# Finite Element Approach for Coupled Striplines Embedded in Dielectric Material

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#### Abstrak

Pada makalah ini, metode elemen hingga (FEM) dihadirkan untuk analisis kuasi-statis dua dimensi (2D) dua struktur stripline tergandeng dan terlindung pada perangkat mikroelektronik. Pada metode yang diusulkan, nilai kapasitansi per satuan panjang dan induktansi per satuan panjang dari dua striplines tergandeng secara vertikal terlindung dan dua striplines tergandeng terlindungi yang ditanam material dielektrik ditentukan secara khusus. Hasil simulasi yang diperluas disajikan, dan beberapa hasil perbandingan dengan metode lain diberikan, dan ditermukan bahwa metode yang diusulkan memiliki unjuk kerja yang baik. berada dalam perjanjian baik. Lebih dari itu, spektral kuasi-TEM untuk distribusi potensi kedua striplines terlindung dan tergandeng juga ditentukan.

*Kata kunci:* metode elemen hingga, striplines tergandeng dan terlindungi, kapasitansi per satuan panjang, induktansi per satuan panjang

#### Abstract

In this paper, we present finite element method (FEM) to investigate the quasi-static analysis of two dimensional (2D) shielded two coupled stripline structures for microelectronic devices. In the proposed method, we specifically determine the values of capacitance per unit length and inductance per unit length of shielded two vertically coupled striplines and shielded two coupled striplines embedded in dielectric material. Extensive simulation results are presented, and some comparative results are given by other methods and found them to be in excellent agreement. Furthermore, we determine the quasi-TEM spectral for the potential distribution of these shielded two coupled striplines.

**Keywords**: Finite element method, shielded coupled striplines, capacitance per unit length, inductance per unit length

## 1. Introduction

Coupled striplines modeling for advanced microelectronic devices is a growing area of interest. Today, the desire of computational accuracy in planar microwave devices and circuits becomes an essential part for engineers and designers. The computation of the capacitance per unit length and inductance per unit length of quasi-static shielded coupled stripline are of considerable significance in the design of microwave circuits.

Several methods used for analyzing the coupled striplines structure include the method of lines (MoL) [1-2], the moment method (MoM) [3], the integral-equation technique (IET) [4], the Green's Functions [5], the method of images [6], the equivalent electrodes method (EEM) [7].

In this work, we propose the finite-element method (FEM) in the quasi-static analysis of the shielded two vertically coupled striplines and shielded two coupled striplines embedded in dielectric material. We show that FEM is especially suitable for the computation of electromagnetic fields in strongly inhomogeneous media. Furthermore, it has high computation accuracy and fast computation speed [8-11]. Although, we use FEM to compute the capacitance per unit length, inductance per unit length, of quasi-static shielded coupled stripline, and then compare some of the results of our modeling with the previous methods, then identify the potential distribution spectra of the integrated circuits.

## 2. Theory for the Problem Formulation of Coupled Striplines in Dielectric Media

The models are designed in two-dimensional (2D) using electrostatic environment in order to compare our results with some of the other available methods. In the boundary

condition of the model's design, we use ground boundary which is zero potential (V=0) for the shield. We use port condition for the conductors to force the potential or current to one or zero depending on the setting. Also, we use continuity boundary condition between the conductors and between the conductors and left and right grounds.

The quasi-static models are computed in form of electromagnetic simulations using partial differential equations. Recently, with the advent of integrated circuit technology, the coupled striplines consisting of multiple conductors embedded in a multilayer dielectric medium have led to a new class of microwave networks. Multiconductor transmission lines have been utilized as filters in microwave region which make it interesting in various circuit components. For coupled multiconductor striplines, it is convenient to write:

$$Q_i = \sum_{j=1}^{m} C_{sij} V_j$$
 (i = 1, 2, ...., m) (1)

where  $Q_i$  is the charge per unit length,  $V_j$  is the voltage of j th conductor with reference to the ground plane,  $C_{sij}$  is the short circuit capacitance between i th conductor and j th conductor.

The short circuit capacitances can be obtained either from measurement or from numerical computation. From the short circuit capacitances, we obtain

$$C_{ii} = \sum_{j=1}^{m} C_{sij}$$
<sup>(2)</sup>

where  $C_{ii}$  is the capacitance per unit length between the *i* th conductor and the ground plane. Also,

$$C_{ij} = -C_{sij}, \qquad j \neq i \tag{3}$$

where  $C_{ij}$  is the coupling capacitance per unit length between the *i* th conductor and *j* th conductor. The coupling capacitances are illustrated in Fig. 1.



Figure 1. The per-unit length capacitances of a general m-conductor transmission line

For m-strip line, the per-unit-length capacitance matrix [C] is given by

$$[C] = \begin{bmatrix} C_{11} & -C_{12} & \cdots & -C_{1m} \\ -C_{21} & C_{22} & \cdots & -C_{2m} \\ \vdots & \vdots & & \vdots \\ -C_{m1} & -C_{m2} & \cdots & C_{mm} \end{bmatrix}$$
(4)

The inductance and capacitance per unit length of multiconductor transmission lines are related as

$$[L] = \mu_0 \varepsilon_0 [C_0]^{-1}$$
<sup>(5)</sup>

where [L] is the inductance matrix,  $\mathcal{E}_0$  the permittivity of free space or vacuum,  $\mu_0$  the permeability of free space or vacuum, and  $[C_0]^{-1}$  the inverse matrix of the capacitance of the multiconductor transmission line when all dielectric constants are set equal one.

# 3. Results and Discussion

Conductors cannot be considered to have negligible thickness in packaging structures, printed circuit boards and many microwave structures. Their thickness has to be taken into account for the calculation of capacitance in order to give accurate results. The accuracy is essential in studying the interactions of transmission lines to obtain useful information for the ability of the line to transmit signals at high frequencies (in GHz region) with low distortion and minimum crosstalk. Therefore, we choose to use very small thickness (0.01 mm) for all the conductors.

The significant advantages of printed circuits are somewhat offset by the electromagnetic complexity of the structure, because its inherent inhomogeneous nature makes accurate calculations difficult. We compared our results with the integrated equations technique (IET), the method of moment (MoM) and the method of lines (MoL).

In this paper, we consider two different models. First case, investigates the modeling of the shielded two vertically coupled striplines embedded in dielectric material. We identify the quasi-TEM spectral for the potential distribution of the designed model and its mesh. Also, we computed the values of capacitances and inductance per unit length the model. Indeed, we compare the values of capacitances with the other methods. For second case, we illustrate the modeling of shielded two coupled striplines embedded in dielectric material. We identify the quasi-TEM spectral for the potential distribution of the designed model and its mesh. In addition, we computed the values of capacitances and inductance per unit length. Also, we compare the capacitances with the other methods.

# 3.1. Shielded Two Vertically Coupled Striplines Embedded in Dielectric Material

In this section, we illustrate the modeling of the shielded two vertically coupled striplines embedded in dielectric material by focusing on the potential distribution and meshing. Furthermore, we compute the capacitance per unit length and inductance per unit length of the coupled striplines.

In Figure 2, we show the cross section for shielded two vertically coupled striplines and the geometry is enclosed by a 3.4 X 1mm shield with the following parameters:

 $\mathcal{E}_r$  = dielectric constant = 1 and 9.5

 $W_1$  = width of the stripline 1 = 1.4mm

 $W_2$  = width of the stripline 2 = 1mm

 $H_1$  = height from stripline 1 and stripline 2 to the upper side and lower side of the shield respectively = 0.4mm

 $H_2$  = distance between the two striplines = 0.2mm

S = distance between the stripline 1 and right/left side of the shield = 1mm

 $a = (W_1 - W_2)/2 = 0.2mm$ 

t = thickness of the striplines = 0.01mm



Figure 2. Cross section of the shielded two vertically coupled striplines embedded in dielectric material

From the model, we generate the finite element mesh plot as in Figure 3. Table 1 shows the statistical properties of the mesh.



Figure 3. Mesh plot of the shielded two vertically coupled striplines embedded in dielectric material

Table 1. Mesh statistics of the shielded two vertically coupled striplines embedded in
dielectric material

Items	Value
Number of degrees of freedom	9789
Total Number of mesh points	2260
Total Number of elements	4466
Triangular elements	4466
Quadrilateral	0
Boundary elements	402
Vertex elements	12

Figure 3 shows the 2D surface potential distribution for the model; while the contour plot is presented in Figure 4. Figure 5 presents the comparison analysis of potential distribution of the model with and without dielectric substrate. It observed that the peak value of electric potential is same as the dielectric is placed in the substrate when we use the first conductor as input.



Figure 3. 2D surface potential distribution of the shielded two vertically coupled striplines embedded in dielectric material



Figure 4. Contour plot of the shielded two vertically coupled striplines embedded in dielectric material

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Figure 5. Potential distribution of the shielded two vertically coupled striplines embedded in dielectric material from (x, y) = (0, 0) to (x, y) = (3.4, 1) mm

Table 2 shows the finite element results for the capacitance per unit length of shielded two vertically coupled striplines compared with the work of previous investigation using integrated equation technique (IET) and method of lines (MoL). They are in good agreement.

Table 2. Values of the capacitance coefficient (in F/m) for shielded two vertically coupled

stri	plines in air ( $\mathcal{E}_{o}$ =	$= 8.854 \times 10^{-12} F / n$	<i>n</i> )
Capacitance	IET [Ref. 4]	MoL [Ref. 1]	Our work
$C_{_{11}}$ / $arepsilon_{_o}$	11.434	11.4142	11.9557
$C_{_{12}}$ / $arepsilon_{_o}$	-5.954	-5.9672	-6.3256
$C_{21}$ / $arepsilon_o$	-5.954	-5.9672	-6.3256
$C_{_{22}}$ / $arepsilon_{_o}$	9.303	9.3305	9.7124

We extend the model by computing the capacitance coefficient (in pF/m) and inductance (in  $\mu$  H/m) for shielded two vertically coupled striplines in dielectric material ( $\varepsilon_r = 9.5$ ) as shown in Table 3.

Table 3. Values of the capacitance coefficient (in pF/m) and inductance (in  $\mu$  H/m) for shielded two verticelly coupled striplings in dielectric material (c = 9.5)

two vertically coupled striplines in dielectric material ( $\varepsilon_r = 9.3$ )			
Capacitance	Our work	Inductance	Our work
$C_{11}$	1006.015	$L_{11}$	0.1602
$C_{12}$	-532.3175	$L_{12}$	0.1043
$C_{21}$	-532.3175	$L_{21}$	0.1043
$C_{22}$	817.3067	$L_{22}$	0.1971

# 3.2. Shielded Two Coupled Striplines Embedded in Dielectric Material

In this section, we illustrate the designing and modeling of the shielded two coupled striplines embedded in dielectric material by focusing on the calculation of capacitance and inductance per unit length. In Figure 6, we show the cross section for shielded two coupled striplines and the geometry is enclosed by a 16 X 3mm shield with the following parameters:

 $\mathcal{E}_r$  = dielectric constant = 9.5

 $W_1$  = width of the stripline 1 = 3mm

 $W_2$  = width of the stripline 2 = 3mm

 $H_1$  = height from stripline 2 to lower side of the shield = 0.5mm

 $H_2$  = distance between the two striplines = 0.5mm

 $H_3$  = height from stripline 1 to the upper side of the shield = 2mm

 $S_1$  = distance between the stripline 1 and right side of the shield and the distance between the stripline 2 and the right side of the shield = 4mm

 $S_2$  = distance between the striplines = 2mm

t = thickness of the striplines = 0.01 mm

From the model, we generate the finite element mesh plot as in Figure 7. Table 4 shows the statistical properties of the mesh. While, Figure 8 shows the 2D surface potential distribution for the coupled stripline; although the contour plot is presented in Figure 9 of the model. Figure 10 presents the electric potential plot as a function of arc-length.



Figure 6. Cross section of the shielded two coupled striplines embedded in dielectric material



Figure 7. Mesh of the shielded two coupled striplines embedded in dielectric material

Table 4. Mesh statistic	s of the shielded two	vertically	coupled	striplines	embedded	d in
	dielectric mat	erial				

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Items	Value
Number of degrees of freedom	5120
Total Number of mesh points	1233
Total Number of elements	2283
Triangular elements	2283
Quadrilateral	0
Boundary elements	185
Vertex elements	12



Figure 8. 2D surface potential distribution of shielded two coupled striplines embedded in dielectric material



Figure 9. Contour plot of the potential distribution of shielded two coupled striplines embedded in dielectric material with port 1 as input



Figure 10. Potential distribution of shielded two coupled striplines embedded in dielectric material using port 1 as input along a line from (x, y) = (0, 0) to (x, y) = (16, 3) mm

Table 5 shows the finite element results for the capacitance per unit length of shielded two coupled striplines compared with the work of previous investigations using the method of moment (MoM) and the method of lines (MoL). They are in good agreement.

Table 5. Values of the capacitance coefficient (in F/m) for shielded two coupled striplines

	ma	aterial	
Capacitance	MoM [Ref. 3]	MoL [Ref. 1]	Our work
$C_{_{11}}$ / $\varepsilon_{_o}$	60.4924	59.2746	60.5640
$C_{12}$ / $arepsilon_o$	-1.0447	-1.0567	-1.1740
$C_{_{21}}$ / $arepsilon_{_o}$	-1.0447	-1.0567	-1.1740
$C_{_{22}}$ / $arepsilon_{_o}$	88.4797	88.1186	88.6231

We computed inductance per unit length of the model computing and compare the results with the MoM and MoL methods, we found then in excellent agreement as shown in Table 6.

	mai	enal	
Capacitance	MoM [Ref. 3]	MoL [Ref. 1]	this work
$L_{\!_{11}}$ / $\mu_o$	0.16178	0.1603	0.1569
$L_{ m 12}$ / $\mu_o$	0.0019	0.0019	0.0021
$L_{ m 21}$ / $\mu_o$	0.0019	0.0019	0.0021
$L_{ m 22}$ / $\mu_o$	0.1106	0.1078	0.1072

Table 6.	Values of the inductance (in H/m) for shielded two coupled striplines in dielectric		
matorial			

#### 4. Conclusion

In this paper, we have presented the quasi-static of two-dimensional shielded two vertically coupled striplines and shielded two coupled striplines embedded in dielectric material using FEM. The results obtained using finite element method for the capacitance per unit length and inductance per unit length agrees well with those found in the other methods. We found them to be very close. In addition, we determine the quasi-TEM spectral for the potential distribution of the models.

#### References

- [1] Zitouni A, Bourdoucen H, Djoudi TN. Quasi-static Mol-based Approach for the Analysis of Multilayer Transmission Line Structures. *International Journal of Numerical Modeling: Electronic Networks, Devices and Fields.* 1997; 10: 209-216.
- [2] Papachristoforos A. *Method of Lines for Analysis of Planar Conductors with Finite Thickness*. IEE Proc. Microwave Antennas and Propagation. 1994; 141(3): 223-228.
- [3] Wei C, Harrington RF, Mautz JR, Sarkar TK. Multiconductor Transmission Lines in Multilayered Dielectric Media. *IEEE Transaction on Microwave Theory and Techniques*. 1984; 32(4): 439-450.
- [4] Kammler DW. Calculation of Characteristic Admittances and Coupling Coefficient for Strip Transmission Lines. *IEEE Transaction on Microwave Theory and Techniques*.1968; 16(11): 925-937.
- [5] Li YL, Liu CH. Simplified Green's Functions for Calculating Capacitance and Inductance of Multiconductor Transmission Lines in Multilayered Media. IEE Proc. Microwave Antennas and Propagation. 1994; 141(2):141-144.
- [6] Bontzios YI, Dimopoulos MG, Hatzopoulos AA. An Evolutionary Method for Efficient Computation of Mutual Capacitance for VLSI Circuits based on the Method of Images. *Simulation Modeling Practice* and Theory. 2011; 19: 638-648.
- [7] Milovanovic A, Kopprivica B. Calculation of Characteristic Impedance of Eccentric Rectangular Coaxial Lines. *Przeglad Elektrotechniczny (Electrical Review)*. 2012; 88: 260-264.
- [8] Illias HA, Bakar AHA, Mokhlis H, Halim SA. Calcualtion of inductance and Capacitance in Power System Transmission Lines using Finite Element Analysis Method. *Przeglad Elektrotechniczny* (*Electrical Review*). 2012; 88: 278-283.
- [9] Musa SM, Sadiku MNO, Clark JV. Finite Element Analysis for Electromagnetic Parameters of Multiconductor Interconnects in Multilayered Dielectric Media. International Journal of Research and Reviews in Computer Science. 2011; 2(6): 1300-1304.
- [10] Musa SM, Sadiku MNO. Computer-aided Modeling of Circular Conductors. *Microwave and Optical Technology Letters*. 2009; 51(5): 1366-1372.
- [11] Musa SM, Sadiku MNO. Finite Element Analysis of Multiconductor Transmission Lines Multilayered Dielectric Media," Microwave and Optical Technology Letters. 2008; 50(10): 2743-2747.