# Cost Allocation of Transmission Losses in Electric Market Mechanism

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## Abstrak

Tulisan ini mengusulkan sebuah metoda baru untuk menghitung alokasi biaya rugi-rugi tramsmisi, berdasarkan pada suatu harga energi tertentu. Suatu model matematik dalam paper ini dikembangkan dengan memanipulasi persamaan jaringan guna memisahkan rugi-rugi dalam jaringan transmisi. Model ini menggunakan injeksi daya komplek dan tidak menggunakan pendekatan dan asumsi dalam menentukan alokasi biaya rugi-rugi. Perhitungan dimulai dari hasil perhitungan aliran daya kemudian diteruskan untuk menghitung distribusi daya dari sebuah generator ke setiap beban dan saluran. Akhirnya, perhitungan secara terpisah dilakukan untuk mendapat rugi-rugi dan alokasi biaya rugi rugi tersebut. Metoda yang diusulkan ini mudah dimengerti dan diterapkan. Hasil uji coba pada sistem IEEE 14-bus menunjukan bahwa metoda ini konsisten sesuai perkiraan namun sedikit berbeda dengan beberapa metoda yang menjadi acuan.

Kata kunci: alokasi biaya, harga marjinal, pemisahan rugi-rugi, transmisi

#### Abstract

This paper proposes a new method to calculate cost allocation of transmission losses based on a certain price of energy. A mathematic model is developed by manipulating of the network equation to separate losses. This model uses complex power injection and, does not use approximations and assumptions in determining the cost allocation of losses. The calculation begins from the results of load flow calculation and it is continued to calculate power distribution from a generator to every load and line. Finally, the separating of losses and cost allocation of losses are calculated. The proposed method is easy to be understood and applied. An illustration results on IEEE 14-bus system show that the method is consistent with expectancies and slightly different from several referenced methods.

Key words: cost allocation, marginal price, separation of losses, transmission

## 1. Introduction

Transmission network (transmission) is a main component of electric power system because its function is to transmit energy from sources to loads. In the market machanisme, transmission is open access so that cost allocation of transmission usage is an important issue. Determination of cost transmission was proposed in some publications [1-5], but cost allocation of transmission losses does not get faily formulation. In this paper will propose a new method to determine cost allocation of transmission losses.

Transmission losses (or losses) in electric network (in service) always exist because wire resistance can't be ignored. In practice it is shown that losses can reach 10% of total generated power. On monopoly system (integrated system), a cost allocation of losses has not become a serious attention. However, in competitive system, cost allocation has become a problem that has to be fairly solved. The problem is how to fairly share losses to all competitive participants, both as supplier (generator) and as demand (load).

A few methods [2] and [6], of the cost allocation of losses have been published in some references like pro-rata, proportional sharing (PS), incremental transmission loss (ITL) and Z-

bus. The descriptions below are general features of the methods. Pro-rata technique is a general method that has been applied in a few countries. Allocated loss component is based on active power level that is injected to bus from generator or load. In this method, bus location in transmission does not influence to calculation results. As an example of electric market used the method can be found at Mainland Spanish [7], England and Wales [8].

The cost allocation of losses of the ITL method [8], is based on change of loss coefficients that have three characteristics. First, distribution losses of the ITL method can have positive or negative value, and it indicates as a cross subsidy. Second, cost allocation of losses depends on the choice of swing generator. Third, strike application of the coefficients can cover more losses than the actual ones.

The PS method [7] and [9], is the simplest method to determine the cost allocation of losses. It is only to satisfy the first Kirchhoff law and need some other assumptions. Its principle is proportional power sharing with an assumption that losses are divided proportionaly among the entire power injection at each bus in the grid. Then, the cost of losses is allocated to all generators and all loads with the share figure of 50-50.

The Z-bus method [10], begins from the solving of load flow and its next step is to determine distribution of losses at each bus that is based on bus impedance, where the bus impedance is obtained from bus admittance matrix.

Distribution of losses is an important matter that has to be determined in electric business in competitive market. Principally, it is very difficult to determine the distribution of losses that should be shared by all generators and all loads because the losses are function of quadratic current. Until now, it is to be a fact that losses can't be separated because of its function. So, fair and perfect calculation in the cost allocation of losses has never been published.

In the reference [10], a few characteristics are needed in order to achieve the aim, i.e. to produce fair cost allocation of losses. Generally, the characteristics have to meet six conditions as followed:

- (i) To reflex power or current injection at each bus.
- (ii) To reflex relative position of each bus in the transmission.
- (iii) To reflex both network topology and voltage-current correlation.
- (iv) More simple to be understood and implemented.
- (v) Able as incentive or disincentive to generator and load, it is depended on power capacity and its location in the transmission.
- (vi) Have to be consistent with the calculation result of load flow.

In this paper, it has been proposed a new method to calculate cost allocation of losses that has to be endured by each generator and each load. It was presented in this paper. The developed method has met the six conditions above.

#### 2. Research Method

As an illustration of cost calculation of losses, it is used a simple system as seen in Figure 1. From this figure, generator at bus-1 (G1) produces power of 116 MW, the 50 MW of which is absorbed directly by load at bus-1 (L1). Then the power of 66 MW is sent to load at bus-2 (L2), and 60 MW of it is received by L2. The generator at bus-2 (G2) generates power of 40 MW, which is absorbed directly by L2. So, this system produces losses of 6 MW. The losses are only caused by G1 and L2, where L1 and G2 do not cause losses. If it is used portion 50-50 between generators and loads, distribution losses in the illustration are 3 MW for each G1 and L2, and 0 MW for G2 and L1.

To determine the cost allocation of losses in the proposed method, it is begun from the calculation result of load flow, [11]. Next step is to distribute losses to all generators and all loads, and then to determine the losses distribution for each load if grid is supplied by one generator. For m-bus are connected to generators, distributed losses is

$$P_t^r = \sum_{i=1}^m P_{Gi}^r \tag{1}$$

where:  $P_t^r$  is total losses.  $P_{Gi}^r$  is loss for generator at bus-i.



From the load flow calculation results (the base case) it can be known that the total injection current for bus-i, i.e. consists of generator and load current. It is expressed as

$$I_i = I_i^G - I_i^D \tag{2}$$

where:  $I_i$  is total current at bus-i  $I_i^G$  is generator current at bus-i  $I_i^D$  is load curent at bus-i.

Current flows to load at bus-i is

$$I_i^D = \left(\frac{S_i^D}{V_i}\right)^* = y_i^D V_i \tag{3}$$

where:  $S_i^D$  is complex power of load at bus-i.

 $V_i$  is voltage at bus-i.

$$y_{i}^{D} = \frac{P_{i}^{D} - jQ_{i}^{D}}{\left|V_{i}\right|^{2}}$$

Whereas bus current in the matrix form is

$$\begin{bmatrix} I_{bus} \end{bmatrix} = \begin{bmatrix} Y_{bus} \end{bmatrix} \begin{bmatrix} V_{bus} \end{bmatrix}$$
(4)

where:  $I_{bus}$  is a bus current vector.  $Y_{bus}$  is a bus admittance matrix. V<sub>bus</sub> is a bus voltage vector.

Equation (1) that had been changed into matrix and then it is substituted into equation (3) and yield

$$\left[I_{bus}^{G} - I_{bus}^{D}\right] = \left[Y_{bus}\right] \left[V_{bus}\right]$$
(5)

or

$$\begin{bmatrix} I_{bus}^G \end{bmatrix} = \begin{bmatrix} Y_{bus} \end{bmatrix} \begin{bmatrix} V_{bus} \end{bmatrix} + \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} I_{bus}^D \end{bmatrix}$$
(6)

where: [I] is a unit matrix

The following step, equation (2) that had been changed to matrix form is substituted into equation (6) and yield

$$\begin{bmatrix} I_{bus}^G \end{bmatrix} = \begin{bmatrix} Y_{bus} \end{bmatrix} \begin{bmatrix} V_{bus} \end{bmatrix} + \begin{bmatrix} y_{bus}^D \end{bmatrix} \begin{bmatrix} V_{bus} \end{bmatrix}$$
(7)

From the equation (7) is yielded voltage bus, that is

$$\begin{bmatrix} V_{bus}^G \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{bus}^{\#} \end{bmatrix} \begin{bmatrix} I_{bus}^G \end{bmatrix}$$
(8)

where:  $[Z_{bus}^{\#}] = ([Y_{bus}] + [y_{bus}^{D}])^{-1}$ 

From last the equation is applied superposition technique to determine current flow in all lines that is sent by individual generator's current. For example, for generator's current at bus-i, the equation (8) yields equation (9).

$V_1^{Gi}$	Γ	$Z_{11}^{*}$	$Z_{12}^{\#}$	٠	٠	٠	$Z_{1n}^{\#}$	0	
		$Z_{21}^{\#}$	$Z_{21}^{\#}$	٠	٠	•	$Z_{2n}^{\#}$	0	
•		•	•	•	٠	•	0	0	
•	=	•	•	٠	٠	•	0	$I_i^G$	
•		•	٠	•	•	•	0	0	
$V_n^{Gi}$		$Z_{n1}^{\#}$	$Z_{n2}^{\#}$	•	٠	•	$Z_{nn}^{\#}$	0	

Equation (9) is a superposition technique solution to calculate voltage bus that is only influenced by the generators at bus-i. It can be derived from the equation (9) that yields equation (10).

$$V_i^{Gi} = Z_{ii}^{\#} I_i^G \tag{10}$$

then can be calculated generator's current at bus-i that is received by load at bus-j, that is

$$I_j^{Di} = V_j^{Gi} y_j^D \tag{11}$$

Finally, contribution power from generator at bus-i to load at bus-j can be calculated as this following:

$$Sd_{j}^{Gi} = V_{j} \left( I_{j}^{Di} \right)^{*}$$
(12)

where:  $Sd_{j}^{Gi}$  is complex power at bus-j from generator at bus-i.  $V_{j}$  is voltage at bus-j from the base case.

From equation (12) is got active power, that is

$$Pd_{i}^{Gi} = \text{Real}\left(Sd_{i}^{Gi}\right) \tag{13}$$

So, from equation (13) is got active power contributions of generator at bus-i to all loads that is written in the equation (14) below.

$$Pg_{i}^{D} = \sum_{j=1}^{n} Pd_{j}^{Gi}$$
(14)

where:  $Pg_i^D$  is total active power of generator at bus-i that reaches to all loads.

 $Pd_{j}^{Gi}$  is active power from generator at bus-i that reachs to load at bus-j. *n* is bus number.

By equation (14) can be calculated total power for each generator that reaches to all loads, so that losses can be separated. For example, for generator at bus-i and the resulted total losses is

$$\Pr_i^G = Pg_i^D - Pg_i \tag{15}$$

where:  $Pg_i$  is active power injection to bus-i, it is get from the base case.  $Pg_i^D$  is total active power that is received by all loads, it is only supplied by generator at bus-i.

If  $\lambda$  is energy price, distribution cost allocation of losses can be determined with the following results. For generator-i:

$$Br_i^G = \frac{1}{2} \Pr_i^G \lambda$$
(16)

For load-j:

$$Br_j^D = \sum_{i=1}^m \frac{Pd_j^{Gi}}{2Pg_i^D} \lambda$$
(17)

# 3. Results and Analysis

#### 3.1. Case study

A cost allocation of losses method that was proposed, a new method, it has been tested and its result was compared with a few alternative methods from references. In this paper was took a case study, i.e. typical IEEE 14-bus system, [10], with two different supply conditions. The used evaluation in the reference methods are based on unit power in MW and their cost allocation in unit dollar per-hour, for an energy price 50 \$/MWH.

	Losses 13.5 MW and $\lambda$ =50 \$/MWh							
	Methods							
#	Pg	Pd	new	Z-bus	Pro- P	-rata I	Ps	ITL
	[MW]	[MW]	[\$/h]	[\$/h]	[\$/h]	[\$/h]	[\$/h]	[\$/h]
1	232,7	0,0	344,7	382	323	275	324	307
2	40,0	21,7	-5,6	8	25	32	15	48
3	0,0	94,2	136,0	139	131	116	144	146
4	0,0	47,8	69,9	42	66	58	63	63
5	0,0	7,6	11,3	4	11	9	8	9
6	0,0	11,2	9,2	24	16	51	12	16
7	0,0	0,0	0,0	0	0	0	0	0
8	0,0	0,1	-4,4	1	0	33	0	0
9	0,0	29,5	45,5	26	41	41	39	34
10	0,0	9,0	14,0	9	12	13	14	10
11	0,0	3,5	5,4	3	5	5	5	4
12	0,0	6,1	9,1	5	8	7	8	9
13	0,0	13,5	20,5	13	19	17	19	16
14	0,0	14,9	22,4	22	21	19	26	16
tot	272,7	259,1	678,0	678,0	678,0	678,0	678,0	678,0

Table 1 consists of the calculation results for first condition, i.e. generator at bus-1 generates 232.7 MW and generator at bus-2 generates 40.0 MW. In this condition was

produced total losses in the network is 13.5 MW. Column-1 consists of bus number, column-2 consists of generated power of individual generator at each bus, column-3 consists of capacity of individual load at each bus, and column-4 up to column-9 is the calculation results of the 6 methods. For all methods use same unit marginal price, i.e. 50 \$/MWH.

Table 2 consists of cost allocation calculation results of losses on the second condition, i.e. added generator at bus-8 and in this condition occur total losses 6.2 MW. Aim of the second condition is to look influence cost allocation of losses against spreading of generators in the system.

		Methods						
#	Pg	Pd	Now	7 huo	Pro-rata		DC	171
			new	Z-bus	Р	I	FJ	112
	[MW]	[MW]	[\$/h]	[\$/h]	[\$/h]	[\$/h]	[\$/h]	[\$/h]
1	125,3	0,0	84,5	116	80	72	111	90
2	40,0	21,7	0,3	4	12	11	11	26
3	0,0	94,2	61	124	60	57	92	79
4	0,0	47,8	31	13	31	28	17	25
5	0,0	7,6	4,9	1	5	5	4	4
6	0,0	11,2	8,1	23	7	25	8	10
7	0,0	0,0	0	0	0	0	0	0
8	100,0	0,1	69,2	-9	64	59	32	44
9	0,0	29,5	18,9	3	19	20	0	7
10	0,0	9,0	5,7	3	6	6	1	3
11	0,0	3,5	2,2	1	2	2	2	2
12	0,0	6,1	3,9	5	4	4	6	5
13	0,0	13,5	8,7	11	9	9	16	8
14	0,0	14,9	9,6	15	10	9	8	5
tot.	265,3	259,1	308	308	308	308	308	308

Table	2. Calculation results of 6 methods on the second condition
	Losses 6.2 MW and $\lambda = 50$ \$/MWb

## 3.2. Analyses

Table 1 and Table 2 consist of cost allocation component of losses for each bus from different methods. The all methods use an energy price, 50 \$/MWH. Column-4 until column-9 consists of cost allocation of losses for each bus from the 6-methods. The brief definitions of the methods such as New, Z-bus, pro-rata, PS, ITL.

From the above table s show that calculation result of pro-rata and PS method is always having positive value. It is caused by results of two methods only influenced of injected power or current to system without caring its direction. Results of ITL and Z-bus method can be negative or positive value; it is depended on characteristic and condition of system. Both Table 1 and Table 2, ITL method gives positive value for all buses. But Z-bus method on the Table 1 gives negative value at bus-8, i.e. -9 \$/h. whereas, the new method in the Table 1 has negative value at the bus-2, i.e. -5.6 \$/h, and at the bus-8, i.e. -4.4 \$/h.

In the Table 1, generator in bus-1 supplies 85% of total power of the system, so the generator has always maximum cost allocation of losses among all methods. The next maximum is bus-3 that has load 36% of total load of the system. The new method allocates 136 \$/h to bus-3 and light different with result of Z-bus method, i.e. 139 \$/h. Most different results are occurred at bus-2 and bus-8, where the new method allocates negative cost and other methods allocate positive cost. At bus-2 is generated power 40.0 MW and load at this bus is 21.7 MW, so total power that is sent to grid is 18.3 MW. In fact, the sent power from bus-2 causes to reduce losses, so that responsibility of bus-2 to losses is negative value. It is also occurred at bus-8 with load 0.1 MW that is supplied dominant from generator at bus-2 with negative allocation losses. Whereas other buses have variation cost allocation of losses that depend on applied method, the case study has shown very much different that is enough significant relatively.

Table 2 is calculation results of second condition, i.e. new generator is connected to bus-8 of the 14-bus system with output power 100 MW and the others have been maintained like Table 1. Objective of this case is to look influencing of cost allocation of losses when

generators spread in the grid. The consequence, total losses will go down drastically under 50% (6.2 MW). Reduction of the losses is also able to change cost allocation of losses each bus with enough significant. For example, generator in bus-1 that generates power 47% of total power of system, where Z-bus method gives cost allocation of losses -9 \$/h (negative value). Whereas the new method gives cost allocation of losses 69.2 \$/h (positive value) and it is not far different with the others, unless Z-bus method.

The new method focuses to separate losses, where the losses will be distributed to each generator and each loads. Generally, other methods focus to current or power injection into bus. There is different focus here that give results of cost allocation of losses is enough different signification from the new method. Strength of this new method is good illustration mathematically to separate losses without approximations and assumptions that is expressed by equation (15). This equation is also depended on complex power of all loads, complex power of all generators and grid characterization.

From the case study, portion of cost allocation of losses (between generators and loads) is variation, like to be shown by Table 3 and Table 4, just the new method and ITL method have balance portion. From two cases above, Z-bus method has different portion between case-1 and case-2, i.e. in the case-1 Table 3 has 58-42 portion and in the case-2 Table 4 has 36-48 portion. The others is not changed their portion between case-1 and case-2.

Table 3. Comparison of portion from cost allocation of losses between generators and loads for case-1 (losses of system is 13.5 MW)

No.	Method	Generators [%]	Loads [%]
1	New	50	50
2	Z-bus	58	42
3	Pro-rata:P	51	49
4	Pro-rata:I	45	55
5	PS	50	50
6	ITL	52	48

Table 4. Comparison of portion from cost allocation of losses between generators and loads for case-2 (losses of system is 6.2 MW)

No.	Method	Generators [%]	Loads [%]
1	New	50	50
2	Z-bus	36	64
3	Pro-rata:P	51	49
4	Pro-rata:I	46	54
5	PS	50	50
6	ITL	52	48

In the fact, the generated power by all generators is more than the received power by all loads. Here is occur unbalance power between generators and loads that is caused by losses. This will influence portion of cost allocation of losses between generators and loads. The 50-50 portion can be used with reason that losses is caused by generators and loads, it was explained by Figure 1 above.

#### 4. Conclusion

A new method for determining cost allocation of losses that can be used in the electric business through market mechanism, both competition and contract bilateral, had been proposed and tested. The basic calculation is based on manipulating of network equation that starts from results of the load flow solution. This method is consistent with the six conditions above that accommodate complex power of generators and loads, and line complex impedance. In this method is also shown separating of losses both among generators and among loads, it is solved without using approximations and assumptions. This is strength of the proposed method, where separating of losses has not been able to be solved perfectly [2]. One addition again, this

method has simple formulation and easy applied like that be shown by solution of two case studies above.

The case studies show that the proposed method produces cost allocation of losses that consistent with expectation, both for each generator and load. Cost allocation of losses on the method (proposed method) is enough significant different when it is compared with the other methods, mainly Z-bus method. But portion (of cost allocation of losses between generators and loads) is very close to three methods, i.e. Pro-rata method, PS method and ITL method. The four methods have portion that is not changed although different case applied like that is shown by Table 3 and Table 4.

The portion will influence calculation result of the proposed method. Determining the portion between generators and loads needs a consideration fairly. This paper uses 50-50 portion (proposed by reference methods) that can be received with a reason that generators and loads cause all together losses, like that be shown by Figure 1.

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