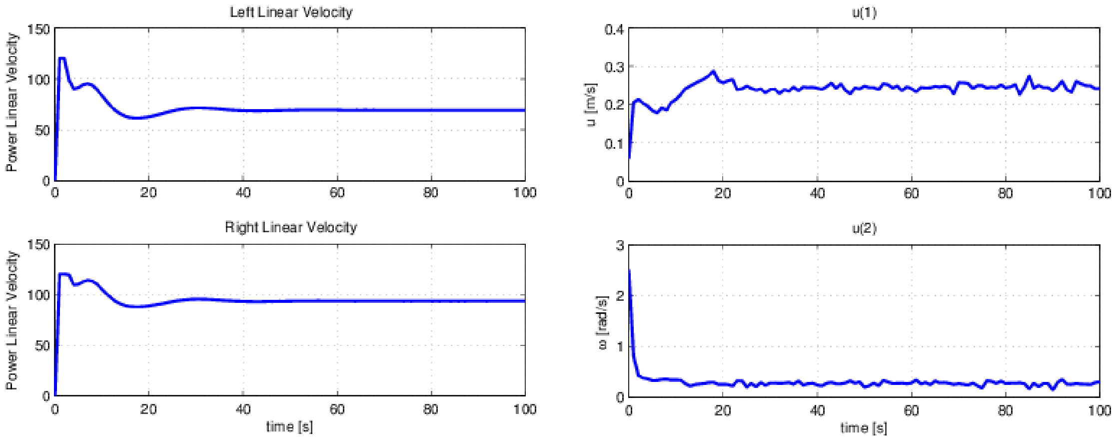


(a) (b)

Figure 4. Simulation results (a) The trajectory followed by the mobile robot, (b) The linear velocity and angular velocity

4.1. Comparative Study

The second experiment is designed to compare between the Backstepping approach proposed in [13] and PI/backstepping, we carried out the experiment with same circular track reference trajectory of 2 meters of radius and same initial posture $P_0 = (x, y, \theta) = (0{,}0{,}0)$. Figure 6 presents the results of the structure control with the evolution the distance error using the approach and backstepping controller, it can be see that the PI/backstepping is the most stable and accurate method comparing the backstepping approach. Now in Table 1, we have the numerical results: the mean error, variance and standard deviation of trajectory using backstepping and PI/backstepping approaches, we notice that the PI/backstepping gives good results in accuracy, by the numerical results its can show the effectiveness of this algorithm. One of the advantages of this approach is that they can usually to compensate for the dynamics not modeled as hysteretic damping, wheel tire diameters and vibrations.



(a) (b)

Figure 5. Experimental results (a) Power linear velocity, (b) Control law inputs

Table 1. Numerical comparison results

	Backstepping		PI/backstepping	
Trajectory	X(meter)	Y(meter)	X(meter)	Y(meter)
Mean Error (m)	0.1359	0.0551	0.1169	0.0090
Variance,(m2)	0.0482	0.0045	0.0516	0.0003
Standard deviation	0.2195	0.0671	0.2271	0.0181

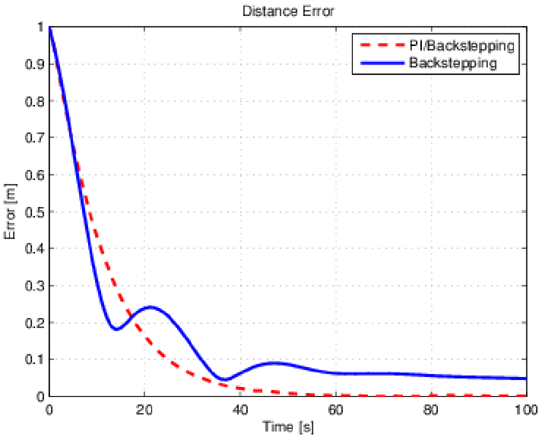


Figure 6. Distance errors for backstepping and PI/backstepping.

5. Conclusion

The paper addressed the control structure of mobile robot type unicycle. The proposed approach is based on the backstepping technique combined with integral action; the stability study of the system is demonstrated with Lyapunov method, the design controller is implemented on an Arduino mobile robot. The integral part of algorithm is added to eliminate the tracking errors and the disturbances and dynamics not modeled as wheel and tire diameters, mass, inertia, etc, the comparative results explain the advantages the proposed PI or Backstepping controller in precision and stability. The force estimator design can be added to guarantee the robustness of the controller. This work can be applied to remote control using the virtual reality [16], and augmented reality [17]. We intend also to improve the system in an environment with obstacles.

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