# Nine-Phase Induction Motor Dynamic Model Based On 3x9 Transformation Matrix 

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#### Abstract

Abstrak Analisis model dinamik motor induksisembilan fasa menjadi sulit karena berbasis pada matrik $9 \times 9$ dan sebuah rangkaian trafo terkopel secara magnetik. Agar model dinamik dalam bentuk qdn pada motor induksi sembilan fasa diperoleh dengan mudah, sederhana, cepat dan konsisten, maka analisis motor berbasis pada matrik transformasi $3 \times 9$ dan rangkaian ekuivalen motor model T. Matrik transformasi $3 \times 9$ disubstitusikan ke persamaan dalam bentuk abc agar diperoleh persamaan matrik dalam bentuk qdn. Persamaan qdn menghasilkan rangkaian ekivalen qd yang sama dan rangkaian ekivalen $n$ berbeda dengan metode yang berbasis pada rangkaian trafo terkopel secara magnetik. Hasil simulasi menunjukkan bahwa karakteristik dinamik motor induksi 9 -fasa yang berbasis pada model $T$ dan rangkaian terkopel secara magnetik mempunyai respon dinamik yang sama untuk torka dan kecepatan.


Kata kunci: Matrik qdn ordo 3x9, rangkaian ekuivalen trafo model T, model dinamik qdn


#### Abstract

Analysis of the dynamic model nine-phase induction motor becomes difficult because based on a 9x9 matrix and a circuit of magnetically coupled transformer. In order to gain the qdn dynamic model of the nine-phase induction motors in easy, simple, quick and consistent way, the motor analysis will based on $3 \times 9$ transformation matrix and the equivalent circuit $T$ model. The $3 \times 9$ transformation matrix qdn is substituted into the equation in form abc so that the matrix equation of qdn is obtained. Then qdn equation results a similar qd equivalent circuit, which has different methods $n$ that is, based the circuit of magnetically coupled transformer. Simulation results show that the dynamic characteristics of 9-phase induction motor based on the $T$ model and magnetically coupled circuit have similar torque and speed dynamic response.


Keywords: $3 \times 9$ matrik qdn, T model of motor equivalent circuit, qdn dynamic model

## 1. Introduction

A dynamic model of qdn for induction motor is required to determine the characteristics of the starting conditions of motor until it gets steady state. All dynamic induction models of motor have been introduced by using the analysis based on the circuit of 1-phase transformer, which is magnetically coupled by stator and rotor frequency references [1]-[3]. A single-phase transformer circuit can easily be converted into $T$ models of electrical circuit by primary/secondary reference because it has similar frequency. Mutual reactance of transformer under steady-state conditions is constant because all primary and secondary winding of the transformer are silent.

However, an induction motor is very different from the transformer, the stator winding is stationary and the rotor winding is moving. In steady-state conditions, mutual reactance of the motor should have a function to manage rotation speed of the rotor which is then assumed to be constant to simplify the analysis. In addition, all of the parameters and frequency references refer to each part of the motor. While stator and rotor reactance are the leakage reactance as given in [4]. Thus, magnetically coupled circuits for 3-phase induction motor may not produce the equivalent circuit of the transformer T models [5]. Similarly, the equivalent circuit for the motor with higher phase becomes much more complex and may not be a T model of the equivalent circuit of the motor.

Meanwhile, all the motor parameters obtained from testing the motor are based on the equivalent circuit $T$ model. All parameters are expressed in the stator reference [6][7]. Mutual inductance can be obtained from the mutual reactance. Inductance of the stator and rotor can be obtained from leakage reactance the motor. At T model, the frequency of the rotor follows the stator frequency. Thus, there is a difference base in the analysis and testing of induction motors.

To generate dynamic model in term qdn, a square matrix is required according to the phase number of induction motors [8]. The transformation matrix 9-phase system has been introduced to anticipate the development of equipment that is considered to produce large power and small dimensions [9][13]. Transformation matrix for the 9-phase induction motors has $9 \times 9$ orders. Strong effort is required to obtain dynamic model of the 9 -phase induction motors in term qdn.

A $3 \times 9$ matrix transformation can be used to simplify the form the analysis of the equation of 9 -phase systems and can speed up calculations with the same result. Purpose of simplification can be achieved when a small number of matrix elements are used in the analysis. Transformation matrix used in the modeling of 9 -phase induction motor has a line of three. Thus, the form of the transformation matrix 9 -phase system is much simpler than the $9 \times 9$ transformation matrix.

Therefore, it will be simple, consistent and fast to obtain the analysis of the dynamic modeling of 9 -phase induction motor using $3 \times 9$ transformation matrix based on the motor equivalent circuit T model. This dynamic model using qdn transformation is based on equivalent circuit T model by reference to the stator. Simulation is used to test the simple dynamic model by a synchronous reference frame. Motor load is made from full to half-full load. A dynamic response obtained from the simulations shows that there is no difference between the proposed model with the dynamic model by synchronous and rotor reference frame.

## 2. Research Method

### 2.1. A Dynamic Model of the nine-phase induction motors in term qdn

A nine-phase system in term abc that can represent nine-phase voltage, current and flux has the following equations:

$$
\begin{align*}
& f_{a 1}=F_{m} \cos \omega t  \tag{1}\\
& f_{b 1}=F_{m} \cos (\omega t-2 \pi / 9)  \tag{2}\\
& f_{c 1}=F_{m} \cos (\omega t-4 \pi / 9)  \tag{3}\\
& f_{a 2}=F_{m} \cos (\omega t-2 \pi / 3)  \tag{4}\\
& f_{b 2}=F_{m} \cos (\omega t-8 \pi / 9)  \tag{5}\\
& f_{c 2}=F_{m} \cos (\omega t+8 \pi / 9)  \tag{6}\\
& f_{a 3}=F_{m} \cos (\omega t+2 \pi / 3)  \tag{7}\\
& f_{b 3}=F_{m} \cos (\omega t+4 \pi / 9)  \tag{8}\\
& f_{c 3}=F_{m} \cos (\omega t+2 \pi / 9) \tag{9}
\end{align*}
$$

Coordinate transformation of $f_{a b c}$ to $f_{q d n}$ in term matrix that follows the pattern of Park. The transformation of $\mathbf{T}(\theta)$ in term $3 x 9$ matrix has been stated [14]:

$$
\mathbf{T}(\theta)=\sqrt{\frac{2}{9}}\left[\begin{array}{ccc}
\cos \theta & \sin \theta & 1 / \sqrt{2}  \tag{10}\\
\cos (\theta-2 \pi / 9) & \sin (\theta-2 \pi / 9) & 1 / \sqrt{2} \\
\cos (\theta-4 \pi / 9) & \sin (\theta-4 \pi / 9) & 1 / \sqrt{2} \\
\cos (\theta-2 \pi / 3) & \sin (\theta-2 \pi / 3) & 1 / \sqrt{2} \\
\cos (\theta-8 \pi / 9) & \sin (\theta-8 \pi / 9) & 1 / \sqrt{2} \\
\cos (\theta+8 \pi / 9) & \sin (\theta+8 \pi / 9) & 1 / \sqrt{2} \\
\cos (\theta+2 \pi / 3) & \sin (\theta+2 \pi / 3) & 1 / \sqrt{2} \\
\cos (\theta+4 \pi / 9) & \sin (\theta+4 \pi / 9) & 1 / \sqrt{2} \\
\cos (\theta+2 \pi / 9) & \sin (\theta+2 \pi / 9) & 1 / \sqrt{2}
\end{array}\right]
$$

A circuit model of 9 -phase induction motor consist of nine equivalent circuits of the same, and can be described in an equivalent circuit $T$ model in a single-phase that opens at the rotor by stator reference as shown in Figure 1. Thus, all the parameters of the motor have the same references that are stator reference. $L_{s}$ and $L_{r}$ value is the combined value between mutual inductance with inductance leakage stator or rotor. The voltage is expressed in deffrential matrix equation as follows:


Figure 1. Motor equivalent circuit T model

$$
\begin{align*}
& \mathbf{v}_{s}=\mathbf{r}_{s} \mathbf{i}_{s}(t)+\frac{d \lambda_{s}(t)}{d t}  \tag{11}\\
& \mathbf{v}_{r}=\frac{\mathbf{r}_{s}}{s} \mathbf{i}_{r}(t)+\frac{d \lambda_{r}(t)}{d t} \tag{12}
\end{align*}
$$

In Equations 11 and 12, the resistances are diagonal matrix by order of $9 \times 9$ while the voltage, current and flux linkages are $1 \times 9$ order of column matrix. Where $\lambda_{1}(t)=\boldsymbol{i}_{s}(t)\left(L_{\mid s}+L_{m}\right)+$ $\boldsymbol{i}_{r}(t) L_{m}$ and $\lambda_{2}(t)=\boldsymbol{i}_{r}(t)\left(L_{\mid r}+L_{m}\right)+\boldsymbol{i}_{s}(t) L_{m}$. If $L_{s}=L_{\mid s}+L_{m}$ and $L_{r}=L_{m}+L_{\mid r}$ then Equations 11 and 12 can be decomposed in the flux linkage as:

$$
\begin{equation*}
\lambda_{1}(t)=\lambda_{s}(t)+\lambda_{m r}(t) \tag{13}
\end{equation*}
$$

where,

$$
\lambda_{s}(t)=\mathrm{L}_{s s} \mathbf{i}_{s}(t)=\left[\begin{array}{ccccccccc}
L_{s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_{s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & L_{s} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_{s} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L_{s} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & L_{s} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & L_{s} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{s} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{s}
\end{array}\right] \dot{i}_{s}(t)
$$

$$
\boldsymbol{\lambda}_{n r}(t)=\mathbf{L}_{m r} \mathbf{i}_{r}(t)=\left[\begin{array}{ccccccccc}
L_{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & L_{m} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_{m} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L_{m} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & L_{m} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & L_{m} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{m} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{m}
\end{array}\right] \dot{\mathbf{i}}_{r}(t)
$$

and

$$
\begin{equation*}
\lambda_{2}(t)=\lambda_{r}(t)+\lambda_{m s}(t) \tag{14}
\end{equation*}
$$

where,

$$
\begin{aligned}
& \lambda_{r}(t)=\mathbf{L}_{r} \mathbf{i}_{r}(t)=\left[\begin{array}{ccccccccc}
L_{s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_{s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & L_{s} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_{s} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L_{s} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & L_{s} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & L_{s} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{s} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{s}
\end{array}\right] \mathbf{i}_{r}(t) \\
& \lambda_{m s}(t)=\mathbf{L}_{m s} \mathbf{i}_{s}(t)=\left[\begin{array}{ccccccccc}
L_{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_{n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & L_{n} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_{m} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L_{m} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & L_{m} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & L_{m} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{m} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{m}
\end{array}\right] \mathbf{i}_{s}(t)
\end{aligned}
$$

Equations 11 and 12 are multiplied by $\mathrm{T}\left(\theta_{\mathrm{s}}\right)$ and modify on the right of the equation respectively to produce the following equation:

$$
\begin{align*}
& \mathbf{v}_{q d n s}=\mathbf{r}_{q d n s} \mathbf{i}_{q d n s}(t)+\mathbf{T}\left(\theta_{s}\right) \mathbf{T}\left(\theta_{s}\right)^{-1} \frac{d \boldsymbol{\lambda}_{q d n 1}(t)}{d t}+\mathbf{T}\left(\theta_{s}\right) \frac{d \mathbf{T}\left(\theta_{s}\right)^{-1}}{d t} \boldsymbol{\lambda}_{q d n 1}(t)  \tag{15}\\
& \mathbf{v}_{q d n r}=\frac{\mathbf{r}_{q d n r}}{s} \mathbf{i}_{q d n r}(t)+\mathbf{T}\left(\theta_{s}\right) \mathbf{T}\left(\theta_{s}\right)^{-1} \frac{d \boldsymbol{\lambda}_{q d n 2}(t)}{d t}+\mathbf{T}\left(\theta_{s}\right) \frac{d \mathbf{T}\left(\theta_{s}\right)^{-1}}{d t} \boldsymbol{\lambda}_{q d n 2}(t) \tag{16}
\end{align*}
$$

Multiplication of transformation matrix in term qdn by its inverse matrix is done to produce identical matrix expressed as follows:

$$
\mathbf{T}\left(\theta_{\mathrm{s}}\right) \mathbf{T}\left(\theta_{\mathrm{s}}\right)^{-1}=\left[\begin{array}{lll}
1 & 0 & 0  \tag{17}\\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]=\mathrm{I}
$$

The derivative of inverse of transformation matrix in term qdn can be expressed as:

$$
\frac{d \mathbf{T}\left(\theta_{\mathrm{s}}\right)^{-1}}{d t}=\omega_{s} \sqrt{\frac{2}{9}}\left[\begin{array}{ccc}
-\sin \theta_{\mathrm{s}} & \cos \theta_{\mathrm{s}} & 0 \\
-\sin \left(\theta_{\mathrm{s}}+2 \pi / 9\right) & \cos \left(\theta_{\mathrm{s}}+2 \pi / 9\right) & 0 \\
-\sin \left(\theta_{\mathrm{s}}+4 \pi / 9\right) & \cos \left(\theta_{\mathrm{s}}+4 \pi / 9\right) & 0 \\
-\sin (\theta+2 \pi / 3) & \cos \left(\theta_{\mathrm{s}}+2 \pi / 3\right) & 0 \\
-\sin \left(\theta_{\mathrm{s}}+8 \pi / 9\right) & \cos \left(\theta_{\mathrm{s}}+8 \pi / 9\right) & 0 \\
-\sin \left(\theta_{\mathrm{s}}-8 \pi / 9\right) & \cos \left(\theta_{\mathrm{s}}-8 \pi / 9\right) & 0 \\
-\sin \left(\theta_{\mathrm{s}}-2 \pi / 3\right) & \cos \left(\theta_{\mathrm{s}}-2 \pi / 3\right) & 0 \\
-\sin \left(\theta_{\mathrm{s}}-4 \pi / 9\right) & \cos \left(\theta_{\mathrm{s}}-4 \pi / 9\right) & 0 \\
-\sin \left(\theta_{\mathrm{s}}-2 \pi / 9\right) & \cos \left(\theta_{\mathrm{s}}-2 \pi / 9\right) & 0
\end{array}\right]
$$

where $\theta_{s}=\omega_{s} t$, multiplication of transformation matrix in term qdn by its derivative inverse matrix produces as follows:

$$
\mathbf{T}\left(\theta_{\mathrm{s}}\right) \frac{d \mathbf{T}\left(\theta_{\mathrm{s}}\right)^{-1}}{d t}=\omega_{\mathrm{s}}=\omega_{s}\left|\begin{array}{ccc}
0 & 1 & 0  \tag{18}\\
-1 & 0 & 0 \\
0 & 0 & 0
\end{array}\right|
$$

Equations 17 and 18 are substituted into equations 15 and 16 to obtain:

$$
\begin{align*}
& \mathbf{v}_{q d n s}=\mathbf{r}_{q d n s} \mathbf{i}_{q d n s}(t)+\frac{d \lambda_{q d n 1}(t)}{d t}+\omega_{\mathbf{s}} \lambda_{q d n 1}(t)  \tag{19}\\
& \mathbf{v}_{q d n r}=\frac{\mathbf{r}_{q d n r}}{s} \mathbf{i}_{q d n r}(t)+\frac{d \lambda_{q d n 2}(t)}{d t}+\omega_{s} \lambda_{q d n 2}(t) \tag{20}
\end{align*}
$$

$\mathbf{r}_{\text {qdns }}$ and $\mathbf{r}_{\text {qdnr }}$ are $3 \times 3$ diagonal matrixes
In Equations 19 and 20, the resistances are $3 \times 3$ diagonal matrix while the voltage, current and flux linkage are $1 \times 3$ column matrix. Thus, the flux linkage in term qdn is obtained from multiplication equations 13 and 14 with the transformation matrix of 9 -phase system in term qdn such as:

$$
\begin{align*}
& \lambda_{q d n 1}(t)=T\left(\theta_{s}\right) L_{s} T\left(\theta_{\mathrm{s}}\right)^{-1} \mathbf{T}\left(\theta_{\mathrm{s}}\right) \mathbf{i}_{\mathrm{s}}(\mathbf{t})+\mathbf{T}\left(\theta_{\mathrm{s}}\right) \mathbf{L}_{\mathrm{mr}} \mathbf{T}\left(\theta_{\mathrm{s}}\right)^{-1} \mathbf{T}\left(\theta_{\mathrm{s}}\right) \mathbf{i r}_{\mathrm{r}}(\mathbf{t}) \\
& \lambda_{\text {qdn1 }}(\mathbf{t})=\mathbf{L}_{\text {qdns }} \mathbf{i}_{\text {qdns }}(\mathbf{t})+\mathbf{L}_{\text {qdn_mr }} \mathbf{i}_{\text {qdnr }}(\mathbf{t})  \tag{21}\\
& \lambda_{q d n 2}(t)=T\left(\theta_{\mathrm{s}}\right) \mathbf{L}_{\mathrm{r}} \mathbf{T}\left(\theta_{\mathrm{s}}\right)^{-1} \mathbf{T}\left(\theta_{\mathrm{s}}\right) \mathbf{i}_{\mathrm{r}}(\mathbf{t})+\mathbf{T}\left(\theta_{\mathrm{s}}\right) \mathbf{L}_{\mathrm{ms}} \mathbf{T}\left(\theta_{\mathrm{s}}\right)^{-1} \mathbf{T}\left(\theta_{\mathrm{s}}\right) \mathbf{i}_{\mathrm{s}}(\mathrm{t}) \\
& \lambda_{q d n 2}(t)=\mathbf{L}_{\text {qdnr }} \mathbf{i}_{\text {qdnr }}(\mathbf{t})+\mathbf{L}_{\text {qdn_ms }} \mathbf{i}_{\text {qdns }}(\mathbf{t}) \tag{22}
\end{align*}
$$

where $\mathbf{L}_{\text {qdns }}, \mathbf{L}_{\text {qdnr }}, \mathbf{L}_{\text {qdn_ms }}$ dan $\mathbf{L}_{\text {qdn_mr }}$ can be expressed as:

$$
\mathbf{L}_{\text {qdns }}=\left[\begin{array}{ccc}
L_{s} & 0 & 0 \\
0 & L_{s} & 0 \\
0 & 0 & L_{s}
\end{array}\right], \mathbf{L}_{\text {qdnr }}=\left[\begin{array}{ccc}
L_{r} & 0 & 0 \\
0 & L_{r} & 0 \\
0 & 0 & L_{r}
\end{array}\right] \text { and } \mathbf{L}_{\text {qdn_ms }}=\mathbf{L}_{\text {qdn_mr }}=\left[\begin{array}{ccc}
L_{m} & 0 & 0 \\
0 & L_{m} & 0 \\
0 & 0 & L_{m}
\end{array}\right]
$$

Part derivative of Equations 19 and Equation 20 is replaced by 22, 23 and 18 (matrix $\omega_{\mathrm{s}}$ ) to obtain the equation as follows:

$$
\begin{equation*}
\mathbf{v}_{q s}=\mathrm{r}_{s} \mathrm{i}_{q s}(t)+\mathrm{L}_{l s} \frac{d \mathrm{i}_{q s}(t)}{d t}+\omega_{\mathrm{s}} \lambda_{d s}(t)+\mathrm{L}_{m} \frac{d\left[\mathrm{i}_{q s}(t)+\mathrm{i}_{q r}(t)\right]}{d t} \tag{23}
\end{equation*}
$$

$$
\begin{align*}
& \mathbf{v}_{q r}=\frac{\mathrm{r}_{r}}{s} \mathrm{i}_{q r}(t)+\mathrm{L}_{l r} \frac{d \mathrm{i}_{q r}(t)}{d t}+\omega_{\mathrm{s}} \lambda_{d r}(t)+\mathrm{L}_{m} \frac{d\left[\mathrm{i}_{q s}(t)+\mathrm{i}_{q r}(t)\right]}{d t}  \tag{24}\\
& \mathbf{v}_{d s}=\mathrm{r}_{s} \mathrm{i}_{d s}(t)+\mathrm{L}_{l s} \frac{d \mathrm{i}_{d s}(t)}{d t}-\omega_{\mathrm{s}} \lambda_{q s}(t)+\mathrm{L}_{m} \frac{d\left[\mathrm{i}_{d s}(t)+\mathrm{i}_{d r}(t)\right]}{d t}  \tag{25}\\
& \mathbf{v}_{d r}=\frac{\mathrm{r}_{r}}{s} \mathrm{i}_{d r}(t)+\mathrm{L}_{l r} \frac{d \mathrm{i}_{d r}(t)}{d t}-\omega_{\mathrm{s}} \lambda_{q r}(t)+\mathrm{L}_{m} \frac{d\left[\mathrm{i}_{d s}(t)+\mathrm{i}_{d r}(t)\right]}{d t}  \tag{26}\\
& \mathbf{v}_{n s}=\mathrm{r}_{s} \mathrm{i}_{n s}(t)+\mathrm{L}_{l s} \frac{d \mathrm{i}_{n s}(t)}{d t}+\mathrm{L}_{m} \frac{d\left[\mathrm{i}_{n s}(t)+\mathrm{i}_{n r}(t)\right]}{d t}  \tag{27}\\
& \mathbf{v}_{n r}=\frac{\mathrm{r}_{r}}{s} \mathrm{i}_{n r}(t)+\mathrm{L}_{l r} \frac{d \mathrm{i}_{n r}(t)}{d t}+\mathrm{L}_{m} \frac{d\left[\mathrm{i}_{n s}(t)+\mathrm{i}_{n r}(t)\right]}{d t} \tag{28}
\end{align*}
$$

From equations 23 to 28 can be established equivalent circuit of 9-phase induction motors in qdn model as shown in Figure 2a, 2 b and 2c.

Electromechanical torque generated in the motor in term qdn can be expressed as:

$$
\begin{equation*}
T_{e}=\frac{p}{2}\left(\lambda_{q r} i_{d r}(t)-\lambda_{d r} i_{q r}(t)\right) \tag{29}
\end{equation*}
$$

Substitute part of flux rotor linkage in Equation 22 into Equation 29 to obtain a new equation as in Equation 30.

$$
\begin{equation*}
T_{e}=\frac{p}{2} L_{m}\left(i_{q s} i_{d r}(t)-i_{d s} i_{q r}(t)\right) \tag{30}
\end{equation*}
$$


(a) The motor equivalent circuit in term $q$

(b) The motor equivalent circuit in term $d$

(c) The motor equivalent circuit in term $n$

Figure 2. The motor equivalent circuit in term qdn

Electromechanical torque equation is also obtained from the mechanical equations by neglecting rotor friction and load friction as follows:

$$
\begin{equation*}
T_{e}=T_{L}+J \dot{\theta}_{m} \tag{31}
\end{equation*}
$$

where: $J$ is load inertia
$T_{L}$ is load torque

### 2.2. Simulation

Equations 19 and 20 are the basic simulation of 9-phase induction motors. However, the desired result is the flux linked to the stator and the rotor, so that Equations 19 and 20 are substituted into the qdn modified equations 32 and 33 as follows:

$$
\begin{align*}
& \frac{d \lambda_{q d s}(t)}{d t}=\mathbf{v}_{q d s}-\mathbf{r}_{s} \mathbf{i}_{q d s}(t)-\omega_{\mathrm{s}} \lambda_{q d s}(t)  \tag{32}\\
& \frac{d \lambda_{q d r}(t)}{d t}=\mathbf{v}_{q d r}-\frac{\mathbf{r}_{r}}{s} \mathbf{i}_{q d r}(t)-\omega_{\mathrm{s}} \lambda_{q d r}(t) \tag{33}
\end{align*}
$$

where $\omega_{\mathrm{s}}$ expressed as:

$$
\omega_{\mathrm{s}}=\left[\begin{array}{ccc}
0 & \omega_{s} & 0  \tag{34}\\
-\omega_{s} & 0 & 0 \\
0 & 0 & 0
\end{array}\right]
$$

Equations 32, 33, and 34 are forms of the basis of the simulation model in term qdn for stator flux and rotor flux of 9 -phase induction motor as shown in Figure 3. All initial conditions are zero in motor Equation.


Figure 3. Motor flux linkage, (a) Stator flux linkage, (b) Rotor flux linkage

From Equations 21 and 22, the flux can be expressed in term current when $\lambda=L i$, so by combining both the stator flux and the rotor flux equations and then insert the inductance values in term qdn the equation 24 is obtained.

As desired outputs and inputs are respectively, the motor current and the motor flux. Therefore, the motor flux equations must be adapted to the needs so that it looks like the Equation 36.

$$
\lambda_{q d n}=\left[\begin{array}{c}
\lambda_{q s}  \tag{35}\\
\lambda_{d s} \\
\lambda_{n s} \\
\lambda_{q r} \\
\lambda_{d r} \\
\lambda_{n r}
\end{array}\right]=\left[\begin{array}{cccccc}
L_{s}+L_{m} & 0 & 0 & L_{m} & 0 & 0 \\
0 & L_{s}+L_{m} & 0 & 0 & L_{m} & 0 \\
0 & 0 & L_{s}+L_{m} & 0 & 0 & L_{m} \\
L_{m} & 0 & 0 & L_{r}+L_{m} & 0 & 0 \\
0 & L_{m} & 0 & 0 & L_{r}+L_{m} & 0 \\
0 & 0 & L_{m} & 0 & 0 & L_{r}+L_{m}
\end{array}\right]\left[\begin{array}{l}
i_{q s} \\
i_{d s} \\
i_{n s} \\
i_{q r} \\
i_{d r} \\
i_{n r}
\end{array}\right]
$$

Simulations obtained from Equation 36 can be seen in Figure 4. Figure 4 has already shown the stator flux and rotor flux act as input, while the stator and rotor current act as output.

$$
\mathbf{i}_{q d n}=\left[\begin{array}{c}
i_{q s}  \tag{36}\\
i_{d s} \\
i_{n s} \\
i_{q r} \\
i_{d r} \\
i_{n r}
\end{array}\right]=\left[\begin{array}{cccccc}
L_{s}+L_{m} & 0 & 0 & L_{m} & 0 & 0 \\
0 & L_{s}+L_{m} & 0 & 0 & L_{m} & 0 \\
0 & 0 & L_{s}+L_{m} & 0 & 0 & L_{m} \\
L_{m} & 0 & 0 & L_{r}+L_{m} & 0 & 0 \\
0 & L_{m} & 0 & 0 & L_{r}+L_{m} & 0 \\
0 & 0 & L_{m} & 0 & 0 & L_{r}+L_{m}
\end{array}\right]^{-1}\left[\begin{array}{l}
\lambda_{q s} \\
\lambda_{d s} \\
\lambda_{n s} \\
\lambda_{q r} \\
\lambda_{d r} \\
\lambda_{n r}
\end{array}\right]
$$



Figure 4. Simulation of motor current as a function of the motor flux

Simulation of the motor is not complete before the slip is obtained. The slip that occurs in motor is according to the following equation 37 :

$$
\begin{equation*}
s=\frac{\omega_{s}-\omega_{m}}{\omega_{s}} \tag{37}
\end{equation*}
$$

where $\omega_{m}$ is the rotor angular velocity in electrically
When the input is synchronous rotor speed, the output is the slip. Simulation of slip model can be seen in Figure 5.


Figure 5. Simulation of slip as a function angular velocity

The last simulation of 9 -phase induction motors is electromechanical parts. The Equation that contributes to this section is equations 30 and 31. Both equations are not described as inputs or outputs according to the simulation. Adjustments are made in order to obtain the mechanical angular speed of the motor by giving input of an electromechanical torque and a load torque. The simulation of electromechanical torque can be seen in Figure 6.


Figure 6. Simulation of the mechanical angular velocity as a function of torque

Simulation of the induction motor can be made by combining simulation of motor parts to form a simulation of 9 -phase induction motor as a whole. Simulation of induction motor is shown in Figure 7.

Table1. Parameter induction motor [15]

| Parameters | Nine-phase <br> induction motor |
| :---: | :---: |
| $\mathrm{R}_{\mathrm{s}}$ | 0,47 |
| $\mathrm{X}_{\mathrm{s}}$ | 0,69 |
| $\mathrm{R}_{r}^{\prime}$ | 0,53 |
| $\mathrm{X}_{r}^{\prime}$ | 0,69 |
| $\mathrm{X}_{\mathrm{m}}$ | 13,42 |
| $\mathrm{~V}_{\mathrm{s}}$ | 73 V |



Figure 7. Simulation of 9-phase induction motor based on equivalent circuit T model

## 3. Results and Discussion

In this paper, the induction motor is a nine-phase induction motor with very low voltage. This motor was developed from a 3-phase induction motor for electric car need with the goal of a high level of safety. Motor data are taken from previous research as presented in Table 1. The number of poles and the inertia motors respectively are 2 and $0.025 \mathrm{~kg}-\mathrm{m}^{2}$.

To determine the dynamic characteristics of 9 -phase induction motor it is done by making a simulation of the analytical results. Then the motor data in Table 1 are included in the simulation by different loading treatment in the stator reference frame. The motor load is in fullload conditions at start and then it is changed to a half-full load. Stator and rotor reference
frames that have been developed previously are used as validation. Source voltage of 9-phase induction motor is 9 -phase voltage symmetry that is divided in groups of 3-phase, each different phase angle of $40^{\circ}$ as shown in Figure 8. After that, the source voltage is transformed to the coordinates of the reference that the stator qdn can generate $v_{q}=225 \mathrm{~V}, v_{d}=0 \mathrm{~V}$ and $v_{n}=0 \mathrm{~V}$ as shown in Figure 9.

Stator and rotor produced currents in term qdn can be seen in Figures 10 and 11. Rotor currents show a negative value to the stator current, which means that the actual current is unidirectional due to the opposite rotor current to the direction given. Just at the start, the stator current $\left(\mathrm{i}_{\mathrm{d}}\right)$ is higher than rotor current $\left(\mathrm{i}_{q}\right)$. Then when approaching steady-state current, for a moment, the stator current ( $\mathrm{i}_{\mathrm{d}}$ ) turns lower than the rotor ( $\mathrm{i}_{\mathrm{q}}$ ) current. At steady-state with full load, the stator current $\left(\mathrm{i}_{\mathrm{d}}\right)$ is higher than the iq current. When the stator current $\left(\mathrm{i}_{\mathrm{d}}\right)$ is still lower than $i_{q}$ current and does not change position despite the motor load, the ( $\mathrm{i}_{\mathrm{d}}$ ) is lowered to halffull load.


Figure 8. $v_{\mathrm{abc}}$ output voltage which is the result of the transformation of the term $\mathrm{v}_{\mathrm{qdn}}$ by $\omega=0$


Figure 9. $v_{\mathrm{qdn}}$ output voltage which is the result of the transformation of the term $\mathrm{v}_{\mathrm{abc}}$ by $\omega=\omega_{\mathrm{s}}$


Figure 10. Motor current $i_{\text {qdn }}$ by $\omega=\omega_{\mathrm{s}}$


Figure 11. Motor current $i_{\text {qdn }}$ by $\omega=\omega_{\mathrm{s}}$ in the area of load changes (from full to half load)

Characteristics of 9-phase induction motor obtained from the simulation for synchronous reference frame can be seen in Figure 12 and 13. The transient response of angular speed of the proposed method is no over-shoot and in the same time achieving steady-state conditions at full load compared to models using synchronous reference and rotor reference. The load decrease does not cause oscillations in response of angular speed. Meanwhile, the transient response of the oscillating torque equally and the same time achieve steady-state conditions at full load compared to the synchronous reference frame. The load decrease does not cause oscillations in response of torque and angular speed. The proposed model has the same maximum torque of the models using synchronous reference. At steady-state with full load, the torque generated is similar between the proposed motor model with a model that uses synchronous reference and rotor reference. In areas of change, from full to half load, the torque on both motors is down and the speed is faster. Both torque and speed responses have a large value. Both motors have no overshoot and oscillation when the load changes. They remain the same in next steady-state conditions.


Figure 12. Characteristics of torque and angular speed vs. time in the refference frame $\omega_{\mathrm{s}}$


Figure 13. Characteristics of torque and angular speed vs. time by the refference frame $\omega_{s}$ in the starting area

## 4. Conclusion

Analysis of qdn dynamic model of 9-phase induction motor indicates that at the qd equivalent circuit has no difference with the previous method, but in the neutral equivalent circuit $(\mathrm{n})$, there is a parameter mutual inductance between the stator to the rotor. From the simulations, it can be seen that the synchronous reference frame, compared to the previous method, shows the characteristic torque and angular speed of 9-phase induction motor reaching steady-state in the same time to produce the same maximum torque. The transient response does not occur of oscillation and over-shoot. Dynamic characteristics of 9-phase induction motor based on the T model and magnetically coupled circuit have similar torque and speed dynamic response. In the circuit-based model of T , torque and angular speed response with similar maximum torque achieve steady-state rather than the model based on magnetically coupled circuit.

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