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Bayesian Segmentation in Signal with Multiplicative Noise Using Reversible Jump MCMC

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Abstract

This paper proposes the important issues in signal segmentation. The signal is disturbed by multiplicative noise where the number of segments is unknown. A Bayesian approach is proposed to estimate the parameter. The parameter includes the number of segments, the location of the segment, and the amplitude. The posterior distribution for the parameter does not have a simple equation so that the Bayes estimator is not easily determined. Reversible Jump Markov chain Monte Carlo (MCMC) method is adopted to overcome the problem. The Reversible Jump MCMC method creates a Markov chain whose distribution is close to the posterior distribution. The performance of the algorithm is shown by simulation data. The result of this simulation shows that the algorithm works well. As an application, the algorithm is used to segment a Synthetic Aperture Radar (SAR) signal. The advantage of this method is that the number of segments, the position of the segment change, and the amplitude are estimated simultaneously.

Keywords*: Reversible Jump MCMC, Bayesian, Multiplicative Noise, Signal Segmentation*

1. Introduction

Signal processing with additive noise has been investigated by several researches, for example Gustafsson *et al.* [1]. In many applications it is often found that signal is disturbed by multiplicative noise. Some researches have also discussed signal with multiplicative noise, such as Ullah *et al.* [2], Osoba and Kosko [3], Tian *et al.* [4], and Dong *et al.* [5]. Ullah *et al.* [2] used a variational approach to restore images with multiplicative noise. Osoba and Kosko [3] used the noisy Expectation-Maximization Algorithm for multiplicative noise injection. Tian *et al.* [4] used an adaptive fractional-order method to eliminate multiplicative noise. Dong *et al.* [5] proposed a method using a sparse analysis model for signal with multiplicative noise. In signal segmentation with multiplicative noise, generally the number of segments is unknown and must be estimated based on the data. This paper proposes the segmentation of signal with multiplicative noise when the number of segments is unknown.

Let N be the many pixels contained in a line from the SAR image. The equation of the line can be expressed in the following form (Suparman *et al.* [6], Tourneret *et al.* [7]) :

$$y_t = r_t z_t \qquad t = 1, 2, \cdots, N \tag{1}$$

with y_t is the intensity of the measured SAR image, r_t is SAR intensity, and z_t is a multiplicative noise. In various SAR images, including agricultural images, the properties of r_t and z_t can be defined as follows (Oliver and Quegan [8]) :

a. SAR intensity r_t is a step function. The equation can be written as :

$$r_t = h_K \qquad n_K < t \le n_{K+1}$$

with K = 0, 1, …, K_{max}. Here, n_K is the position of the height change of K^{th} step. (with agreement $n_0 = 0$ and $n_{K+1} = N$) and h_K is the height of K^{th} step, and K is the number of steps.

b. Multiplicative noise z_t is given in the form of a random variable that follows the gamma distribution with the mean L and variance 1/L, written $z_t \sim G(L, L)$,

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$$f(z_t) = \frac{L^L}{\Gamma(L)} z_t^{L-1} \exp -[Lz_t] \qquad t = 1, 2, \cdots, N$$

Here, L is the number of measurements. The value of L is known. Based on the data y_t (t = 1, 2, ..., N), then the value of the parameter

K, $n^{(K)} = (n_1, n_2, \ldots, n_{K+1})$ and $h^{(K+1)} = (h_0, h_1, \ldots, h_K)$.

will be estimated. To estimate the value of these parameters, a hierarchical Bayesian approach is used.

2. Research Method

2.1. Hiearchical Bayesian

The Bayesian approach [9] is a method for estimating parameter values $\theta = (K, n^{(K)}, h^{(K)})$ which is based on the information from the data y_n (expressed in the probability distribution $f(y|\theta)$) and information from the parameter θ (expressed in the prior distribution $\pi(\theta)$). Due to a multiplicative noise $z_t \sim G(L, L)$, then the probability distribution for y_t can be written as :

$$f(y|\theta) \propto \prod_{i=0}^{K} h_i^{L\tau(n_i, n_{i+1})} \exp -\left[\frac{L\omega(y, n_i, n_{i+1})}{h_i}\right] \tag{2}$$

with $\tau(a, b) = b - a$, $\omega(y, a, b) = \sum_{n=a+1}^{b} y_n$, and symbol "$\propto$" means "proportional to".

To use the Bayesian approach, the prior distribution for the parameter θ should be determined. Prior distribution for parameter θ is taken the same as in Suparman *et al.* [6]. Suppose K_{max} is the maximum number of steps. The K is assumed to follow a Binomial distribution with parameter λ. The prior distribution for K can be written as

$$\pi(K | K_{maks}, \lambda) \propto \lambda^K (1-\lambda)^{K_{maks}-K} \quad K = 0, 1, \ldots, K_{max}. \tag{3}$$

For the value of K given, $n^{(K)}$ is assumed to follow the following distribution :

$$\pi(n^{(K)} | K) \propto \prod_{i=0}^{K} (n_{i+1} - n_i - 1) \tag{4}$$

and $h^{(K)}$ follows the inverse gamma distribution with parameters α dan β. Prior distribution for $h^{(K}$ can be written as

$$\pi(h^{(K+1)} | K, \alpha, \beta) \propto \prod_{i=0}^{K} h_i^{\alpha-1} \exp -[\frac{\beta}{h_i}] \tag{5}$$

The problem that arises is the presence of hyperparameter $\varphi = (\lambda, \alpha, \beta)$ in the above prior distributions. To simplify the problem, in Suparman *et al.* [6] value φ is known. In this paper, as in Tourneret *et al.* [7] hyperparameter φ is seen as a random variable with a given distribution, here λ follows a uniform distribution at interval (0,1) and β follows Jeffrey distribution. The value α is taken relatively small.

Suppose that $\pi(\theta, \varphi | y)$ is a posterior distribution for θ. By using Bayes's theorem, the posterior distribution $\pi(\theta, \varphi | y)$ can be expressed as the multiplication of the probability distribution for y_i and prior distribution for (θ, φ) :

$$\pi(\theta, \varphi | y) \propto f(y | \theta) \times \pi(\theta | \varphi) \times \pi(\varphi) \tag{6}$$

The estimation of the parameter θ will be based on the posterior distribution. Because the shape of the posterior distribution $\pi(\theta|\,y)$ is very complex, it is difficult to estimate parameter value θ. To overcome this problem, the Reversible Jump MCMC method is adopted.

2.2. Reversible Jump MCMC Method

Suppose that $M=(\theta,\varphi)$ is a Markov chain. The MCMC method is a sampling method. This sampling method makes a homogeneous Markov chain $M_1, M_2, \cdots, M_m$ which satisfies periodic and irreducible properties such that $M_1, M_2, \cdots, M_m$ can be considered as a random variable that follows the distribution $\pi(\theta,\varphi|\,y)$ [10]. The chain Markov $M_1, M_2, \cdots, M_m$ can be used to estimate the parameter M. To realize it, Gibbs algorithm is adopted which consists of two stages:

a. Simulate the distribution $\pi(\varphi|\,\theta,y)$
b. Simulate the distribution $\pi(\theta|\varphi,y)$

The Gibbs algorithm is used to simulate the distribution $\pi(\varphi|\,\theta,y)$. Hybrid algorithm is used to simulate the distribution $\pi(\theta|\varphi,y)$. This hybrid algorithm combines the Reversible Jump MCMC algorithm [11] to simulate parameter $\pi(K,n^{(K)}|\,\varphi,y)$ and algoritma Gibbs to simulate parameter $\pi(h^{(K)}|\,\varphi,y)$. Reversible Jump MCMC algorithm is an extension of the Metropolis-Hastings algorithm.

2.2.1.Distribution Simulation $\pi(\varphi|\,\theta,y)$

Suppose that $\pi(\varphi|\,\theta,y)$ is a conditional distribution of parameter φ given θ and y. The distribution $\pi(\varphi|\,\theta,y)$ can be expressed as

$$\pi(\varphi|\;\theta,y)\propto\lambda^{K}(1-\lambda)^{K_{maks}-K}\beta^{\alpha(K+1)}\exp-(\beta\textstyle\sum_{i=0}^{K}\;\dfrac{1}{h_i})$$

This distribution is the multiplication of distribution $B(K+1,K_{max}-K+1)$ and $G(\alpha(K+1)+1,\sum_{i=0}^{K}1/h_i)$. The Gibbs algorithm is used to simulate it.

2.2.2. Distribution Simulation $\pi(\theta|\varphi,y)$

Suppose that $\pi(\theta|\varphi,y)$ is a conditional distribution of θ given (φ,y). This conditional distribution can be expressed as

$$\pi(\theta|\varphi,y)\propto\quad C_K^{K_{maks}}\lambda^{K}(1-\lambda)^{K_{maks}-K}\frac{1}{C_{N-2}^{2K+1}}\left(\frac{\beta^{\alpha}}{\Gamma(\alpha)}\right)^{K+1}\prod_{i=0}^{K}(n_{i+1}-n_i)-1$$
$$\prod_{i=0}^{K}\;h_i^{\Phi(\alpha,L,n_i,n_{i+1})-1}\exp-[\frac{\Psi(\beta,L,y,n_i,n_{i+1})}{h_i}]$$

where $\Phi=\alpha+L\tau(a,b)$ and $\Psi(\beta,L,y,a,b)=\beta+L\omega(y,a,b)$.
The conditional distribution $\pi(\theta|\varphi,y)$ is integrated against $h^{(K)}$, it will be obtained

$$\pi(K,n^{(K)}|\varphi,y)\propto\quad C_K^{K_{maks}}\lambda^{K}(1-\lambda)^{K_{maks}-K}\frac{1}{C_{N-2}^{2K+1}}\left(\frac{\beta^{\alpha}}{\Gamma(\alpha)}\right)^{K+1}\prod_{i=0}^{K}(n_{i+1}-n_i)-1$$
$$\prod_{i=0}^{K}\;\frac{\Gamma(\Phi(\alpha,L,n_i,n_{i+1}))}{\Psi(\beta,L,y,n_i,n_{i+1})}$$

On the other hand, we have

$$\pi(h^{(K)} \mid K, n^{(K)}, \varphi, y) \propto \prod_{i=0}^{K} h_i^{\Phi(\alpha, L, n_i, n_{i+1})-1} \exp-[\frac{\Psi(\beta, L, y, n_i, n_{i+1})}{h_i}] \otimes_{i=0}^{K} IG(\Phi(\alpha, L, n_i, n_{i+1}, \Psi(\beta, L, y, n_i, n_{i+1}))$$

The distribution $\pi(\theta|\varphi, y)$ as the multiplication of the distribution $\pi(K, n^{(K)}|\varphi, y)$ and distribution $\pi(h^{(K)} \mid K, n^{(K)}, \varphi, y)$, that is :

$$\pi(\theta|\varphi, y) = \pi(K, n^{(K)}|\varphi, y)\ \pi(h^{(K)} \mid K, n^{(K)}, \varphi, y)$$

To simulate the distribution $\pi(\theta|\varphi, y)$, a Gibb algorithm is used. This Gibb algorithm consists of two stages :

a. Stage 1 : Simulate the distribution $\pi(h^{(K)} \mid K, n^{(K)}, \varphi, y)$
b. Stage 2 : Simulate the distribution $\pi(K, n^{(K)}|\varphi, y)$

To simulate the distribution $\pi(h^{(K)} \mid K, n^{(K)}, \varphi, y)$, the Gibbs algorithm is used. On the other hand, distribution $\pi(K, n^{(K)}|\varphi, y)$ is not explicitly so that the MCMC Reversible Jump algorithm is used to simulate it.

3. Results and Analysis

As an application, this method is applied to segment simulation multiplicative and real multiplicative signals. As in [12], a simulation study was undertaken to confirm the performance of the Reversible Jump MCMC algorithm whether it works well or not. The Case studies are given to provide examples of application of research to solve problems in everyday life.To segment the multiplicative signals of simulation and real multiplicative signals, the Reversible Jump MCMC algorithm is implemented as much as 25 thousand iterations with a 5 thousand burn-in period.

3.1. Multiplicative Signal Simulation

Figure 1 is a simulated multiplicative signal created according to the Equation (1) with N = 250 and L = 5. The value K = 3, vector value $n^{(3)}$ = (75, 125, 200) and vector value $h^{(4)}$ = (1,7,3,5).

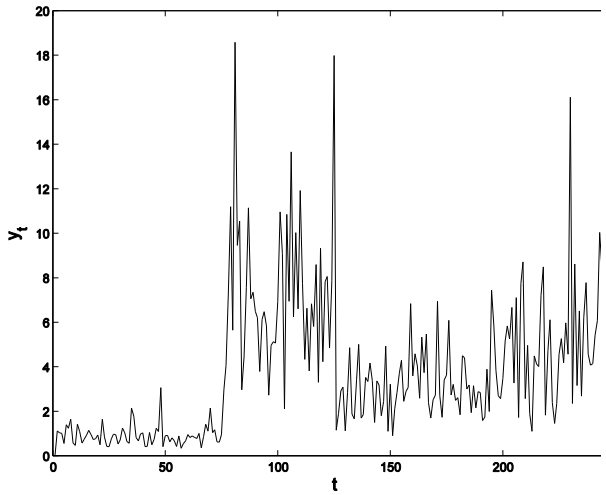

Figure 1. Multiplicative Signal Simulation

Based on the simulation multiplicative signal in Figure 1, the parameter is estimated. The number of segments K, vector $n^{(K)}$ and vector $h^{(K)}$ are estimated by using the Reversible Jump MCMC algorithm. Estimator of K, $n^{(K)}$ and $h^{(K)}$ are

$$\hat{K} = 3,\ n^{(\hat{K})} = (75, 125, 196),\ \text{and}\ h^{(\hat{K})} = (0.9, 7.3, 3.1, 5.1)$$

The histogram for K is given in Figure 2.

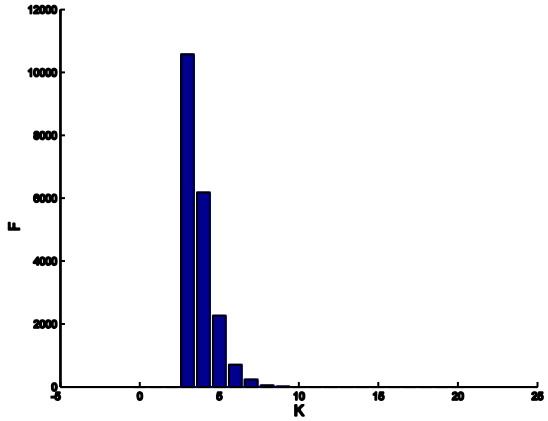

Figure 2. Histogram for K

The signal segmentation generated by the algorithm is presented in Figure 3.

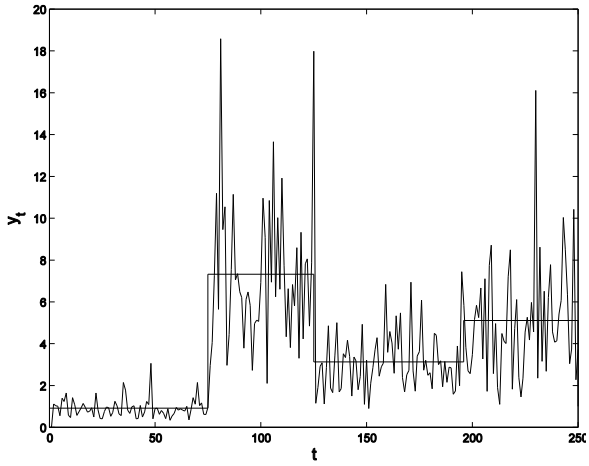

Figure 3. Result of Multiplicative Segmentation of Simulated Signal

If the true parameter value of K, n(K), h(K) is compared against the estimated value of parameter K, $n^{(K)}$, $h^{(K)}$ obtained by algorithm, it appears that the algorithm can work well.

3.2. Real Multiplicative Signal

Now, algorithm is used to segment a line on the real image. The real image used is 480 x 640 as shown in Figure 4. The image taken using the Nokia 3220 mobile phone is a natural scene around the Imogiri Tomb, Yogyakarta Indonesia.

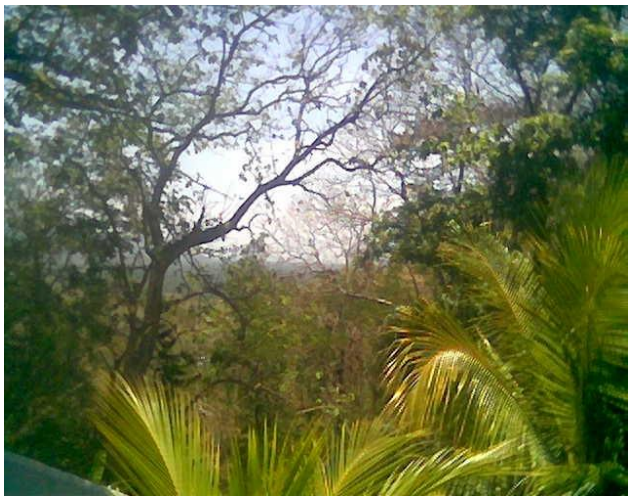

Figure 4. Images of the Natural Scene around the Imogiri Tomb, Yogyakarta Indonesia

The 198th column of the real image is presented in Figure 5. The 198th line will be called a real signal.

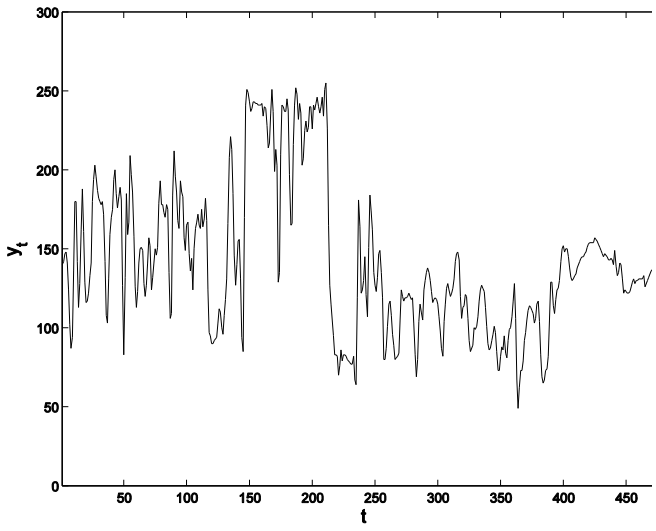

Figure 5. The 198th Column of the Real Image

The Reversible Jump MCMC algorithm is implemented in this real signal. An estimation for the value K, $n^{(K)}$ dan $h^{(K)}$ is given as follows :

$$\hat{K} = 3,\ n^{(\hat{K})} = (145, 213, 394) \text{ and } h^{(\hat{K})} = (151, 217, 107, 139).$$

The histogram for K is given in Figure 6. Because the mode of the histogram is 3, the estimator of the number of segments is 4.

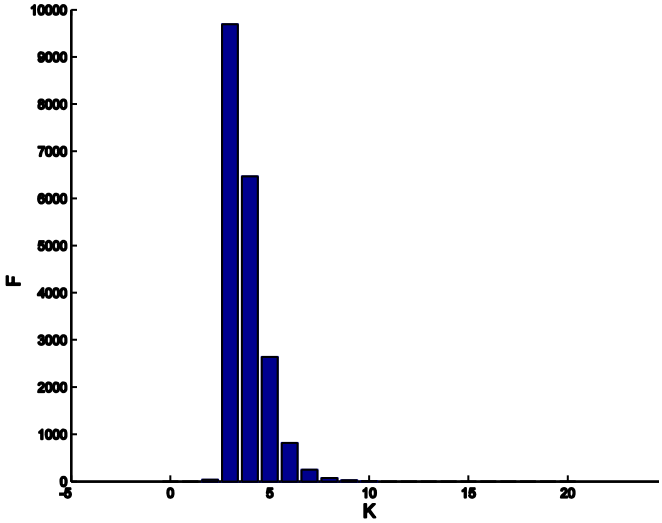

Figure 6. Histogram for K

The results of its segment are presented in Figure 7.

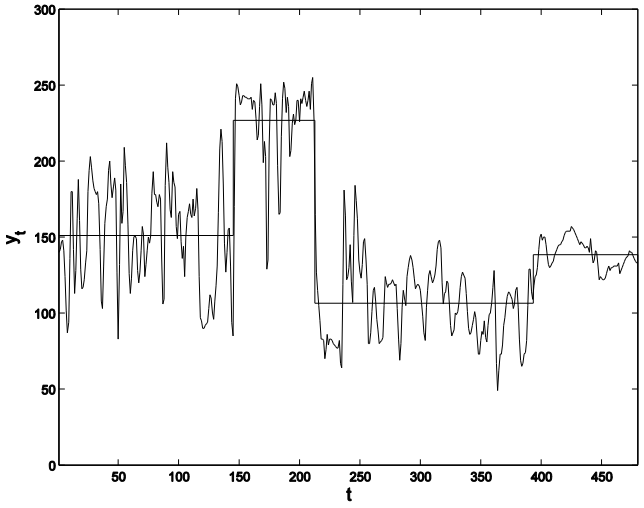

Figure 7. Results of Real Multiplicative Signal Segmentation

4. Conclusion

The above description was a theoretical study of Reversible Jump MCMC algorithm and their applications for segmenting signal models with multiplicative noise. From the simulation results showed that Reversible Jump MCMC algorithm can segment the signal well.

As an algorithm implementation, real signal was drawn from columns in a natural scene around the Imogiri Tomb, Yogyakarta Indonesia. If the Reversible Jump MCMC algorithm is implemented on each row or column of the image it will generate segmentation of the image. The advantage of this method is that the number of segments, the position of the segment change, and the amplitude are estimated simultaneously. The development of a Reversible Jump MCMC algorithm to segment images directly will be an interesting research topic.

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