A Cognitive Radio Spectrum Sensing Algorithm to Improve Energy Detection at Low SNR

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Abstract
Energy detection is among the most popular spectrum sensing method for spectrum sensing due its low complexity. Unfortunately, its performance is poor at low SNR. In this paper we proposed a spectrum sensing method for cognitive radio network that improves the performance of energy detection. The proposed method based on distribution analysis using kurtosis as test statistic. This comes from the fact that distribution of received signal when a channel is occupied will be different from vacant channel. Noise tends to have a Gaussian distribution. Signal which faces multipath fading during the transmission way will have non Gaussian distribution. Sensing algorithm was tested using captured DTV signal. Result shows that our method performs well at low SNR. It achieves probability of detection of 90 % for 10 % Probability of false alarm for low SNR, below -20 dB.

Keywords: cognitive radio, spectrum sensing, DTV signal, kurtosis

1. Introduction

Compare to a conventional radio, cognitive radio introduces two main different features, cognitive capability and reconfigurability [1],[2]. Cognitive radio allows same frequency bands to be used simulaneously by primary user and secondary user [3]. Primary user, as the owner of spectrum license, has the first usage priority. Secondary user may only use the vacant spectrum that is not being used by primary user. This spectrum sharing cognitive radio has been proposed to be used in TV's white space bands opportunistically [4]. A new radio access standard IEEE 802.22 has been issued lately [5]. A cognitive radio platform may transmit at the spectrum holes in the TV bands as long as the channel is not being used by the primary user.

As required by the standard, before transmitting, secondary user has to ensure that its transmission will not cause harmful interference to the primary user by its sensing capability. The spectrum sensing has to accurately detect the existing spectrum holes to avoid harmful interference. In the other hand, mistaken detection will result in lower spectrum holes utilization. The main challenging of cognitive radio spectrum sensing is its requirement to be able to detect a signal even at very low level with a stringent acceptable performance. The performance usually is measured by probability of detection (Pd) and probability of false alarm (Pf). IEEE 802.22 demands a stringent sensing requirement. For maximum probability of false alarm of 10 %, a sensing algorithm should achieve probability of detection 90 % for signal as low as -20 dB signal to noise ratio (SNR) [5],[6]. This means that some licensed signals have to be sensed at a very low SNR. In addition, wireless channel fading and noise fluctuation will also bring difficulties.

Several sensing methods have been proposed to combat the above mentioned challenges such as matched filtering [7],[8], feature detection approach [9,[10] and energy detection [8],[11]-[14]. The matched filtering can maximize the SNR. It can detect signal at very low SNR. However, matched filtering method requires information such as pilot and frame structure of primary signal. For the feature detection method which relies on cyclostationarity, the sufficient signal information must be given as well. The required information introduces complexity in implementation, such as if the cognitive radio has to detect several different primary signals. On the other hand, energy detection does not require information about the signal to be detected. This method exploits energy difference between occupied and vacant channel condition. It compares the energy of received signal with pre-defined threshold. Due to

this low complexity, energy detection is the most preferable method. However, this method would be prone to the false detection since it only relies on the signal energy features [13],[14]. When signal fluctuates or noise is large, this method is likely to fail to distinguish between the absence and the presence of the signal [8],[13]. Eventhough it gives a good performance at high SNR, but its performance is very poor at low SNR.

Many methods have been proposed to overcome the shortcomings of the energy detection approach. Some methods based on the eigenvalues from the covariance matrix of the received signal were proposed in [15]-[17]. Same as in energy detection, the methods also do not require primary signal information. However, the corresponding computational complexities are quite large. In this paper, we proposed a simpler method using 4th order moment, i.e. its kurtosis value. Kurtosis of received signal is used as test statistics. Their values are compared with a predefined threshold to distinguish between occupied spectrum and white space. A threshold is calculated from empirical estimation of system's noise. Without knowledge of signal's parameters, the target of kurtosis analysis is the output of signal's FFT. Our experiment shows that it provides optimal performance, enabling detection at very low SNR primary signal as verified by simulation results. The rest of this paper will describe subsequently about formulation of spectrum sensing problem followed with explanation of proposed sensing method, results of performance evaluation by simulation using real DTV signal and conclusion.

2. Research Method

When spectrum sensing works to detect white space based on the received signal, there will be 2 possible conditions, channel is vacant or occupied. Decision has to be be made if the received signal is comprise of primary signals with inherent noise or it just comprise of sytem noise. Denote the discrete time received signal by r(n) during the sensing stage. The underlying primary signal is denoted by s(n) while w(n) is additive white Gaussian noise (AWGN). Signal received by spectrum sensing is $r(n) = s(n) + w(n)$. There are two possible condtions. If there is no primary signal, detector will only receive noise. If primary user occupied channel by transmitting signals, detector will receive signal and noise. There are 2 different conditions on each time instance, H_0: signal is absent and H_1: signal is present. Suppose there are N samples for detection, the problem can then be modeled as equation of binary hypothesis testing:

$$H_0: r(n) = w(n) \qquad\qquad n = 0, 1, \dots, N-1 \qquad\qquad (1)$$

$$H_1: r(n) = s(n) + w(n) \qquad\qquad n = 0, 1, \dots, N-1 \qquad\qquad (2)$$

It is assumed that signal and noise are uncorrelated and each is an i.i.d. sequence. The spectrum sensing problem is therefore to determine whether the signal *s(n)* exists or not, based on the received signal samples *r(n)* [11],[15].

Spectrum sensing has to be accurately making a decision whether a channel is vacant or occupied. Deciding occupied as vacant will result in prohibited transmission which causes harmfull interference to primary user. While wrong decision of occupied for vacant makes lower spectrum holes utilization. A spectrum sensing method's performance is evaluated by its probability of detection $P_d = P\big((H_1)\big|(H_1)\big)$ and probability of false alarm $P_f = P\big((H_1)\big|(H_0)\big)$. The detection algorithm in spectrum sensing works based on Neyman-Pearson's theorem, which states that the objective is to maximize probability of detection P_d in a fixed probability of false alarm $P_f = \alpha$. The detector makes decision of H1 if it fulfills likelihood ratio test (LRT) of:

$$L(x) = \frac{p(x|H_1)}{p(x|H_0)} > \lambda \qquad\qquad (3)$$

Where threshold λ is calculated by:

$$P_f = \int_{x:L(x)>\lambda} p(\boldsymbol{x}|H_0)dx = \alpha \qquad\qquad (4)$$

Suppose the primary signal is a wide-sense stationary Gaussian random process with variance of σ_s^2 and noise is additive wide Gaussian noise with variance of σ^2, likelihood ratio of $L(x)$ of equation 3 can be expressed as:

$$L(x) = \frac{\frac{1}{\left(2\pi(\sigma_s^2+\sigma^2)\right)^{\frac{N}{2}}} \exp\left(-\frac{1}{2(\sigma_s^2+\sigma^2)}\sum_{n=0}^{N-1} x^2(n)\right)}{\frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left(-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1} x^2(n)\right)} \tag{5}$$

Taking the natural logatitmic of $L(x)$, we will get the log-likelihood ratio:

$$l(x) = \frac{N}{2}\ln\left(\frac{\sigma^2}{\sigma_s^2+\sigma^2}\right) + \frac{1}{2}\frac{\sigma_s^2}{\sigma^2(\sigma_s^2+\sigma^2)}\sum_{n=0}^{N-1} x^2(n) \tag{6}$$

Detector will make dicision of H$_1$ if $\sum_{n=0}^{N-1} x^2(n) > \lambda$. This detector calculates energy of received data and compares it with a predefined threshold. It's called energy detection or radiometer. Energy of the received signal will be composed of energy of signal and noise's variance if the channel is being ouccupied. But if the primary signal is absent it will be equal to noise variance. To get its performance metrix (P$_f$ and P$_d$), we need to define the distribution for both conditions. For the large samples, we can make assumption on the statistical distribution of $T(x)$ for both conditions to be Gaussan, then, the probability of false alarm and the probability of detection can be expressed as:

$$P_d = Q\left(\frac{\lambda-(\sigma_s^2+\sigma^2)}{\sqrt{\frac{2}{N}}(\sigma_s^2+\sigma^2)}\right) \; ; \qquad P_f = Q\left(\frac{\lambda-\sigma^2}{\sqrt{\frac{2}{N}}\sigma^2}\right) \tag{7}$$

Where Q( ) is a q-function. As a class of Nyeman-Person detector, we set the threshold from equation 7 as:

$$\lambda = \left(\sqrt{\frac{2}{N}}Q^{-1}(P_f) + 1\right)\sigma^2 \tag{8}$$

where P_f is set to certain acceptable required value, mostly set at 0,1. We can see from equation 8 that the threshold is a function of the noise's variance and the number of samples. A primary signal in the target frequency band will be easier to be detected if its level is received higher than noise's variance. Signal to noise radio (SNR) is used to measure a ratio between the power level of received signal $(x(n))$ and noise's variance. Probability of detection is a function of the SNR, and can be expresed as:

$$P_d = Q\left(\frac{Q^{-1}(P_f)-\left(\sqrt{\frac{2}{N}}SNR\right)}{(1+SNR)}\right) \tag{9}$$

Energy detection is quite powerful spectrum sensing method when the signal is stronger than noise's variance (high SNR). But, its performance will be poor at low SNR. At low SNR, noise uncertainty makes it impossible to detect the presense of signal even with high number of samples [8],[14].

We consider another feature to improve the energy detection's performance. In wireless communication, received signal will be faded due to multipath channel. If we check the distribution of received signal, mostly they will have a certain non Gaussian such as a Rayleigh distribution. The distribution of the received signal (contaminated with noise) will be different from noise only. Spectrum sensing problem then can be drawn as hypotheses testing of two conditions, a vacant channel as a Gaussian distribution received samples, and occupied as a non-Gaussian distribution:

$$H_0: \quad r(n) \sim N(\bar{r}, \sigma^2); \qquad\qquad H_1: \quad r(n) \neq N(\bar{r}, \sigma^2) \qquad\qquad (10)$$

Then we can deduce condition based on the signal's distribution, a Gaussian or not.
Kurtosis is one of quite easy measure of gaussianity [12]. Kurtosis is defined as:

$$kurt\,(r) = E(r^4) - 3\big(E(r^2)\big)^2 \qquad\qquad (11)$$

Suppose that r has zero mean, and r has been normalized so that its variance is equal to one: $E(r^2) = 1$. Equation 16 will be:

$$kurt\,(r) = E(r^4) - 3 \qquad\qquad (12)$$

Since r is gaussion, its fourth moment equal to:

$$E(r^4) = 3(E[r^2])^2 = 3 \qquad\qquad (13)$$

So, for the samples with Gaussian distribution, they will have kurtosis equal to 0.
The Kurtosis value of Guassian samples will not exactly equal to 0 if the number of samples is not long enough. For the limited samples of N: $\bar{r} = [r_0 \quad r_1 \quad ...... \, r_{N-1}]$, estimation of the kurtosis is:

$$K(r) = \frac{1}{N}\sum_{n=0}^{N-1}(r_n - \bar{x})^4 - 3\left(\frac{1}{N}\sum_{n=0}^{N-1}(r_n - \bar{r})^2\right)^2 \qquad\qquad (14)$$

Since, the sensing methods mostly perform detection in frequency domain, received samples r(n) is transformed to frequency domain R(f) by an FFT block. The result is complex-value samples R(f). The FFT output comprised of real and imaginary parts. For the purpose of out sensing methods, we set the test statistics (T) into 3 kinds, and its respective performance will be evaluated:

$$T_1 = \frac{1}{2}K\big(re(X(f))\big) + \frac{1}{2}K\big(im(X(f))\big); \quad T_2 = K\big(re(X(f))\big); \quad T_3 = K\big(im(X(f))\big) \qquad (15)$$

We performed the non-Gaussianity test for the real parts and imaginary parts. The proposed spectrum sensing algorithm is performed as follows:

Step 1. R(f) is got by performing N-point FFT to the received sample frame x(n). FFT is currently widely used for communication system employing multicarries (OFDM), the system which is also used by digital broadcasting system.
Step 2. Kurtosis is calculated for each frame of M samples of real part as well as imaginary part using equation 20.
Step 3. Calculation of the test statistics (T) using equation 21.
Step 4. Reject the null hypothesis H_0 in favor of the presence of primary signal transmission if T > threshold. Otherwise, accept H_0 and declare the absence of the primary user's signal.

In the application of energy detection for spectrum sensing, initial sensing phase will be started by noise calibration to measure its variance in order to be able to set the threshold. This is also applied in our proposed method. The threshold is taken empirically from noise samples. To get the threshold, the detector will perform detection in the condition of H_0. Detection events will be counted to get the detecton rate which shows the probability of false alarm.. As required by the standard, its value should be under 10 %. Adjustment of threshold will be made until the probability of false alarm achieve 10 %. This threshold then will be taken as detection threshold to be compared with test statistic in equation 10. In this proposed method, the detector works blindly as it doesn't need information of signal parameters.

3. Results and Discussion

The performance was measured from its detection rate in low signal-to-noise ratios (SNR). This shows the sensitivity of proposed method in detecting a low level primary signal

existence. The primary signal source for the experiment was taken from real captured digital television (DTV) signal [20]. The reason for the choise is twofolds, for the type and its characteristics. For the DTV signal type, it represents one the spectrum sharing cognitive radio standard, IEEE 802.22, which operates in the TV whitespace. And the signal which was taken from real broadcasting signals carrier also will represent the real environment of the wave propagation especially its multipath fading channel's effect. The captured signal's frequency is 545 MHz with 6 MHz bandwidth. Given the samping rate 50 MHz and the oversampling factor 8/7 x bandwidth, we can calculate the sample rate or number of samples per second. The output of ADC is samples with speed equal to 6857 samples per 1 ms. Number of samples taken for sensing is particularty important since our method works based on the received samples distribution. Even for the absent of primary signals, the distribution of received signal will not be Gaussian if the number of samples we took is not sufficiently long.

Prior to test the proposed detection method, we need to set up the threshold. As in the energy detection, to define the threshold for our detector, we performed a kind of noise calibration. The purpose of noise calibration in the energy detection is to measure the noise's variance. In order to get it, the energy detector works in the condition of no primary signal (H_0). Once we get its value, the variance's then used to set up the threshold for the detector. In the proposed method, instead of measure the variance, we took the noise's kurtosis. Since the measured signal is taken at the output of fast Fourier transform (FFT), each sample will have real and imaginary parts. Kurtosis values were taken from both parts. According to equation 22, test statistics of T_1, T_2, and T_3 for noise only are calculated. Calculation was performed sequently to each frame of M number of samples. The test statistic for each frame's then compared with a predefined threshold. We set the threshold to 0.04. False decision (H_1) will occur when the test statistic is above the threshold. The number of false decision devided by total number of test statistics shows the false alarm rate or the probability of false alarm (P_f). We set the number of FFT points to 2048 and 8192, the same number used by the DTV standard.
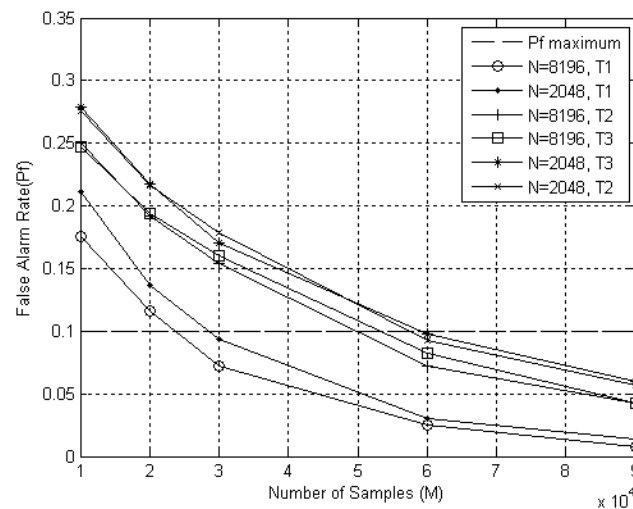

Figure 1. False alarm rate as a function of M

Figure 1 shows the result, the false alarm rate as a function of the number of samples. False alarm rate shows the number of times the detector made wrong decision of vacant channel as occupied. This will miss the opportunity for cognitive radio to maximize spectrum holes ulitization. This will be mainly a function of the threshold. If we set the threshold higher, this will result in lower false alarm, but will also make lower detection rate when later it has to work to detect primary signals. As a general spectrum sensing requirement, false alarm rate should not exceed P_f maximum of 10 %, as shown in the figure as a dashed line. By setting the fixed threshold, we intend to measure the other factors, including number of samples in a frame used in the test statistics calculation and also the effect of FFT size. The six lines in the figure show the result of experiment for each number of FFT points and for 3 test statistics. False

alarm rate will decrease as number of samples increase. In means that for a fixed threshold, increasing number of samples will make detector perform better due to the closer to its Gausian distribution for large samples. As we mentioned before, for the real Gausssian noise samples, output of the FFT will comprise of the real part and imaginary part, each will have Gaussian distribution and its kurtosis should be 0. Since we have 2 kurtosis values of the real and imaginary part, we can use both or omit one of them. From the figure, if we compare the performance of the test statistics, we will get that the test statistic of T_1 performs better than T_2 and T_3. Using the both part by averaging give the highest performance, i.e. the lowest false alarm rate for the same number of samples (M). Since the variance of kurtosis estimates will decrease with the inceasing number of samples (M), we need to get the M minimum which fulfills the requirement. And if we used the T_1, an average kurtosis of the real and the imaginary samples as the test statistic, the result also sugest that the number of sample should at least 30000 to achieve false alarm rate below 10 %.

After we set the threshold and it had been confirmed its respective probability of false alarm is under 10 %, we performed simulation to get the main performance metric, the detection rate. A captured DTV Signal is as primary signal. The main purpose of simulation is to measure the detector's sensitivity. The sensitivity here means how low the level of primary signal can be detected by the proposed method. The primary signal was added by generated sequences of white Gaussian noise. Signal's level was adjusted to represent the intended signal-to-noise ratio (SNR) values particulary to represent the samples value at low SNR. For each samples in a frame, fast Fourier transform (FFT) was performed. Estimation of kurtosis was calculated for each real samples and imaginary samples, and then used for test statistic (T_1). The test statistic of frames (M) of 60,000 samples each was calculated. The test statistics (T_1) for each frame then's compared to the threshold above (0.04). The proposed detector will make decision of channel is being occupied if the test statistic is above the threshold. The total number of test statistics which are above the threshold devide by the total number of frames or the ratio of test statistic above threshold to the all available test statistic values is a detection rate for the respective SNR (probability of detection).
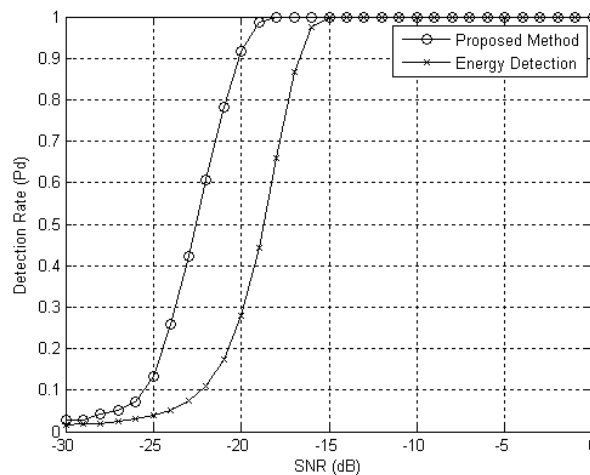

Figure 2. Detection Rate for N-FFT=8,192, M=60,000

Figure 2 shows the result of the experiment. The detection rate of proposed method was compared with the energy detection, both using simulation for several signal to noise ratio (SNR). The propose method's performance's better than energy detection and It also gives acceptable performance. They perform well for low SNR of under -20 dB. As the energy detection measures the energy difference, its performance deteriores rapidly once the primary signal's power is close or below the noise's variance. Our method which based on kurtosis measures the difference of its statistical distribution. The distribution of noise is Gaussian. We can say that in the condition of H_0, the distribution of the FFT output should be Gaussian. The

approach will result in robust result as long as the number of samples is sufficient, so that its distribution will remain as in the original distribution.
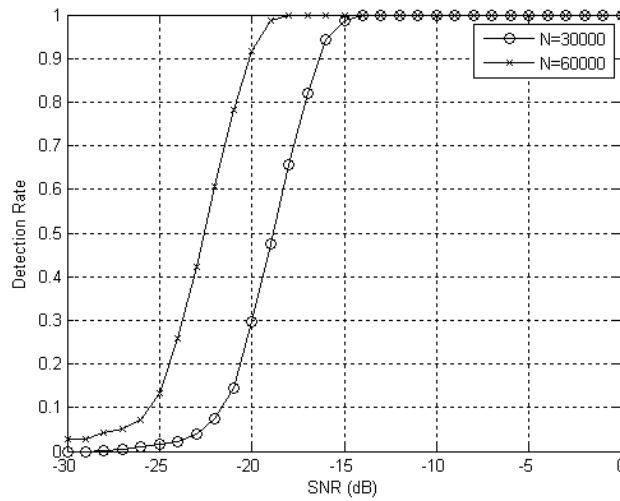

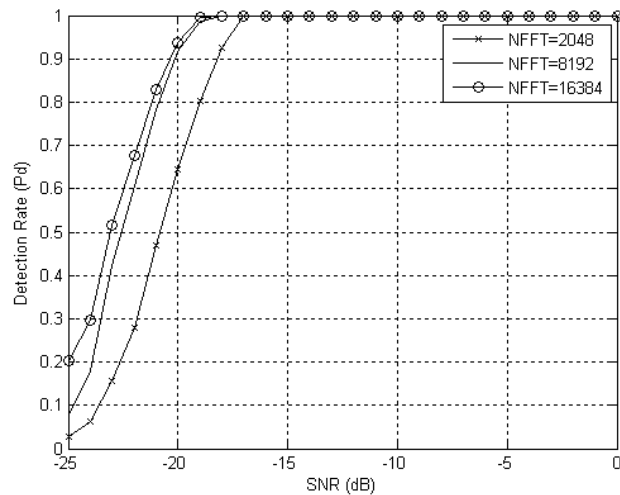Figure 3. Detection rate comparison on number of samples

Figure 4. Detection Rate for Several N-FFTs

Number of samples to calculate the test statistic also has impact on probability of detection. Since the variance of kurtosis estimation is a function of 1/M, we conduct an experiment to compare the performance for different number of samples in a frame (M). If we modify the number of samples to be twice we get a comparison result. In Figure 3, we plot simulation result for M=30,000 and M=60,000 respectively. This figure shows that the sample size of 60,000 is acceptable to detect primary signal at low SNR. The 60,000 samples is equal to sensing time of 10 ms. We also conducted an experiment to measure the effect of FFT. Figure 4 shows the effect of N-FFT to the detector performance. When number of FFT set as variable, we can see that probability of detection increases with the increasing of FFT points. This resul come from the fact that the higher the FFT point the more accurate it will represent the signal in time domain, including its distribution. The N-FFT of 8192 gives acceptable performance. It performs well for low SNR of under -20 dB.

4. Conclusion

In this paper we proposed a new spectrum sensing method based on kurtosis. Our method is able to work blindly without primary signal knowledge. Its performance improves the energy detection and it is comply with the standard requirement without adding much complexity. Two main factors have been investigated, number of FFT points and number of sample per testing frame. Number of FFT points gives more significant impact on the performance with minimum value is 8,192 points. Since threshold is got from real measurement of noise samples, our method can adapt to various noise characteristics.

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