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Energy harvesting half-duplex AF power splitting protocol relay network over rician channel in case of maximizing capacity

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Abstract

In this letter, we propose a novel power splitting protocol for energy harvesting half-duplex AF relaying communication systems. In our proposed system, the relay harvests energy from the source transmissions, by employing adaptive PS protocol, for powering the retransmissions to the destination. The proposed model system is investigated in cases maximize and non-maximize ergodic capacity. Firstly, we perform the analytical mathematical analysis for deriving the integral closed-form expression of the outage probability and the ergodic capacity. Then, the analytical analysis of the system performance can be convinced by Monte-Carlo simulation with helping Mat Lab software. Finally, the numerical analysis provides practical insights into the effect of various system parameters on the system performance of the proposed system. This paper can be considered as a recommendation for the energy harvesting communication network.

1. Introduction

Energy harvesting relay network, which uses a radio frequency (RF) signal for wireless power transfer, has attracted much attention because of prolonging the lifetime of a wireless network. This solution based on the fact that RF signal can carry both energy and information simultaneously and replace or recharge batteries incurs a high cost and can be inconvenient or hazardous (e.g., in toxic environments), or highly undesirable (e.g., for sensors embedded in building structures or inside the human body). At the beginning, it has not been widely used in practice due to the high propagation loss of RF signals. The new techniques in a small cell, transmission using large-scale antenna arrays MIMO, millimeter-wave communications, and the advancements in low power electronics significantly reduce the propagation loss and achieve much higher efficiency energy consumption [1-5]. From that point of view, has a high potential to be widely implemented in the next-generation wireless communication systems. In the last decades, there are many works focused on the WPCNs solution. Such as some papers presented the process energy harvesting through the RF signals in cooperative wireless networks by a MIMO relay system the difference between the energy transfer and the information rates to provide the optimal source and relay precoding. In the literature of the others, the authors investigated multi-user and multi-hop systems for simultaneous information and power transfer with a dual-hop channel with an energy harvesting relay, the transmission strategy depends on the quality of the second link. In these previous papers, the authors only focused on the WPCNs by using only the Rayleigh fading channels or only Rician fading channel [6-12]. For this new model, the question of system performance is still open and is necessary to investigate, and this is the aim of our paper.

In this paper, the system performance analysis of the half-duplex energy harvesting relay network is proposed, analyzed and derived in details. In this analysis, we consider the

amplifier-and-forward at the helping relay and the time splitting (PS) protocol. All the transmission channels are the Rician environment with random distribution signal. Firstly, we perform the analytical mathematical analysis for deriving the integral closed-form expression of the outage probability and the ergodic capacity. Then, the analytical analysis of the system performance can be convinced by Monte-Carlo simulation with helping Mat Lab software. Finally, the numerical analysis provides practical insights into the effect of various system parameters on the system performance of the proposed system. The main contributions of the paper are summarized as follows:

- Propose the system model of the half-duplex energy harvesting relay network for the amplifier-and-forward the half-duplex energy harvesting relay network modes at the helping relay in the PS protocol.
- Derive the integral closed-form expressions of the outage probability and ergodic capacity of the proposed system in the maximize and non-maximize cases.
- The impact of the main system parameters on the system performance is investigated with the Monte Carlo simulation.

The rest of this paper is organized as follows. Section 2 describes the system model and the EH protocol that is used in this paper. Section 3 provides the detailed performance analysis of the system. The numerical results to validate the analysis are presented in section 4. Finally, conclusions are drawn in section 5.

2. System Model

In this section, we consider a half-duplex (HD) relaying network with one source, one destination and one relay as illustrated in Figure 1. Let denote the source is D, the relay is R, and the destination is D. In the proposed model in Figure 1, every node has only one antenna and operates in a half-duplex mode. Then we denote the channel gain between S node and the relay R as h, and between R node and D node as g. Here, both channels are assumed Rician fading channels. Throughout this analysis, the following assumptions are proposed:

- The source node cannot directly transfer energy and information to the destination node because of the weak transmission line. Then these processes are only performed by helping of an intermediate relay.
- The intermediate relay is an energy-constrained node. It first harvests energy from the S node and uses this harvested energy for transmitting the source information to the destination.
- It is assumed that the processing power required by the transmit/receive circuitry at the relay is negligible as compared to the power used for signal transmission from the relay to the destination.
- We have assumed perfect channel knowledge at the destination and assumed negligible overhead for pilot transmission, which is in line with the previous work in this research field [3].

Figure 2 illustrates the energy harvesting and information transmission processes in the proposed system. In which, T denotes the block time of all processes. Here, the S node transfers the energy and information to the R node in the first half-interval time $T/2$. In the first half-interval time $T/2$, the energy harvesting time is ρT and the information transmission time is $(1-\rho)T/2$, which ρ is the power splitting factor and $0<\rho<1$. Finally, the R node transfers the information from the S node to the D node in the remaining half-interval time $T/2$ [13-19].
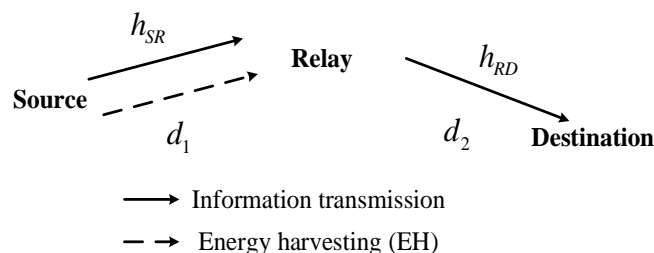

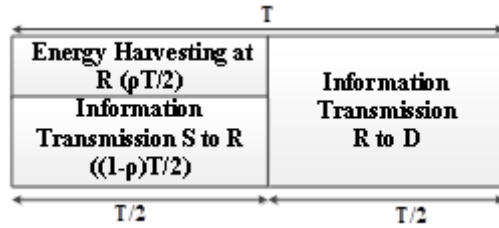
Figure 1. The proposed system model

Figure 2. The energy harvesting and information processing in the proposed system

3. System Performance Analysis

In this paper, we consider a relaying network system, where each terminal operates in half-duplex mode and has a single antenna. The two nodes S and D communicate with each other via the help of relay R over Rician fading channels. There is no direct path between S and D, the amplifier-and-forward modes is proposed in this model [19-26]. In the first interval time, the received signal at the relay can be formulated by the (1):

$$y_r = \frac{1}{\sqrt{d_1^\chi}} \sqrt{(1-\rho)} h_{sr} x_s + n_r \tag{1}$$

where: h_{sr} is the source to relay channel gain, d_1 is the source to relay distance, χ is the path loss exponent, x_s is the transmitted signal at the source, n_r is the additive white Gaussian noise (AWGN) with variance N_0, $0 < \rho < 1$ is power splitting ratio at the relay node, $\mathrm{E}\{|x_s|^2\} = P_s$, $\mathrm{E}\{\bullet\}$: expectation operator, P_s is average transmit power at the source. The harvested power at the relay can be obtained as:

$$P_r = \frac{E_h}{d_1^\chi (T/2)} = \frac{\eta \rho P_s |h_{sr}|^2 (T/2)}{d_1^\chi (T/2)} = \frac{\eta \rho P_s |h_{sr}|^2}{d_1^\chi} \tag{2}$$

where $0 < \eta \leq 1$ is energy conversion efficiency. The received signal at the destination can be given by the following:

$$y_d = \frac{1}{\sqrt{d_2^\chi}} h_{rd} x_r + n_d \tag{3}$$

Here we denote $\mathrm{E}\{|x_r|^2\} = P_r$, and h_{rd} is the relay to destination channel gain, d_2 is the source to relay distance, n_d is the AWGN with variance N_0. In the AF protocol, the transmitted and received signal at the relay has a relationship through the amplifying factor μ, so μ can be expressed as the (4):

$$\mu = \frac{x_r}{y_r} = \sqrt{\frac{P_r}{\frac{(1-\rho)P_s |h_{sr}|^2}{d_1^\chi} + N_0}} \tag{4}$$

by substituting (1), (4) into the (3), we have:

$$y_d = \frac{1}{\sqrt{d_2^\chi}} h_{rd} \mu \left[\frac{1}{\sqrt{d_1^\chi}} \sqrt{(1-\rho)} h_{sr} x_s + n_r \right] + n_d = \underbrace{\frac{\sqrt{(1-\rho)}}{\sqrt{d_1^\chi d_2^\chi}} h_{rd} \mu h_{sr} x_s}_{signal} + \underbrace{\frac{1}{\sqrt{d_2^\chi}} h_{rd} \mu n_r + n_d}_{noise} \tag{5}$$

the end to end signal to noise ratio (SNR) can be calculated as:

$$\gamma_{e2e} = \frac{E\left\{|signal|^2\right\}}{E\left\{|noise|^2\right\}} = \frac{\dfrac{(1-\rho)\mu^2 |h_{sr}|^2 |h_{rd}|^2 P_s}{d_1^\chi d_2^\chi}}{\dfrac{\mu^2 |h_{rd}|^2 N_0}{d_2^\chi} + N_0} \tag{6}$$

Substituting (2) into (6), and after doing some algebra and using the fact that $N_0 << P_r$, so the end to end SNR can be obtained as:

$$\gamma_{e2e} = \frac{\eta\rho(1-\rho)\Psi |h_{sr}|^2 |h_{rd}|^2}{\eta\rho |h_{rd}|^2 d_1^\chi + (1-\rho)d_2^\chi} \tag{7}$$

here we denote $\Psi = P_s / N_0$. Finally, the ergodic capacity of S-D link can be calculated as the following:

$$C_{s,d} = \frac{1}{2}\log_2(1+\gamma_{e2e}) = \frac{1}{2}\log_2\left[1 + \frac{\eta\rho(1-\rho)\Psi |h_{sr}|^2 |h_{rd}|^2}{\eta\rho |h_{rd}|^2 d_1^\chi + (1-\rho)d_2^\chi}\right] \tag{8}$$

Remark

In this analysis, we will consider that h_{sr} and h_{rd} belong to Rician fading channel and they have a random distribution. Then the probability density function (PDF) of a random variable (RV) φ_i where i=1 and 2 can be formulated as in [13]. Where $\omega_1 = |h_{sr}|^2, \omega_2 = |h_{rd}|^2$:

$$f_{\omega_i}(x) = a_i \sum_{l=0}^{\infty} \frac{(b_i K)^l}{(l!)^2} x^l e^{-b_i x} \tag{9}$$

where $a_i = \dfrac{(K+1)e^{-K}}{\lambda_i}, b_i = \dfrac{K+1}{\lambda_i}$, λ_i is the mean value of RV ω_i which i =1 and 2 respectively, and $\lambda_1 = \dfrac{E\left\{|h_{sr}|^2\right\}}{d_1^m}, \lambda_2 = \dfrac{E\left\{|h_{rd}|^2\right\}}{d_2^m}$, K is the Rician K-factor defined as the ratio of the power of the line-of-sight (LOS) component to the scattered components and $I_0(\bullet)$ is the zero-th order modified Bessel function of the first kind. Finally, the cumulative density function(CDF) of RV φ_i where i=1 and 2 can be computed as in [13].

$$F_{\omega_i}(\varsigma) = \int_0^\varsigma f_{\varphi_i}(x)dx = 1 - \frac{a_i}{b_i}\sum_{l=0}^{\infty}\sum_{n=0}^{l} \frac{K^l b_i^n}{l!n!}\varsigma^n e^{-b_i\varsigma} \tag{10}$$

3.1. Maximize Capacity

In this case, it is necessary to obtain the maximum value of ρ for both outage probability γ_{e2e} and ergodic capacity $C_{s,d}$. The following value of ρ maximizes the $C_{s,d}$ S-D link can be formulated as the following:

$$\rho^* = \frac{1}{1 + |h_{rd}|\sqrt{\dfrac{\eta d_1^\chi}{d_2^\chi}}} \tag{11}$$

Proof: See the Appendix A

3.2. Non-maximize

In the non-maximize case, we can formulate the ergodic capacity and the outage probability of the S-D link as the following:

$$C_{s,d} = \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_{\gamma_{e2e}}(\gamma)}{1 + \gamma} d\gamma \tag{12}$$

$$F_{\gamma_{e2e}}(\gamma) = \Pr\left(\frac{\eta\rho(1-\rho)\Psi|h_{sr}|^2|h_{rd}|^2}{\eta\rho|h_{rd}|^2 d_1^\chi + (1-\rho)d_2^\chi} < \gamma \right) = \Pr\left(\frac{\eta\rho(1-\rho)\Psi\omega_1\omega_2}{\eta\rho\omega_2 d_1^\chi + (1-\rho)d_2^\chi} < \gamma \right)$$

$$= \Pr\left[\omega_1 < \frac{\gamma\{\eta\rho\omega_2 d_1^\chi + (1-\rho)d_2^\chi\}}{\eta\rho(1-\rho)\Psi\omega_2} \right] \tag{13}$$

by the similar way as in the maximize case, we have:

$$F_{\gamma_{e2e}}(\gamma) = 1 - \int_0^\infty \frac{a_1 a_2}{b_1} \sum_{l=0}^\infty \sum_{n=0}^\infty \sum^l \frac{K^l b_1^n}{l!n!} \left[\frac{\gamma\{\eta\rho\omega_2 d_1^\chi + (1-\rho)d_2^\chi\}}{\eta\rho(1-\rho)\Psi\omega_2} \right]^n$$

$$\exp\left[-\frac{b_1\gamma\{\eta\rho\omega_2 d_1^\chi + (1-\rho)d_2^\chi\}}{\eta\rho(1-\rho)\Psi\omega_2} \right] \times \sum_{l=0}^\infty \frac{(b_2 K)^l}{(l!)^2} \omega_2^l e^{-b_2\omega_2} d\omega_2 \tag{14}$$

$$F_{\gamma_{e2e}}(\gamma) = 1 - a_1 a_2 e^{\left(-\frac{b_1\gamma d_1^\chi}{(1-\rho)\Psi} \right)} \int_0^\infty \sum_{l=0}^\infty \sum_{k=0}^\infty \sum_{n=0}^\infty \sum^l \frac{K^{l+k} b_1^{n-1} b_2^k}{l!n!(k!)^2} \left[\frac{\gamma}{\Psi} \right]^n \left[\frac{d_1^\chi}{(1-\rho)} + \frac{d_2^\chi}{\eta\rho\omega_2} \right]^n$$

$$\times \exp\left[-\frac{b_1\gamma d_2^\chi}{\eta\rho\Psi\omega_2} \right] \times \omega_2^k e^{-b_2\omega_2} d\omega_2 \tag{15}$$

$$F_{\gamma_{e2e}}(\gamma) = 1 - a_1 a_2 e^{\left(-\frac{b_1\gamma d_1^\chi}{(1-\rho)\Psi} \right)} \sum_{l=0}^\infty \sum_{k=0}^\infty \sum_{n=0}^l \sum_{t=0}^n \frac{K^{l+k} b_1^{n-1} b_2^k}{l!(k!)^2 t!(n-t)!} \left[\frac{\gamma}{\Psi} \right]^n \left[\frac{d_1^\chi}{(1-\rho)} \right]^{n-t} \left(\frac{d_2^\chi}{\eta\rho} \right)^t$$

$$\times \int_0^\infty \omega_2^{k-t} \times e^{-\frac{b_1\gamma d_2^\chi}{\eta\rho\Psi\omega_2}} \times e^{-b_2\omega_2} d\omega_2 \tag{16}$$

then the outage probability can be formulated as:

$$F_{\gamma_{e2e}}(\gamma) = 1 - 2a_1 a_2 e^{\left(-\frac{b_1\gamma d_1^\chi}{(1-\rho)\Psi} \right)} \sum_{l=0}^\infty \sum_{k=0}^\infty \sum_{n=0}^l \sum_{t=0}^n \frac{K^{l+k} b_1^{\frac{2n+k-t-1}{2}} b_2^{\frac{k+t-1}{2}}}{l!(k!)^2 t!(n-t)!} \left[\frac{d_1^\chi}{(1-\rho)} \right]^{n-t}$$

$$\times \left(\frac{d_2^\chi}{\eta\rho} \right)^{\frac{k+t+1}{2}} \times \left(\frac{\gamma}{\Psi} \right)^{\frac{2n+k-t+1}{2}} \times K_{k-t+1}\left(2\sqrt{\frac{b_1 b_2 \gamma d_2^\chi}{\eta\rho\Psi}} \right) \tag{17}$$

finally, by substituting (17) into (12), we also obtain $C_{s,d}$ for the S-D link.

4. Results and Discussion

In this section, some simulation results are presented to investigate the system performances of the proposed system model. Both in theoretical and Monte Carlo simulation results evaluate the system performance analysis. The curves in Figures 3 and 4 correspond to the outage probability and ergodic capacity versus P_S/N_0. In Figures 3 and 4 we set the main system parameters as d_1=0.65, d_2=0.85, K=3, η=0.8, R=0.5, χ =3 and λ_1=λ_2=0.5. In these figure, P_S/N_0 varies from 0 to 25. The results show that the outage probability has a decrease when P_S/N_0 increase from 0 to 25 as shown in Figure 3. In the same way, Figure 4 shows that

the ergodic capacity increases when P_S/N_0 increase from 0 to 25. In both Figures 3 and 4, the analytical and simulation results agree very well with each other.

Figures 5 and 6 illustrate the effect of η on the outage probability and ergodic capacity. It was shown that in Figures 5 and 6 we set P_S/N_0.=15 dB, K=3, ρ=0.3, R=0.5, χ =3 and λ$_1$=λ$_2$=0.5. The outage probability decreases while η increases from 0 to 1 as shown in Figure 5. In another way, the ergodic capacity has a considerable improvement when η increases from 0 to 1 as shown in Figure 6. For cases, the analytical analysis and the Monte Carlo simulation results are the same for all η values.

Moreover, the outage probability and ergodic capacity versus K is shown in Figures 7 and 8, respectively. Similarity, we set $d_1=d_2=1$, P_S/N_0.=10 dB, η=0.8, ρ=0.3, R=0.5, χ =3 and λ$_1$=λ$_2$=0.5. From Figure 7 we see that the outage probability decreases while K varies from 1 to 4. In contrast, the ergodic capacity increase while K varies from 1 to 4 as shown in Figure 8. Furthermore, Figures 9 and 10 show the outage probability and ergodic capacity versus $d_1=d_2$. In the same way, outage probability increases and the ergodic capacity decreases while $d_1=d_2$ varies from 0.5 to 1.5. In all figs, the simulation and analytical results totally agree with each other.
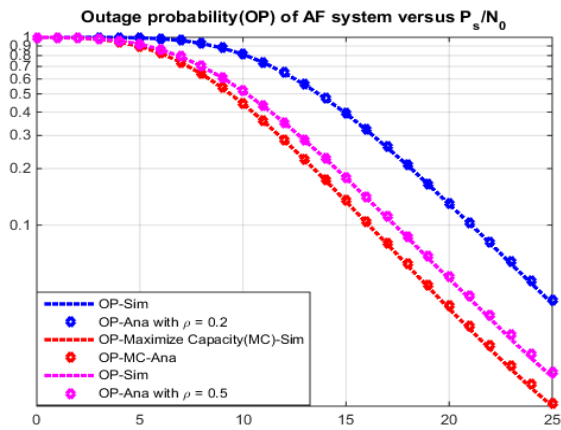

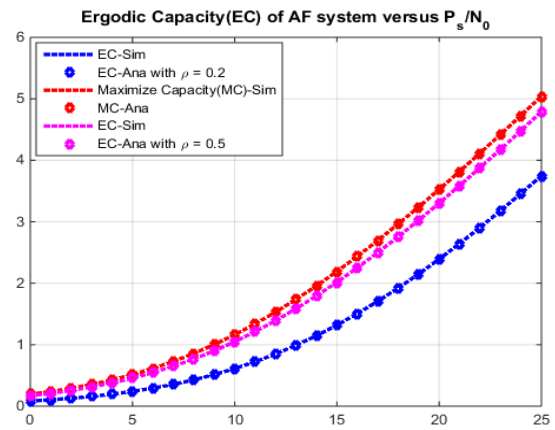Figure 3. Outage probability versus PS/N0
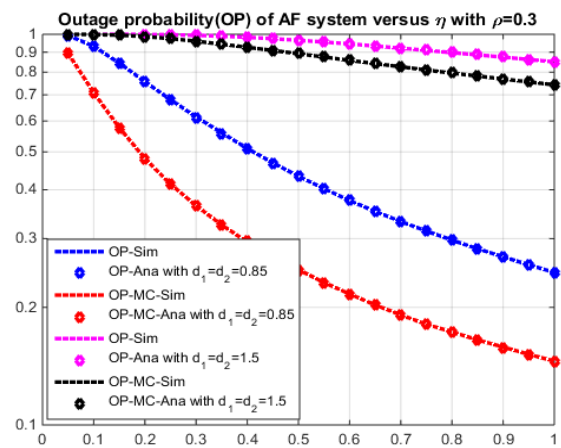

Figure 4. Ergodic capacity versus P_S/N_0
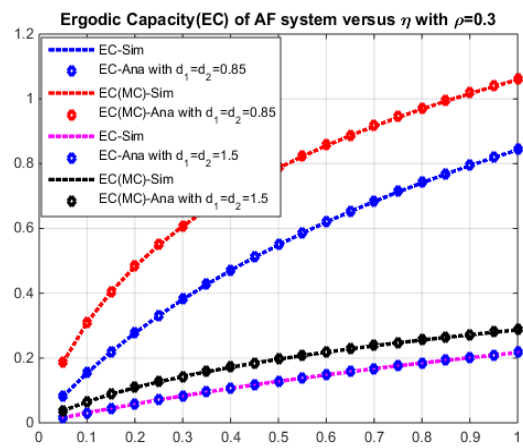

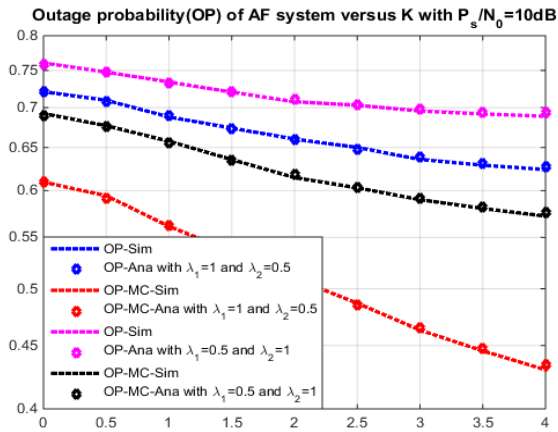Figure 5. Outage probability versus η

Figure 6. Ergodic capacity versus η
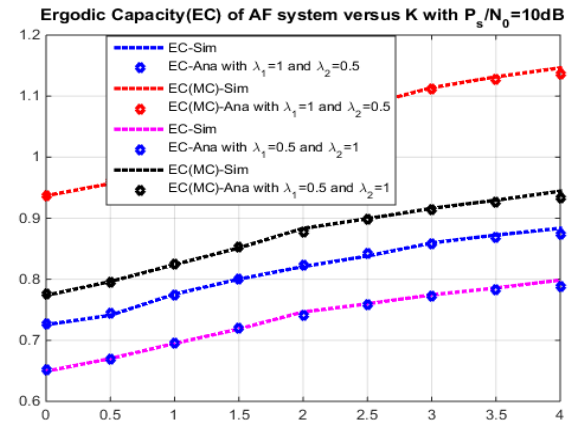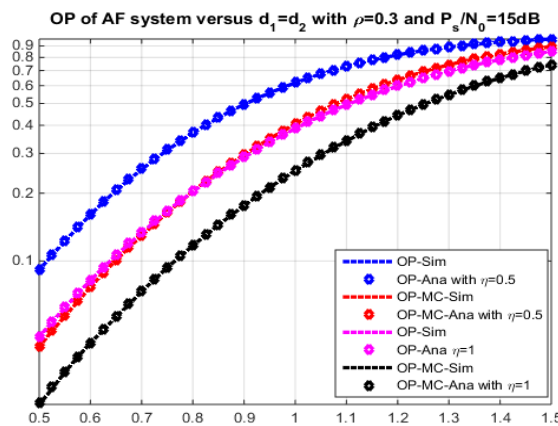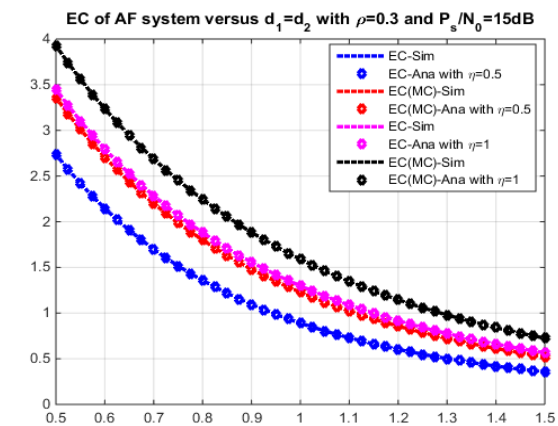
Figure 7. Outage probability versus K

Figure 8. Ergodic capacity versus K

Figure 9. Outage probability versus $d_1=d_2$

Figure 10. Ergodic capacity versus $d_1=d_2$

4. Conclusion

This study proposes the EH half-duplex relaying network in maximize and non-maximize cases of ergodic capacity. In this model, we consider AF relaying, where the energy-constrained relay node harvests energy from the received RF signal from the source and then uses the energy to forward the source's information to the destination. Furthermore, the analytical expressions for the outage probability and ergodic capacity of the proposed model system are derived. By using the Monte Carlo simulation, the research results show that the analytical and simulation results are the same for all possible system parameters.

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Appendix A

It is easy to observe that $\dfrac{\partial^2 \gamma_{e2e}}{\partial \rho^2}$ is negative for $0 < \rho < 1$. Hence, we conclude that γ_{e2e} is a concave function of ρ for $0 < \rho < 1$. We can find the value of ρ which maximize SNR by differentiating the SNR concerning ρ and equate it to zero. After some algebra, we have the following possible solutions for ρ^*:
$$\rho^* = \frac{1}{1 + |h_{rd}|\sqrt{\dfrac{\eta d_1^\chi}{d_2^\chi}}} \quad \text{or} \quad \rho^* = \frac{1}{1 - |h_{rd}|\sqrt{\dfrac{\eta d_1^\chi}{d_2^\chi}}}.$$

With $\rho^* = \dfrac{1}{1 - |h_{rd}|\sqrt{\dfrac{\eta d_1^\chi}{d_2^\chi}}}$ results in a value of $\rho^* > 1$ or $\rho^* < 0$, therefore we choose $\rho^* = \dfrac{1}{1 + |h_{rd}|\sqrt{\dfrac{\eta d_1^\chi}{d_2^\chi}}}$ as

the solution. Replace (12) into (7) we have the value of outage probability in the maximize case as the following:

$$
\gamma_{e2e}^{\max} = \frac{\eta \Psi |h_{sr}|^2 |h_{rd}|^2}{\left(1 + |h_{rd}|\sqrt{\dfrac{\eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}} = \frac{\eta \Psi \omega_1 \omega_2}{\left(1 + \sqrt{\dfrac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}}
\tag{A1}
$$

Then ergodic capacity of the S-D link can be calculated as:

$$
C_{s,d}^{\max} = \int_0^{\infty} f_{\gamma_{e2e}^{\max}}(\gamma) \log_2(1+\gamma) d\gamma = \frac{1}{\ln 2} \int_0^{\infty} \frac{1 - F_{\gamma_{e2e}^{\max}}(\gamma)}{1+\gamma} d\gamma
\tag{A2}
$$

$$
F_{\gamma_{e2e}^{\max}}(\gamma) = \Pr\left(\gamma_{e2e}^{\max} < \gamma\right) = \Pr\left[\frac{\eta \Psi \omega_1 \omega_2}{\left(1 + \sqrt{\dfrac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}} < \gamma\right]
$$

$$
= \Pr\left[\omega_1 < \frac{\gamma \left(1 + \sqrt{\dfrac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}}{\eta \Psi \omega_2}\right] = \int_0^{\infty} F_{\omega_1}\left[\frac{\gamma \left(1 + \sqrt{\dfrac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}}{\eta \Psi \omega_2} \mid \omega_2\right] f_{\omega_2}(\omega_2) d\omega_2
\tag{A3}
$$

where $\gamma = 2^{2R} - 1$ is threshold and R is the source rate of the proposed system. From the (4),(5), we have:

$$
F_{\gamma_{e2e}^{\max}}(\gamma) = 1 - \int_0^{\infty} \frac{a_1 a_2}{b_1} \sum_{l=0}^{\infty} \sum_{n=0}^{l} \frac{K^l b_1^n}{l! n!} \left[\frac{\gamma \left(1 + \sqrt{\dfrac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}}{\eta \Psi \omega_2}\right]^n \exp\left[-\frac{b_1 \gamma \left(1 + \sqrt{\dfrac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}}{\eta \Psi \omega_2}\right]
$$

$$
\times \sum_{l=0}^{\infty} \frac{(b_2 K)^l}{(l!)^2} \omega_2^l e^{-b_2 \omega_2} d\omega_2
\tag{A4}
$$

$$
F_{\gamma_{e2e}^{\max}}(\gamma) = 1 - \int_0^{\infty} \frac{a_1 a_2}{b_1} \sum_{l=0}^{\infty} \sum_{k=0}^{\infty} \sum_{n=0}^{l} \frac{K^{l+k} b_1^n b_2^k}{l! n! (k!)^2} \left(\frac{\gamma d_2^{\chi}}{\eta \Psi \omega_2}\right)^n \left[\left(1 + \sqrt{\frac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2\right]^n
$$

$$
\times \exp\left[-\frac{b_1 \gamma \left(1 + \sqrt{\dfrac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}}{\eta \Psi \omega_2}\right] \times \omega_2^k e^{-b_2 \omega_2} d\omega_2
\tag{A5}
$$

for this analysis, we use the following equations:

$$
\exp\left[-\frac{b_1 \gamma \left(1 + \sqrt{\dfrac{\omega_2 \eta d_1^{\chi}}{d_2^{\chi}}}\right)^2 d_2^{\chi}}{\eta \Psi \omega_2}\right] = \exp\left[-\frac{b_1 \gamma d_2^{\chi}}{\eta \Psi \omega_2}\right] \exp\left[-\frac{2 b_1 \gamma \sqrt{d_1^{\chi} d_2^{\chi}}}{\Psi \sqrt{\omega_2} \sqrt{\eta}}\right] \exp\left[-\frac{b_1 \gamma d_1^{\chi}}{\Psi}\right]
\tag{A6}
$$

furthermore, we apply Taylor series for $\exp\left[-\dfrac{2 b_1 \gamma \sqrt{d_1^{\chi} d_2^{\chi}}}{\Psi \sqrt{\omega_2} \sqrt{\eta}}\right]$ as (A7):

$$\exp\left[-\frac{2b_1\gamma\sqrt{d_1^\chi d_2^\chi}}{\Psi\sqrt{\omega_2}\sqrt{\eta}}\right]=\sum_{m=0}^{\infty}\frac{\left(-\dfrac{2b_1\gamma\sqrt{d_1^\chi d_2^\chi}}{\Psi\sqrt{\omega_2}\sqrt{\eta}}\right)^m}{m!}=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\dfrac{2b_1\gamma\sqrt{d_1^\chi d_2^\chi}}{\Psi\sqrt{\eta}}\right)^m}{m!}\omega_2^{-m/2} \tag{A7}$$

then we have:

$$\exp\left[-\frac{b_1\gamma\left(1+\sqrt{\dfrac{\omega_2\eta d_1^\chi}{d_2^\chi}}\right)^2 d_2^\chi}{\eta\Psi\omega_2}\right]=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\dfrac{2b_1\gamma\sqrt{d_1^\chi d_2^\chi}}{\Psi\sqrt{\eta}}\right)^m}{m!}\omega_2^{-m/2}\times\exp\left[-\frac{b_1\gamma d_2^\chi}{\eta\Psi\omega_2}\right]\times\exp\left[-\frac{b_1\gamma d_1^\chi}{\Psi}\right] \tag{A8}$$

Now by applying the equation $(x+y)^n=\sum_{t=0}^{n}\binom{n}{t}x^{n-t}y^t$ to (A5), the outage probability can demonstrate as follow:

$$F_{\gamma_{e2e}^{\max}}(\gamma)=1-\int_0^{\infty}\frac{a_1a_2}{b_1}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{l}\sum_{t=0}^{2n}\frac{K^{l+k}b_1^n b_2^k(2n!)}{l!n!(k!)^2 t!(2n-t)!}\left(\frac{\gamma d_2^\chi}{\eta\Psi\omega_2}\right)^n\left(\sqrt{\frac{\omega_2\eta d_1^\chi}{d_2^\chi}}\right)^t$$

$$\times\frac{(-1)^m\left(\dfrac{2b_1\gamma\sqrt{d_1^\chi d_2^\chi}}{\Psi\sqrt{\eta}}\right)^m\omega_2^{-m/2}}{m!}\times\exp\left[-\frac{b_1\gamma d_2^\chi}{\eta\Psi\omega_2}\right]\times\exp\left[-\frac{b_1\gamma d_1^\chi}{\Psi}\right]\times\omega_2^k e^{-b_2\omega_2}d\omega_2 \tag{A9}$$

$$F_{\gamma_{e2e}^{\max}}(\gamma)=1-a_1a_2 e^{\left(-\frac{b_1\gamma d_1^\chi}{\Psi}\right)}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{l}\sum_{t=0}^{2n}\frac{(-2)^m K^{l+k}b_1^{m+n-1}b_2^k(2n!)\gamma^{n+m}\left(d_1^\chi\right)^{\frac{m+t}{2}}\left(d_2^\chi\right)^{\frac{2n-t+m}{2}}}{l!n!(k!)^2 t!(2n-t)!m!\Psi^{n+m}\eta^{\frac{2n-t+m}{2}}}$$

$$\int_0^{\infty}\omega_2^{k+t/2-n-m/2}\times\exp\left[-\frac{b_1\gamma d_2^\chi}{\eta\Psi\omega_2}\right]\times e^{-b_2\omega_2}d\omega_2 \tag{A10}$$

apply [3.471,9] of the table of integral [15], the (A10) can be reformulated as:

$$F_{\gamma_{e2e}^{\max}}(\gamma)=1-2a_1a_2 e^{\left(-\frac{b_1\gamma d_1^\chi}{\Psi}\right)}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{l}\sum_{t=0}^{2n}\frac{(-2)^m K^{l+k}b_1^{m+n-1}b_2^k(2n!)\gamma^{n+m}\left(d_1^\chi\right)^{\frac{m+t}{2}}\left(d_2^\chi\right)^{\frac{2n-t+m}{2}}}{l!n!(k!)^2 t!(2n-t)!m!\Psi^{n+m}\eta^{\frac{2n-t+m}{2}}}$$

$$\left(\frac{b_1\gamma d_2^\chi}{\eta\Psi b_2}\right)^{\frac{2k+t-2n-m+2}{4}}\times K_{\frac{2k+t-2n-m+2}{2}}\left(2\sqrt{\frac{b_1 b_2\gamma d_2^\chi}{\eta\Psi}}\right) \tag{A11}$$

finally, the outage probability of the proposed system can be calculated as:

$$F_{\gamma_{e2e}^{\max}}(\gamma)=1-2a_1a_2 e^{\left(-\frac{b_1\gamma d_1^\chi}{\Psi}\right)}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{l}\sum_{t=0}^{2n}\frac{(-2)^m K^{l+k}b_1^{\frac{2n+3m+2k+t-2}{4}}b_2^{\frac{2k-t+2n+m-2}{4}}(2n!)\left(d_1^\chi\right)^{\frac{m+t}{2}}}{l!n!(k!)^2 t!(2n-t)!m!}$$

$$\times\left(\frac{d_2^\chi}{\eta}\right)^{\frac{2n-t+m+2k+2}{4}}\times\left(\frac{\gamma}{\Psi}\right)^{\frac{2n+3m+2k+t+2}{4}}\times K_{\frac{2k+t-2n-m+2}{2}}\left(2\sqrt{\frac{b_1 b_2\gamma d_2^\chi}{\eta\Psi}}\right) \tag{A12}$$

in (A12) $K_v(\bullet)$ is the modified Bessel function of the second kind and v^{th} order by substituting (A12) into (A2) we will have the expression of the ergodic capacity of the S-D link $C_{s,d}^{\max}$. End of Proof.