The Construction and Performance of a Novel Intergroup Complementary Code

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Abstrak

Atas dasar analisis untuk kode antar kelompok komplementer (intergroup complementary, IGC) dan korelasi dengan zona kode komplementer, kode IGC baru telah diusulkan untuk beradaptasi m-ary kode penyebaran orthogonal untuk sistem spektrum atau sistem quasi-synchronous di CDMA. Definisi dan konstruksi kode IGC baru disajikan dan diterapkan dalam makalah ini. Hasil penelitian dan simulasi teoretis menunjukkan bahwa keuntungan utama dari novel kode IGC seperti kode set dari igc baru lebih dari kode igc. Kode nol berkorelasi dengan panjang nol zona korelasi lebih panjang dari kode igc antarkelompok, tetapi lebih pendek dari kode igc antarkelompok. Dengan demikian panjang kode yang sama, kinerja auto-korelasi kode igc yang ditawarkan jauh lebih baik daripada kode IGC.

Kata kunci: kode antar kelompok komplementer, simulasi Matlab, kode zona berkorelasi nol

Abstract

On the basis of the analyses for intergroup complementary (IGC) code and zero correlation zone complementary code, a novel IGC code has been proposed to adapt M-ary orthogonal code spreading spectrum system or quasi-synchronous CDMA system. The definition and construction methods of the new IGC codes are presented and an applied example is given in this paper. Theoretical research and simulation results show that the main advantages of the novel IGC code are as following: The code sets of the novel IGC code is more than IGC code, but shorter than the intergroup IGC code. Under the same code length. The zero correlation zone length is longer than the intergroup IGC code, but shorter than the intergroup IGC code. Under the same code length, the auto-correlation performance of the novel IGC code is better than that of the IGC code, and both are of similar cross-correlation performance.

Keywords: modified intergroup complementary code, Matlab simulation, zero correlation zone code, generalized orthogonal code

1. Introduction

Due to multipath delay and Doppler shift characteristics of wireless channel, CDMA system performance was mainly affected of Inter symbol interference, Multi access interference and Adjacent cell interference. Recently, adopting spreading code with good correlation characteristic and multi-carrier CDMA technology, CDMA system can suppress interference efficiently [1, 2]. Complete Complementary Codes (CC) are multicodes. Though the sum of the squared multiple correlations over the entire displacement range is constantly zero, the number of CC sets is limited to $L^{1/3}$ and here the base number L stands for the overall length of code. In reference 3 to 7, the sets of Perfect Orthogonal Complementary Codes are presented; and it is informed that the sets with the ideal properties of autocorrelation and cross-correlation are arrived at through finding a solution of nonlinear equations, and they have gone beyond the limit of set size and subcode length of CC. Based on PC codes and orthogonal matrix, Intergroup Complementary Codes (IGC), which characterized by the properties of orthogonal codes with Zero Correlation Zone, are constructed. And IGC extend the size of code set [8-11]. Through the analyses, it is found that sidelobes of the periodic auto-correlation function of IGC Codes are greater; ZCZ length is smaller than sub-code length and the ZCZ length of cross-correlation function of intra-class IGC is equal to that of auto-correlation function. In this study, orthogonal codes with Zero Correlation Zone are used to improve IGC; and it turns out that the number of the improved IGC sets is more than that of the primary IGC sets; the properties of autocorrelation superior to those of IGC and the properties of cross-correlation balanced.

2. Intergroup Complementary Code

2.1. The basic concepts of orthogonal complementary code

The term 'Complementary Code' was first proposed by Marcel J. E. Golay. In 1982, the term was extended and named Complete Complementary Code by Suehiro, which contributes to theoretical basis of the application of CC code in CDMA system.

Possessing the ideal properties of auto-correlation and cross-correlation, the CC Code is subject to powerful binding correlations among code length, the number of codes, and the number of support users; besides, the number of code combination is limited; thus the application is restricted. As a superset of CC codes, the Perfect Complementary Code and the Complete Complementary Code are collectively referred to as the Orthogonal Complementary Code labelled as OCC(M, N, K), where M represents the number of the subcode, N subcode length, and K size of set. And both the periodic correlation function and the nonperiodic correlation function are ideal when combining the following conditions [12-14]:

$$C_{i,j}(t) = \sum_{m=1}^{M} \sum_{n=1}^{N} g_{i,m,n} g_{j,m,(n+t)_N}^*$$

$$= \begin{cases} MN, & i = j, n = 0 \\ 0, & others \end{cases}$$

$$R_{i,j}(t) = \sum_{m=1}^{M} \sum_{n=1}^{N-|t|} g_{i,m,n} g_{j,m,n+|t|}^*$$

$$= \begin{cases} MN, & i = j, n = 0 \\ 0, & others \end{cases}$$
(2)

others

2.2. Constructing Procedures of IGC Code

A given code set — M for the number of the subcode, N subcode length, and W_{min} the minimum of ZCZ length, is called Intergroup Complementary Code (IGC), which contains numbers of codes divided into G=M numbers of code groups, i.e. $I^{g}(g = 1, 2, \dots, G)$, and in each code group there are $K/G = N/W_{min}$ numbers of codes. Hence, if ZCZ of IGC code satisfies the properties of delta function distribution, in other words, ZCZ length of CCF of any two codes within groups is W_{\min} , the codes between groups are completely complementary and its ZCZ length is N. The following indicates the constructing procedures [9]:

Step I: Construct an initial binary PC code set denoted by the $C_k (k = 1, 2, \dots, M)$. The length of the *m*th subcode $C_{k,m} = [C_{k,m,1}C_{k,m,2}\cdots C_{k,m,W_{min}}]$ is W_{min} ; the sequence number $m = 1, 2, \cdots, M$.

Step2: Pick a $P \times P$ dimensional orthogonal matrix A satisfying $A = [a_{i,j}], a_{i,j} = \pm 1, (i, j = 1, 2, \dots, N/W_{\min}).$

Step3: By Kronecker product operation, each code of $C_k (k = 1, 2, \dots, M)$ is multiplied by each row of orthogonal matrix A respectively and intergroup orthogonal complementary code labelled as $\Gamma(M, N, W_{\min})$ is obtained. And the *p*th IGC code about *g* code groups is obtained by the operation on the gth initial complementary codes and the pth row of Matrix A and the mth subcode can be represented as $I_{p,m}^g = \left[a_{p,1}C_{g,m}, a_{p,2}C_{g,m} \cdots a_{p,p}C_{g,m} \right]$, where $m = 0, 1, \cdots, M$, $p = 0, 1, \dots, P$ and the length of each subcode is *N*.

2.3. Correlation Properties of IGC Code

IGC code constructed by initial perfect orthogonal complementary code has not only extended the number of PC code sets but has the following desirable properties:

First, for every auto-correlation function of IGC code, on both sides of its zero shift, there is a zero correlation window and the length of either side is W_{\min} ; outside the window nonzero side-lobes are of sparse but equal interval distribution.

Second, as for the cross-correlation function of any two IGC codes in the same intragroup, there is a zero correlation window and the length of either side is W_{\min} ; outside the window nonzero side-lobes are of sparse but equal interval distribution.

Third, cross-correlation functions of any two codes in intergroups are completely complementary. That is to say, cross-correlation function is zero at any point and the ZCZ length of either side is equal to N, the subcode length.

3. Constructing Procedures of Orthogonal Sequence Sets with ZCZ

In a sequence set $G(M,L,W) = \{g_{k,m,n}\}$, if non-periodic correlation functions of sequence g_i and sequence g_i satisfy the following conditions:

$$R_{g_{i},g_{j}}(\tau) = \sum_{k=0}^{L-1} g_{i,k} g_{j,(k+\tau) \text{mod}L}^{*}$$

$$= \begin{cases} \sum_{k=0}^{L-1} |g_{i,k}|^{2}, & i = j, \tau = 0 \\ & i = j, 0 < |\tau| < W_{ii} \\ 0, & \text{or} & i \neq j, |\tau| < W_{ij} \end{cases}$$
(3)

The sequence set is a generalized orthogonal sequence set, or referred to as orthogonal sequence sets with ZCZ. N numbers of orthogonal sequence sets (G^0 , G^1 , L, G^{N-1}) with ZCZ are defined; if any two sets (G^{n_1} and G^{n_2}) satisfy the following conditions [15-17]:

$$R_{g_{i}^{n_{1}},g_{i}^{n_{2}}}\left(0\right) = 0, \quad g_{i}^{n_{1}} \in G^{n_{1}}, g_{j}^{n_{2}} \in G^{n_{2}}$$

$$\tag{4}$$

then sequence sets (G^0 , G^1 , L, G^{N-1}) in Formula (4) are called sequence set as the orthogonal sequence set with ZCZ, where $0 \le i, j < M$.

The following indicates the constructing procedure:

Assume that $a = (a_0, a_1, L, a_{m-1})$ is a perfect sequence whose period is *m* and the autocorrelation functions of the sequences satisfy the following conditions:

$$R_{a}(\tau) = \sum_{i=0}^{m-1} a_{i} a_{(i+\tau) \bmod m}^{*} = \begin{cases} \sum_{i=0}^{m-1} |a_{i}|^{2}, \tau = 0\\ 0, & \text{others} \end{cases}$$
(5)

Meanwhile, $e^k = (e_0^k, e_1^k, L, e_{n-1}^k), (0 \le k < n)$ is an integer sequence and for a $0 \le i < n$ given, there must be a corresponding to match $e_i^k \in Z_m$. By using the sequences *a* and e^k , a sequence set including $B^k = \{b_i^k\} = (b_{i,0}^k, b_{i,1}^k, L, b_{i,n-1}^k)$ can be constructed, where $b_{i,j}^k = a_{(e_i^k+i) \mod m}$.

Let $C = \{c_i\} = (c_{i,0}, c_{i,1}, L, c_{i,n-1}), 0 \le i < n$ be an orthogonal sequence set, and then sequence set B^k and orthogonal sequence set C can be used to construct *n* numbers of sequence sets:

$$\boldsymbol{G}^{k} = \{\boldsymbol{g}_{j}^{k}\} = (\boldsymbol{g}_{j,0}^{k}, \boldsymbol{g}_{j,1}^{k}, \boldsymbol{L}, \boldsymbol{g}_{j,mn-1}^{k}), 0 \le j < n$$
(6)

where, $g_{j,i}^k = b_{q,i \mod n}^k c_{j,i \mod n}$, $0 \le i < mn, q = \lfloor i/n \rfloor$, q is the greatest integer but less than i/n.

If the sequence set G^k which constructed according to formula (6) satisfies the conditions of mutually orthogonal sequence set with ZCZ, then integer sequence e^k must satisfy the following conditions:

$$\mathbf{e}_{j}^{k} = \begin{cases} \left(\frac{jm}{n} + k\right) \mod m, & j < n - k \\ \left(\frac{jm}{n} + k + 1\right) \mod m, & j \ge n - k \end{cases}$$
(7)

where $n \mid m$, and $e^k = (e_0^k, e_1^k, L, e_{n-1}^k), (0 \le k < n)_{\circ}$

It is concluded that for any $0 \le k < n$, suppose that integer sequence satisfies all conditions in formula (7), then every sequence set constructed according to formula (6) is orthogonal sequence set with zero correlation zone, denoted as ZCZ(mn,n,m-2); In addition, when m=n, set G^0, G^1, L, G^{n-2} is mutually orthogonal; when m>n, set G^0, G^1, L, G^{n-1} is also mutually orthogonal. And the correlation values are determined by the following formula:

$$\boldsymbol{R}_{\boldsymbol{G}_{i}^{k},\boldsymbol{G}_{j}^{k}}(\tau) = \begin{cases} \sum_{l=0}^{mn-1} \left| \boldsymbol{G}_{i,l}^{k} \right|^{2}, & i = j, \tau = 0\\ 0, & i \neq j, \tau = 0\\ 0, & 1 \leq \tau \leq m-2\\ 0, & -(m-2) \leq \tau \leq -1 \end{cases}$$
(8)

4. A Novel IGC Code

4.1. Definition and Construction of Novel IGC Code

A given code set $\Psi(L, K, W_{\min}) - L$ for code length, *K* the number of set code and W_{\min} the minimum length of ZCZ, is called novel IGC code. The constructing procedures are as follows:

Step 1: By formula (6) and formula (7), a mutually orthogonal sequence set with ZCZ is constructed. And when m=n and in a set, $\{G^0, G^1, L, G^{n-2}\}$ is taken, i.e. there are *n*-1 numbers of code sets.

Step 2: Evenly divide every code in mutually orthogonal sequence set with ZCZ, which constructed in Step 1, into n segments or n numbers of subcode and the length of each subcode is m.

Step 3: take $a P \times P$ dimensional orthogonal matrix A and $A = [a_{i,j}], a_{i,j} = \pm 1, (i, j = 1, 2, \dots, N/W_{\min})$.

Step 4: By Kronecker product operation, each code of $\{G^0, G^1, L, G^{n-1}\}$ or $\{G^0, G^1, L, G^{n-2}\}$ is multiplied by each row of orthogonal matrix *A* and a novel IGC code $\Psi(L, K, W_{\min})$ is obtained. And the *p*th novel IGC code of the *K* numbers of code group is obtained by the operation on the *g*th orthogonal code with ZCZ and the *p*th row of Matrix *A* and the *q*th subcode can be represented as:

$$I_{p,q}^{g} = [a_{p,1}G_{g,q}, a_{p,2}G_{g,q} \cdots a_{p,p}G_{g,q}]$$
,

where,

 $\boldsymbol{g} = \begin{cases} 0, 1, \dots, n^2 - 1 \text{ (if } m > n \text{ and } n | m) \\ 0, 1, \dots, n^2 - n - 1 \text{ (if } m = n) \text{'}, \text{ and the code length is } \boldsymbol{P} \times \boldsymbol{m}. \end{cases}$ $\boldsymbol{q} = 0, 1, \dots, n - 1, \ \boldsymbol{p} = 0, 1, \dots, \boldsymbol{P} - 1$

4.2. Correlation Properties of Novel IGC Code

The constructed IGC code based on the above steps has the characteristics as follows: (1) Size of Novel IGC Code Set

From the construction mechanism of IGC, the size of novel IGC code set is (9). Now that set size of IGC code set is $P \times n$ and that of novel IGC code set is known in formula (9), it is evident that latter is n or n-1 times larger than the former.

$$\mathcal{K} = \begin{cases} \mathbf{P} \times \mathbf{n}^2, & \text{if } \mathbf{m} > \mathbf{n} \text{ and } \mathbf{n} | \mathbf{m} \\ \mathbf{P} \times (\mathbf{n}^2 - \mathbf{n}), & \text{if } \mathbf{m} = \mathbf{n} \end{cases}$$
(9)

(2) For the auto-correlation function of the novel IGC code, on either side of its zero shift, there is a zero correlation window and the length of one side is $W_{\min} = m - 2$; outside the window nonzero side-lobes are sparse and the number of zero side-lobes is larger than that of nonzero side-lobes.

Demonstration for the above statement: take the *p*th code of the *g*th group I_p^g for an example; Here only the positive offset is proved; as to the negative offset, it can be proved by the symmetry principle. If the positive offset is $xW_{\min} + \tau(|x| < P, x \in Z, |\tau| < W_{\min}, \tau \in Z)$, then the auto-correlation function is:

$$\begin{aligned} R_{g,p}(XW_{\min} + \tau) &= \sum_{q=1}^{n} I_{p,q}^{g} I_{p,q}^{g}(XW_{\min} + \tau) \\ &= \sum_{q=1}^{n} \sum_{l=1}^{P-x} a_{p,l} a_{p,l+x} G_{g,q} G_{g,q}(\tau) + \sum_{q=1}^{n} \sum_{l=1}^{P-x-1} a_{p,l} a_{p,l+x+1} G_{g,q} G_{g,q}(mP - \tau) \\ &= \begin{cases} n(m-2) \sum_{l=1}^{P-x} a_{p,l+x} a_{p,l}, & \tau = 0, 1 \le x < P \\ 0, & 0 < |\tau| < m-1, 0 \le x < P \end{cases} \end{aligned}$$

In this formula, $G_{g,q}(\tau)$ represents sequence G_g with numbers of chip offsetting rightward; and the final step can be derived from correlation properties of ZCZ sequence. Since the auto-correlation function of novel IGC code on [-m+2,-1] and [1,m-2] is zero, there is a ZCZ with the *m*-2 length of one side.

(3) In novel IGC code set, to the cross-correlation of any two codes, there is a zero correlation window and the length of ZCZ of either side is m-2; outside the window nonzero side-lobes are sparse and the number of zero side-lobes is obvious more than that of nonzero side-lobes.

Demonstration for the above statement: Without loss of generality, take the *p*th code I_p^g and the *j*th code of the *g*th group for an example; If where the positive offset is $xW_{\min} + \tau(|x| < P, x \in Z, |\tau| < W_{\min}, \tau \in Z)$, then the cross auto-correlation function is:

$$\begin{aligned} R_{g,p/q}(XW_{\min} + \tau) &= \sum_{q=1}^{n} I_{p,q}^{g} I_{p,j}^{g} (XW_{\min} + \tau) \\ &= \sum_{q=1}^{n} \sum_{l=1}^{P-x} a_{p,l} a_{j,l+x} G_{g,q} G_{g,q}(\tau) + \sum_{q=1}^{n} \sum_{l=1}^{P-x-1} a_{p,l} a_{j,l+x+1} G_{g,q} G_{g,q} (mP - \tau) \\ &= \begin{cases} 0, & 0 < |\tau| < m - 1, 0 \le x < P \\ n(m-2) \sum_{l=1}^{P-x} a_{p,l+x} a_{j,l} 1 \le x < P, \quad \tau = 0 \\ 0, & \tau = 0, x = 0 \end{cases} \end{aligned}$$

The final step can be derived from correlation properties of ZCZ sequence and orthogonality of matrix *A*. The above cross-correlation function on [-m+2, m-2] is zero. Therefore, for novel IGC code, there is an intragroup ZCZ with the *m*-1 length of one side.

4.3. An Example of Novel IGC Code

First, construct an orthogonal sequence set with ZCZ. Let '+' and '-' represent 1 and -1 respectively. With perfect sequence a = (++-+), orthogonal sequence C = (+++-,++++,++++), $e^0 = (0,1,2,3)$, $e^1 = (1,2,3,1)$, $e^2 = (2,3,1,2)$, by using formula

(6) and formula (7), *n*-1=3 numbers—only one provided here—of mutually orthogonal sequence set with ZCZ can be constructed as follows:

 $G_0^0 = (++--,+-+,-++++)$ $G_1^0 = (++++,+--+,-++,++--)$

 $G_2^0 = (+--+,++++,--++,+-+-)$ $G_3^0 = (-+-+,-++,++++,-++-)$

Second, take orthogonal matrix A labelled as $\begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}$, and according to the

constructing principle of novel IGC code, the following can be obtained:

5. Simulation and Result

In Figure 1, auto-correlation function is given by taking for an example Ψ_1 . As can be seen from the Figure 1, for the auto-correlation function, there is a ZCZ whose length is 2, two side-lobes on offset 3 and offset -3; and in a code period, the number of outside window zero side-lobes is 19 and that of nonzero side-lobes is 10. In Figure 2, cross-correlation function of Ψ_1 and Ψ_2 is given.



Figure 1. Periodic auto-correlation of novel IGC code 1.

As seen from the Figure 2, there is a ZCZ whose length is 3; and in a code period, the number of outside window zero side-lobes is 17 and that of nonzero side-lobes is 12. In Figure 3, auto-correlation properties of IGC code with the same code period are given. In Figure 4, cross-correlation properties of IGC code are given. As can be seen from the graphs, for the auto-correlation function, there is a ZCZ whose length is 2; the values of its side-lobes are greater and the greatest is up to 28; and in a code period, the number of outside window zero side-lobes is 16 and that of nonzero side-lobes is 14. And for the cross-correlation function, there is also a ZCZ whose length is 2; the max value of cross-correlation is 8; and in a code period, the number of outside window zero side-lobes is 18 and that of nonzero side-lobes is 12.



Figure 2. Periodic cross-correlation of novel IGC code 1 and 2.



Figure 3. Periodic auto-correlation of IGC code 1.



Figure 4. Periodic cross-correlation of IGC code 1 and 2.

By the correlation characteristics of novel IGC code and IGC code, it is found that under the same periodic condition, ZCZ length of the novel IGC is more than that of the IGC code; autocorrelation side-lobes of the novel IGC reduce obviously; the number of outside window zero side-lobes is more than that of the IGC code; and cross- correlation characteristics of the two

The Construction and Performance of a Novel Intergroup Complementary ... (Wenzhun Huang)

types of code are well-matched. Therefore, under the same periodic condition, the characteristic of the novel IGC code is superior to those of the IGC code.

6. Conclusion

Taking advantage of orthogonal code with ZCZ and orthogonal matrix, the study constructs a new type of orthogonal sequence set with ZCZ ---- the novel IGC code. With a more numbers of sets and better correlation characteristics, the novel IGC code is not only applicable to the new code M-ary orthogonal code spread spectrum system or quasi-synchronous CDMA system but offers a new method for the generation of orthogonal code set.

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