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Maximization Network Throughput based on Maximal Flow for Single-Source Two-Destinations Multicast

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Abstract

For guaranteeing all multicast destination nodes receiving the source information with their maximal flow respectively and obtaining the network maximal throughput, a heuristic algorithm based on network coding, Maximal Flow for Single-source Two-destinations Multicast (MFSTM) is proposed to maximize the network throughput. By calculating the each destination's maximal flow, the number of link-disjoint paths which equals to destination's maximal flow, are searched for each destination to construct the network coding graph. A heuristic algorithm based on network coding is designed to delete the redundant link in the network coding graph and guarantee the network throughput maximization. Comparing the traditional maximal multicast stream algorithm based on network coding, the simulation results show that the MFSTM algorithm makes two destinations receive the information at the speed of their maximal flow respectively, and decode the source node information at each destination node successfully.

Keywords: *multicast networks; network coding; network throughput; maximal flow; heuristic algorithm*

1. Introduction

Growth with the applications of point to multipoint applications, such as high-definition Internet television, video conferencing, distributed video games and storage data networks, multicast applications have increased rapidly in future networks [1]-[2]. It was proved that the multicast makes network throughput drop much [3]. And it is difficult to reach the network maximal throughput for multicast. Network coding was proposed by Ahlswede et al in 2000 to maximize the single source multicast capacity, which lets all destination nodes receive information from source node at the maximal multicast speed [4]. Li et al proved that linear network coding can get the multicast capacity [5]. Koetter et al provided a construction of linear network coding by the algebraic method [6]. References [7]-[9] further study the applications of linear network coding in the network. Reference [10]-[12] proposed many maximal multicast stream algorithm based on network coding (NC) algorithm to increase the network throughput. But previous researches about maximal multicast capacity based on network coding can only ensure that all of the destination nodes receive the information at the same speed, the maximal multicast capacity, which makes some nodes can not achieve their maximal receiving capability (node's maximal flow) and keeps them from getting the network maximal throughput. According to maximal-flow-minimal-cut theory, when each destination receives information at the speed of their maximal flow respectively, the network can get the maximal throughput. But in the actual network, it is usual that the multicast capacity is not more than each node's maximal flow.

Figure 1 shows the relationship of the maximal multicast capacity and node's maximal flow. In the network, the capacity of each link is just 1, multicast's source node is S and destinations are t_1 and t_2 respectively. According to the maximal-flow-minimal-cut theory, we can calculate the maximal flows of destination nodes t_1 and t_2 are 4 and 2, respectively. So the maximal multicast capacity is 2, which is equal to minimum value of 4 and 2. In Figure 1, according to the link-disjoint route principle, the destinations t_1 and t_2 can only receive information at maximal speed 1. When the network coding is used, the destinations t_1 and t_2 can receive the information at maximal speed 2. And it is less than the maximal receive capability of destination t_1. How to make all destination nodes receive information from source at speed of their maximal flow value and ensure decoding information correctly at each destination is an urgent problem [13]. In this paper, we propose MFSTM algorithm, a heuristic algorithm based

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on network coding to construct the network coding graph, which can make two multicast destination nodes receive the information from source node at speed of their maximal flow respectively and can decode information correctly at each destination.

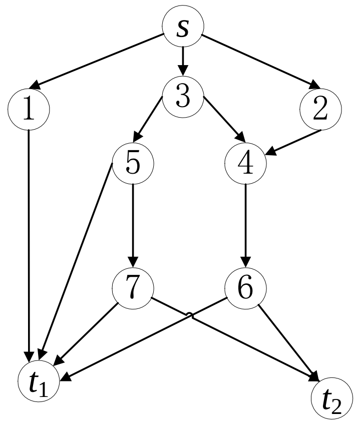
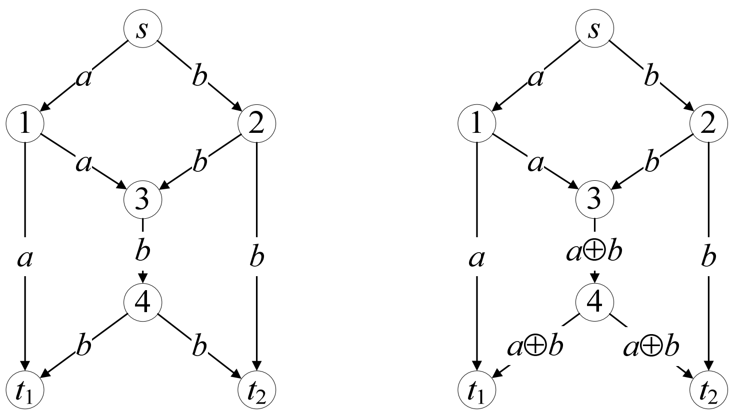


Figure 1. Schematic of multicast capacity

2. Maximal flow of single-source two-destination multicast

Network coding can be used to improve the multicast capacity and save the required link for transmitting information from source node to each destination. Figure 2 shown a classic butterfly network, which include one source node *S* and two destinations t_1 and t_2, the capacity of every link is just 1. The traditional multicast route is selected the edge separation route to transmit the information, where the intermediate nodes in the network only replicate and forward the received information. In Figure 2(*a*), destination nodes t_1 and t_2 can receive information at speed of 2 and 1 unit capacity at a time, respectively. However, if network coding is allowed at nodes, i.e., node 3 in Figure 2(*b*), destination nodes t_1 and t_2 can receive information at speed of 2 simultaneously by XOR information *a* and *b* at node 3. At node 3 in Figure 2(b), two arrival information *a* and *b* from upstream node 1 and 2 are operated as information $a \oplus b$ by XOR. Then at both destinations t_1 and t_2, information $a \oplus b$ is decoded as *a* and *b* by $a \oplus (a \oplus b) = b$ and $b \oplus (a \oplus b) = a$, respectively. So, destinations t_1 and t_2 can receive information *a* and *b* at the same time in Figure 2(b).



(*a*) Traditional way of routing (*b*) Network coding

Figure 2. A classic butterfly network

The above introduced is the process of network coding operation for each destination with the same receiving speed. But when some multicast destinations have different receiving speed, how to utilize network coding to make each destination receive the information at their own speed in accordance of the destination's maximal flow value from the source node, respectively. Many researchers have verified it is a NP-hard problem and it is almost impossible to reach.

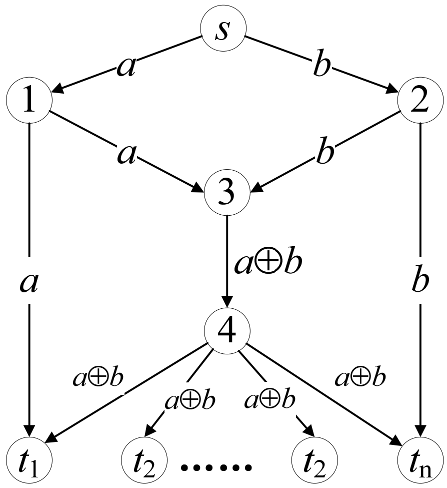
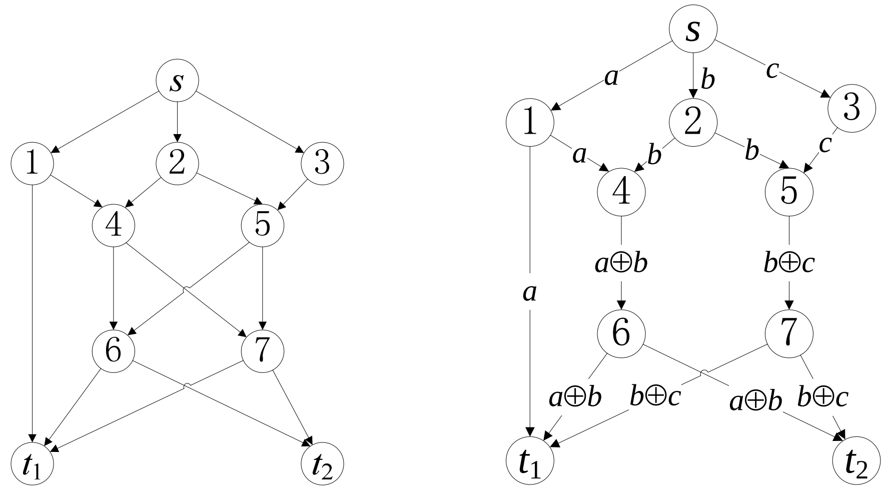


Figure 3. A special network

In Figure 3, we can find that not at all single-source *n*-destinations multicast can make all of the destinations receive the information from source node with each destination's maximal flow, and decode the information correctly, where *n* is greater than 2. In Fig.3, the maximal flow of destination nodes t_1 and t_n are 2, and the maximal flow of destination nodes t_2 ,..., t_{n-1} are 1, respectively. If the destination nodes t_1 and t_n want receive two of information at one time, link (3, 4) needs transmit encoding information $a \oplus b$, and destination nodes t_2, ..., t_{n-1} are unable to decode the received information successfully. So in this paper, we study the maximal flow multicast capacity for single-source two-destinations multicast, and propose a heuristic method based on graph compression to makes each destination receive the information from source node with their destination's maximal flow.

In this paper, a directed acyclic multicast network is modeled as graph G=(V,E), where V and E represent nodes and links, respectively, capacity of all links is unit 1. The maximal flow of destination nodes t_1 and t_2 is set as m and n, respectively ($m \geq n$). In order to realize t_1 and t_2 receiving the information from source node with m and n respectively and decode information successfully, we need to look for m link-disjoint paths and n link-disjoint paths for destination nodes t_1 and t_2, respectively. But, in accordance with the above approach, only destination t_1 can guarantee decoding the m information correctly, but destination t_2 cannot decode n information. For example, in Figure 4(*a*), the maximal flow is 3 and 2 for destination nodes t_1 and t_2, respectively. Therefore, we need to look for 3 link-disjoint paths for destination t_1, which are s-1-t_1, s-2-4-6-t_1 and s-3-5-7-t_1. Two link-disjoint paths are searched for destination t_2, which are s-1-4-6-t_2 and s-2-5-7-t_2. The transmission routes of destinations t_1 and t_2 are shown in Figure 4(*b*), where information *a*, *b* and *c* are transmitted from source node *S*.



(*a*) Original network (*b*) Information transmission routes

Figure 4. Sketch of maximal flow transmission

In Figure 4(*b*), the destinations t_1 can decode the *a*, *b* and *c* information successfully according to Eq. (1).

$$\begin{cases} a = y_1 \\ a + b = y_2 \\ b + c = y_3 \end{cases} \tag{1}$$

Where y_1, y_2 and y_3 are the encoding information received by destination node t_1. There have three linear independence equations with three unknown variables *a*, *b* and *c* in Eq.(1). So, the destination node t_1 can decode three information successfully [13], which are *a*, *b* and *c* in Eq.(1).

Similarly, the decoding equation of destination node t_2 is shown as follows

$$\begin{cases} a + b = y_4 \\ b + c = y_5 \end{cases} \tag{2}$$

Where y_4 and y_5 are the encoding information received by destination node t_2. But, we can find two linear independence equations with three unknowns variables *a*, *b* and *c* in Eq.(2), so the destination node t_2 can't decode the information transmitted by source node *S* and can not obtain the 2 unit information, in terms of maximal flow of destination t_2 at a time.

As discussed above, if there have *k* linear independence equations for *k* unknown variables for the destination node *t*, the destination node *t* can decode the *k* information successfully. So, in this paper, a method is proposed to ensure that *k* linear independence equations only with *k* information for the destination node *t* in single source two destinations multicast.

3. Maximize the network throughput based on maximal flow for single source two destinations multicast

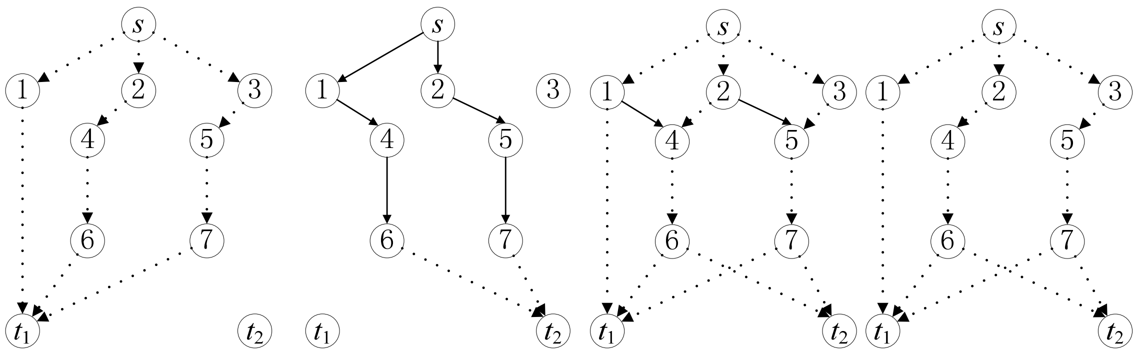
Assumes that the maximal flow of destinations t_1 and t_2 are *m* and *n*, respectively, where *m* is not less than *n*. Firstly, *m* link-disjoint paths are searched for destination node t_1. Then, *n* link-disjoint paths are searched for destination node t_2. Finally, paths in the above link-disjoint paths, which make *n* linear independence equations with *n* unknown variables for the destination node t_2, are deleted from the above link-disjoint paths. The process of maximization the network throughput based on maximal flow for single source two destinations multicast (MFSTM) as follows.

Step 1: Look for *m* link-disjoint paths for destination t_1, and constructing G_1 graph by the above *m* link-disjoint paths, where dotted lines are used to represent the edges in the graph G_1 in Figure 5 (a).

Step 2: Look for *n* link-disjoint paths for destination t_2, and construct the graph G_2 by the above *n* link-disjoint paths, where the solid lines are used to represent the edges in the graph G_2 but not connect to the destination node in Figure 5 (b).

Step 3: Graph G_3 is constructed by link union of G_1 and G_2, which is shown in Figure 5 (c). If the line is both in G_1 and in G_2, the line in G_1 is selected to construct the G_3.

Step 4: Put the solid line links of network G_3 into the set *e*, where $e=\{e_1,\ldots,e_k\}$, e_i is the *i*-th side solid line of network G_3 ($i \in [1, k]$), *k* is the number of solid lines in G_3. Choose one line of the set *e* to be deleted in network G_3. Check whether the maximal flow of destination t_2 in G_3 is *n*. If yes, delete the selected line in G_3, and update the network G_3 and set *e*. Else if the maximal flow of destination t_2 in G_3 is not *n*, then reserve the selected line in network G_3, and change the line with the dotted line in G_3, and update the network G_3 and set *e*. Until end of each line in set *e* are checked.



(*a*) G_1 graph (*b*) G_2 graph (*c*) G_3 graph (*d*) G_4 graph

Figure 5. Example process of MFSTM

Figure 5 shows an example of implementation process of MFSTM, where node *s* is the source node and the nodes 8 and 9 are destination nodes. Since the maximal flow of the destination nodes 8 and 9 is 3 and 2, respectively.

Step 1: Look for 3 link-disjoint paths to destination node 8 and construct graph G_1, shown in Figure 5 (*a*);

Step 2: Look for 2 link-disjoint paths to destination nodes 9 and construct graph G_2, shown in Figure 5 (*b*);

Step 3: Construct network G_3 by link union of G_1 and G_2, shown in Figure 5 (*c*);

Step 4: In Figure 5 (c), set *e*={(1, 4), (2, 5)}. To delete the line (1, 4) in Figure 5 (*c*), we find the maximal flow of destination node 9 is still 2, so the line (1, 4) is deleted in Figure 5 (*c*). Similarly, line (2, 5) should be deleted. Finally, the network, which makes each destination receive the maximal flow, is constructed in Figure 5 (*d*).

In Figure 5 (*d*), we can find paths *s*-1-t_1, *s*-2-4-6-t_1 and *s*-3-5-7-t_1 for destination t_1, and find the paths *s*-2-4-6-t_2 and *s*-3-5-7-t_2 for destination t_2. So, destinations t_1 and t_2 can receive information with their maximal flow respectively.

In the following description, we illustrate that the MFSTM can make one-source two-destinations multicast get the maximal throughput, which lets each destination receive information at speed of their maximal flow respectively. We assume the network is abstracted as G, the two destinations are t_1 and t_2, maximal flow of destinations t_1 and t_2 are *m* and *n*, respectively, where *m* is not less than *n*, the source node is *S*. We assume that the destination t_2 can receive *n* + *k* number of information transmitting from source node *S* using MFSTM algorithm, where m-$n \geq k \geq 1$. The network coding operations should be used among the n link-disjoint paths. The reason is that only n link-disjoint paths are searched for destination t_2 using MFSTM, and the input link number of destination t_2 is n. So, the extra k information should share the n established link-disjoint paths to transmit the information from source node S. There must exist the case shown in the Fig.6. m link-disjoint paths for destination t_1 are denoted as a_1, …, a_m respectively. n link-disjoint paths are denoted as b_1, …, b_n respectively in Figure 6. For destination t_2, if destination t_2 receives $n + k$ information from source node S, the network constructed by MFSTM must exist not less than one node, such as A, whose input link is more than 1 and information carrying by each link is encoded at node A by performing network coding. The link (*A, B*) transmits the encoding information $a_1 \oplus a_2$. So, destinations t_2 receives 2 information by one link b_3, which are a_1 and a_2. There have k similar encoding links as above (*A*, *B*). According to the MFSTM algorithm, the dotted link lines not belong to the m link-disjoint paths. So, if we delete the dotted link line, the maximal flow of destination t_1 is m also and the maximum flow of destination t_2 is still n. It is conflicting with the network coding graph constructed by MFSTM algorithm, which say if we delete any one link in the graph, the maximal flow of destination node t_1 or t_2 should decrease. Therefore, the assumption of $n + k$ information received by destination t_2 was rejected due to not decrease the maximal flow of destination t_2. The number of information received by destination t_2 is not more than n.

Simultaneously, in the network coding graph, which includes the n link-disjoint paths searched by MFSTM algorithm, the maximal flow of destination t_2 is n. If we allow the encoding link exist in the network coding graph, such as $a_1 \oplus a_2$ on link $a_1 \oplus a_2$, the number of information received by destination t_2 is not less than n.

From the above two cases, we know, the number of information received by destination t_2 just equals to n by MFSTM algorithm.

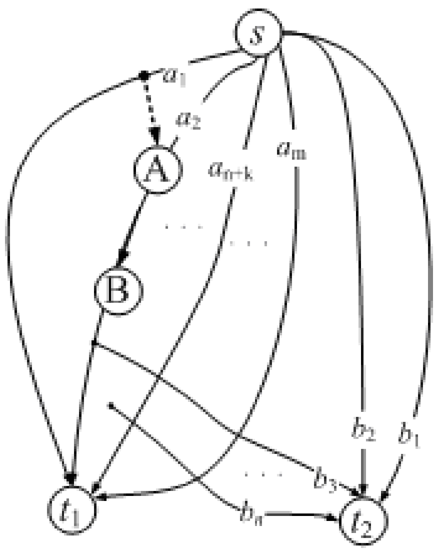


Figure 6. Simplified network constructed by link-disjoint paths

4. Simulation and Discussion

In order to make the simulation network topology closer to reality network, we take Salama model to produce random networks, which are similar to reality network. The number of network nodes is 20, 25, 30, 35 and 40 with two destinations when 100 random networks are produced in the simulation, respectively. The average network throughput obtained by MFSTM is compared with the maximal multicast stream algorithm based on network coding (NC) and the maximal flow theory. Table 1 shows the simulation results of NC and MFSTM.

Table 1. Average network throughput

Number of nodes	NC	MFSTM	Maximal flow theory
20	4.05	6.48	6.48
25	3.89	5.98	5.98
30	3.57	5.08	5.08
35	3.51	4.83	4.83
40	3.45	4.74	4.74

Table 1 shows that the network average throughput of MFSTM is greater than of NC in the 5 types of networks. And the network average throughput of MFSTM algorithm is equal to the network throughput calculated by maximal flow theory. The result shows that the MFSTM can guarantee the two destinations receiving the information at the speed of their node's maximal flow.

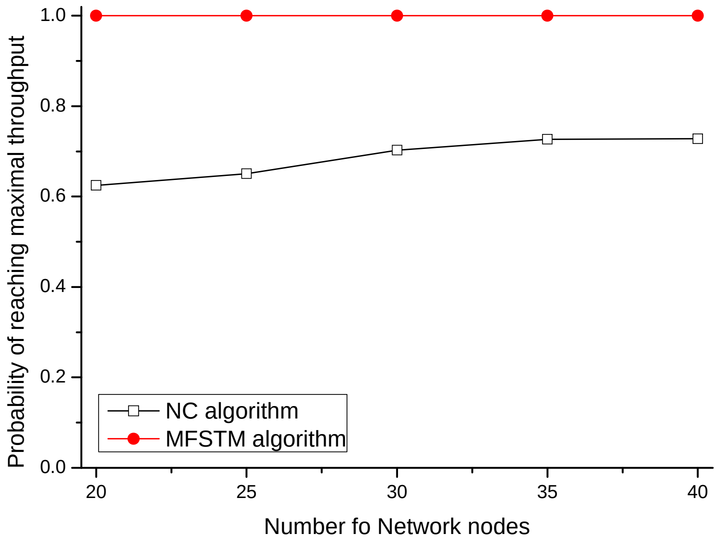


Figure 7. Probability of maximal throughput for NC and MFSTM algorithm

Figure 7 shows that the average network throughput of NC algorithm only about 60%~70% of maximal network throughput which is calculated by maximal flow theory, while the MFSTM can get the 100% of maximal throughput. So, the MFSTM can get the network maximal throughput, which equals to the value of maximal flow theory.

5. Conclusion

Multicast makes it is much difficult to get the network maximal throughput. The paper studies a heuristic algorithm, MFSTM algorithm, to make the multicast of single-source two-destinations to get the maximal network throughput. The network coding graph is build by the link-disjoint paths of each destination with respect to destination's maximal flow. The process of network coding graph construction is to guarantee each destination at the speed of destination's maximal flow using heuristic principle. Compared with the traditional maximal multicast stream algorithm based on network coding (NC) algorithm, the proposed MFSTM can get network maximal throughput of maximal flow theory.

Acknowledgements

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