TELKOMNIKA, Vol.11, No.3, September 2013, pp. 555~562
ISSN: 1693-6930, accredited A by DIKTI, Decree No: 58/DIKTI/Kep/2013
DOI: 10.12928/TELKOMNIKA.v11i3.1085

Reconstruction of Planar Multilayered Structures using Multiplicative-Regularized Contrast Source Inversion

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Abstrak

Metoda nir-destruktif adalah cara yang mampu untuk memberikan informasi mengenai kondisi dalam dari suatu objek, tanpa ada aksi pengerusakana. Metoda ini bekerja berdasarkan data yang didapatkan dari perhitungan atau pengukuran besaran eksternalnya. Problem rekonstruksi struktur datar yang berlapis ditawarkan dengan bantuan data gelombang penyebaran di sisi depan dan belakang struktur yang diamati. Masalah seperti ini adalah problem inversi, yang bersifat ill-posed. Sehingga dalam penyelesaiannya, selain metoda inversi matriks, juga diperlukan suatu prosedur regularisasi. Regularisasi multiplikatif yang digunakan memiliki kemampuan dalam mendeteksi perubahan parameter dielektrika yang selayaknya. Dengan bantuan metoda inversi Gauss-Newton, didapatkan nilai permitivitas lapisan, yang akan meminimalisir suatu fungsi harga tertentu. Dalam pengamtan yang dilakukan beberapa struktur memerlukan lebih lapisan dengan ketebalan dan jumlah lapisan yang berbeda-beda. Beberapa struktur memerlukan diskretisasi yang lebih halus atau jumlah langkah iterasi yang lebih banyak.

Kata kunci: *fungsi Green, gelombang electromagnetika, problem inversi, regularisasi multiplikatif, sumber kontrast*

Abstract

There is an increasing interest to have an access to hidden objects without making any destructive action. Such non-destructive method is able to give a picture of the inner part of the structure by measuring some external entities. The problem of reconstructing planar multilayered structures based on given scattering data is an inverse problem. Inverse problems are ill-posed, beside matrix inversion tools, a regularization procedure must be applied additionally. Multiplicative regularization was considered as an appropriate penalty method to solve this problem. The Gauss-Newton inversion method as an optimization procedure was used to find the permittivity values, which minimized some cost functions. Several dielectric layers with different thickness and profiles were observed. Some layers needed more discretization elements and more iteration steps to give the correct profiles.

***Keywords**: contrast source, electromagnetic waves, Green's function, inverse problems, multiple regularization*

1. Introduction

There is an increasing interest in using the mathematical model of inverse problems for getting understanding of many practical applications. Examples of such applications are ranging from detecting the shape and location of cracks inside a wall, through exploration of geophysical problems, to detecting of breast cancers [1-3]. These inverse problems make use of electromagnetic fields for determining the inner structure of objects, whose parts are distinguished by different dielectric constants or permittivities. By exposing electromagnetic fields to the structures, these inhomogeneous objects cause scattering fields. The scattering fields generated by objects with known permittivities can be analytically or numerically calculated by well known methods [4, 5, 6, 7], this problem is called direct or forward problems. The calculation of direct problems is straightforward and easy. On the other side, if the data about scattering fields is given, and the permittivity profile of the structure must be determined, the situation is different. Considering these inverse problems theoretically, leads to a mathematical expression which contains Fredholm integral equation of the first kind. The Fredholm equation of the first kind has the nature of ill-posedness. An ill-posed equation has at least one of the following characters: no solution exists, the solutions are not unique and the

Received March 25, 2013; Revised June 26, 2013; Accepted July 11, 2013

solution is not stable against noise [8]. Mathematically, the system matrix governing the problem has a very large condition number. In order to solve the inverse problems meaningfully, calculation methods must be combined with some regularization strategies; truncated singular value decomposition (TSVD), Tikhonov regularization, total variation (TV) or multiplicative regularization (MR), just to name a few [9]. Shea et al [10] use the Finite Difference Time Domain (FDTD) combined with TSVD to make some observations according to biomedical tissues. In [11] Charbonnier et al use total variation formulation, which is the basis of some multiplicative regularization (MR), to enhance the quality of image restoration through the ability for detecting some edges in image profile. Gilmore et al [12] compare some regularization strategies based on Born approximation, Tikhonov regularization and multiplicative regularization. The multiplicative regularization has the ability to detect sharp variation in permittivities. Kilic et al [13] show an approach combining an integral equation method and multiplicative regularization for problems inside a rectangular waveguide. Aly et al [14] solve similar problems in coaxial structures using the super-resolution technique root multiple-signal classification. Another strategies are combinations of some nature-inspired optimization methods, like genetic algorithms, neuronal networks etc [15, 16].

In this paper, an inverse problem combining the integral equation method and multiplicative regularization is considered to determine the dielectric properties of planar multilayered objects located in free space. The multilayered walls are illuminated by incident fields of different frequencies. On the front and back side of the walls, the scattering fields are calculated analytically by means of reflection and transmission of electromagnetic fields at and through multilayered walls [4]. This synthetic data is given for reconstructing the wall profile. Some structures consisting of different layers, thickness and permittivities are considered and the results are compared with the original structures.

2. Problem Statement

In an inverse problem, a certain inhomogeneous region is exposed by electromagnetic fields. In this work we concentrate the attention to planar multilayered structures, which consist of layers with unknown thickness and material properties, see Figure 1. Due to interaction between electromagnetic fields and materials some scattering fields are generated and observed for solving the inverse problem. In our case, scattering fields on the same side like the incident field are called reflected fields, and on the contrary side called transmitted fields. The superposition of the incident $E_{inc}(z)$ and scattering fields $E_s(z)$ yields the total field $E(z)$.

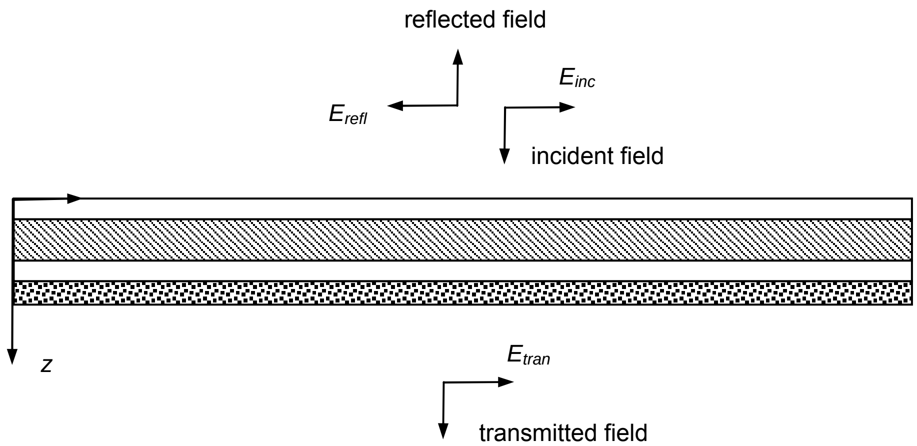

Figure 1. Scattering of electromagnetic field due to interaction with inhomogeneous materials.

The scattering fields must fulfill the Helmholtz partial differential equation [4]

$$\nabla^2 E_s(z) + k_o^2 E_s(z) = -k_o^2 v(z) E(z), \tag{1}$$

which connects the scattering fields, the contrast $v(z)$ and the total fields. The constrast is defined as

$$v(z) = \varepsilon_r(z) - 1 - j\frac{\sigma(z)}{\omega\varepsilon_o} \quad (2)$$

The solution of the above Helmholtz equation is given by the following data equation [1]

$$E_s(z) = k_o^2 \int_D G(z,z')v(z')E(z')dz' , \quad (3)$$

which equates the unknown quantities, contrast and total field, with the data, the scattering fields. The coordinate z in eq. (3) represents observation points outside the inhomogeneous region and $G(z,z')$ is the Green's function given for electromagnetic field in free space as [4,13]

$$G(z,z') = -\frac{j}{2k_o} e^{-jk_o|z-z\prime|} \quad (4)$$

The integration is carried out along the inhomogeneous region *D*. In order to make the problem formulation complete we need the so-called object equation, which combines the incident field and scattered field to get the total field

$$E(z) = E_{inc}(z) + k_o^2 \int_D G(z,z')v(z')E(z')dz' \quad (5)$$

Again the integration in eq. (5) is calculated along the domain *D*, however we set the observation points now inside the domain *D*.

Eq. (3) and (5) play the central role in solving the unknown quantities, the contrast $v(z)$ and the total field $E(z)$. The incident field $E_{inc}(z)$ is generally known, and the scattering field $E_s(z)$ can be approximately measured or if no measurement exists, it can be synthetically calculated. In some parts of the equations we have the unknowns as a product of each other, so that we can define additionally a fictive quantitive the so-called contrast source $w(z)$ [1]

$$w(z') = v(z')E(z') \quad (6)$$

3. Multiplicative-Regularization Contrast Source Inversion (MR-CSI)

Solving the above described inverse problem with computers begins with discretization of the inhomogeneous structures. The layers are discretized so fine, that inside each discretization element the contrast and the field can be assumed to be constant. Eq. (5) becomes

$$[E] = \{[I] - [G_S]\cdot[v]\}^{-1}[E_{inc}]. \quad (7)$$

$[G_S]$ is a matrix, whose elements describe the integration of Green's function in a discretization element with observation point inside the integration domain. Initially, the algorithms begins with a guess value for contrast, let be v_1 = 1.0 for all layers. So that with eq. (7) we can calculate initial value for the total field. Eq. (6) gives the initial value for the source contrast w. For the further iteration steps, the contrast will be improved by

$$[v_n] = [v_{n-1}] + \alpha_n[\Delta v_n]. \quad (8)$$

Δv_n is the search direction and α_n is the distance [17], which can be calculated by the described algorithm below. Using the point-matching method for each scattering data eq. (3) leads to a functional to be minimized by a certain optimization algorithm

$$C = \|d\|^2 \tag{9}$$

with
$$[d] = [E_s] - [G_D][v][E]^2 \tag{10}$$

The symbol $\|\cdot\|$ denotes L^2-norm. $[G_D]$ is a matrix, whose elements describe the integration of Green's function in a discretization element with observation point outside the integration domain. In order to find the minimum of the cost functional, the Gauss-Newton Inversion method as an optimization method is used [17], which makes use the derivative of the cost functional during the computation intensively. A derivative of the cost functional is given in form of the following Frechet derivative with respect to the variable v

$$[C_v] = [G_D]\{[I] - [G_S]\cdot[v]\}^{-1}[E] \tag{11}$$

The system matrix in eq. (10) is principally ill-conditioned, and due to noises contaminating the value of scattered fields, the minimum of this functional leads to physically meaningless solution. In order to alleviate this problem, in this paper we use multiplicative regularization given by a multiplication between the above functional and a penalty function

$$C_M = C \cdot C_{MR} \ , \tag{12}$$

which is defined according to [1] as

$$C_{MR} = \int_\Omega \frac{|\nabla v|^2 + \delta_n^2}{b_n^2}\, d\Omega' \tag{13}$$

with $b_n^2 = N(|\nabla v_n|^2 + \delta_n^2)$ and $\delta_n^2 = \frac{C}{dz^{3/2}}$, N is the number of discretized elements, dz is the width of the discretization small element and ∇v_n is the gradient of the contrast.

Applying the regularized cost functional (12) in Gauss-Newton optimization algorithm, yields the following equation, which can be solved to get the search direction [12]

$$([C_v^*]\cdot[C_v] - C\cdot[\mathcal{L}])[\Delta v_n] = [C_v^*]\cdot[d] + C\cdot[\mathcal{L}]\cdot[v] \tag{14}$$

$[C_v^*]$ is the adjoint of the Frechet derivatives $[C_v]$ and the operator $[\mathcal{L}]$ represents the discrete form of $\nabla\cdot(b_n^2\nabla)$ operator and provides an edge-preserving regularization. If the one specific region of observed region is homogeneous, the weight b_n^2 will be almost constant, therefore the operator $[\mathcal{L}]$ will be approximately equal to $b_n^2\nabla^2$, which favors smooth solution. On the other hand, if there is a discontinuity, the corresponding b_n^2 will be small. The discontinuity will not be smoothed out [12].

The distance of variation α_n is calculated by a line search algorithm [16]. Line search methods work well for finding a minimum of a quadratic function. They tend to fail miserably when searching a cost surface with many minima, because the vectors can totally miss the area where the global minimum exists. Here, we use the Polak-Ribiere formulation [17]

$$\alpha_n = \alpha_{n-1} - \frac{\|[d]\cdot([C_v]_n - [C_v]_{n-1})\|}{C} \tag{15}$$

4. Results and Analysis

In this section, the algorithm is verified by several cases. Some test structures consisting of one or more layers with known permittivities and thicknesses are given. The incident field impinges the planar multilayered wall perpendicular, and is generated in frequency intervall of 9 GHz to 10 GHz (101 frequency points). The scattered data, i.e. reflected and transmitted fields, can be calculated analytically [4]. This scattered data is used in the following reconstruction process. In this way we can avoid the so-called ‘inverse crimes’ [18].

4.1. One-layer structures

Based on synthetic data from analytical calculation, four dielectric layers ε_r = 2 with different thicknesses are observed, the thickest ist 18 mm and the thinnest is 6 mm. As described in section 3, the observed region (z=0 .. 20 mm) is discretized in 40 small elements, in each of the elements constant permittivity is assumed. Beginning with ε_r = 1 for all 40 small elements, the inverse procedure refines the permittivity values iteratively. Figure 2 shows the reconstructed profiles along the observed region and figure 3 gives the cost (eq. 9) at each iteration. For thicknesses D=18 mm, 14 mm and 10 mm, the cost decreases very fast and at 10th-iteration the value is already smaller that 10^{-6}.

The profiles for D=18 mm (stared curve), D=14 mm (curve with circles), and D=10 mm (dashed line curve) can be reconstructed very well. The deviation from the exact value (1.0 or 2.0) is maximal 0.004. The profile for D=6 mm (solid curve in figure 2) is actually calculated also very good (with maximal error of about 0.016), however the the cost obtained is just below 0.1 (solid curve in figure 3). The reconstruction of thin layer is indeed a very challenging problem, it is the question about the resolution of the reconstruction.

In order to verify the capability of the algorithm to detect thin layer, in following a dielectric layer (ε_r = 2) with thickness 4 mm is observed. The whole observed region is 20 mm thick, and discretized into N = 20 small equally thick elements. It means, at frequency 10 GHz, a small element of the thickness 1 mm is much more thinner than the wavelength inside the dielectric layer (21.21 mm). Theoretically N = 20 should be sufficient.

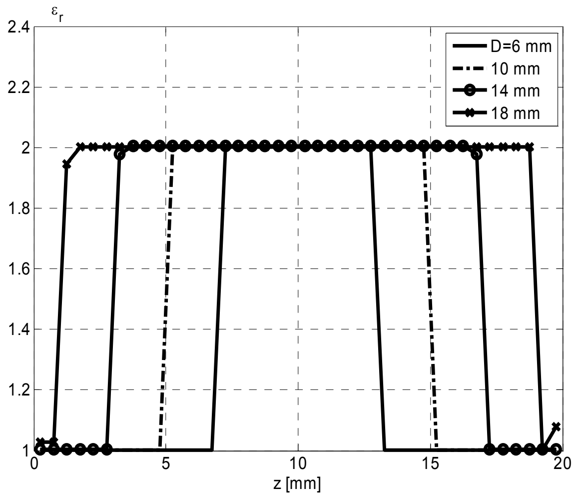

Figure 2. Reconstructed profile of dielectric layer with ε_r = 2 and different thicknesses.

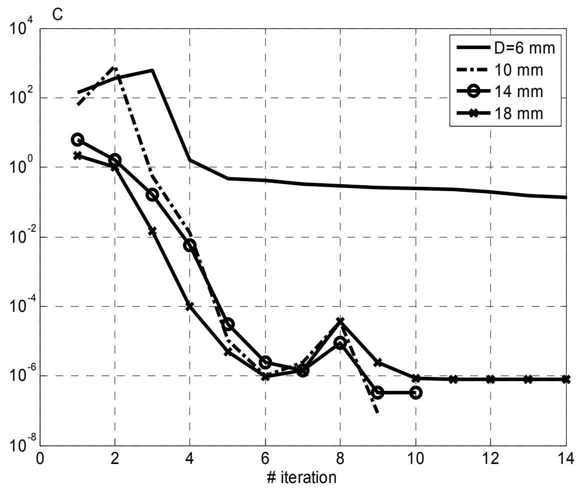

Figure 3. Convergence of the reconstruction of dielectric layer (ε_r = 2) different layers.

Figure 4 show the reconstructed profile for N = 20 (curve with circles). Instead of a profile with constant permittivity ε_r = 2 at position intervall from 8 mm to 12 mm, and ε_r = 1 outside this intervall, we find relative good approxilatiom of the profile, the cost function C > 1.0. Increasing the discretization to N = 40, we get better profile, with the cost function of about 0.05 (Figure 5, curve with triangles at iteration 15). We guess, that the Gauss-Newton inversion method catched the local minimum of the cost function, so at iteration 18 we try to use new start values for the contrast v. However, the cost function has higher value (> 1.0), so we stopped the process.

For N=80 (dashed curve in Figure 4 and 5), the dielectric layer still cannot be reconstructed correctly, the cost function is about $5x10^{-3}$. At the last, we try to use higher discretization elements (N=160). The solid curve in Figure 4 and 5 shows very good result and the cost function is about $4x10^{-5}$.

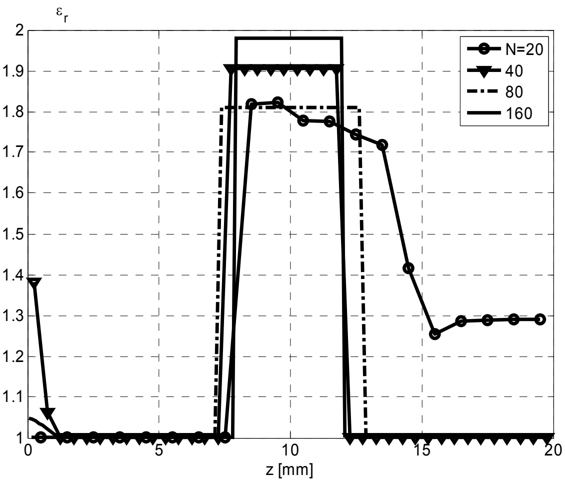

Figure 4. Reconstruction of 4 mm dielectric layer with ε_r = 2 by different discretization.

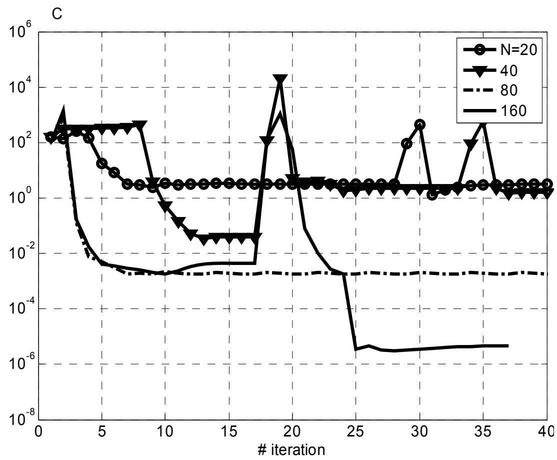

Figure 5. Convergence of the reconstructing process of 4mm-dielectric layer (ε_r = 2).

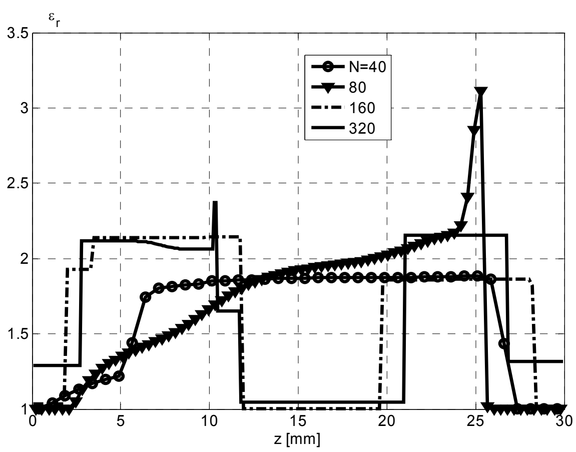

Figure 6. Reconstruction of three-layer dielectric by different discretization.'

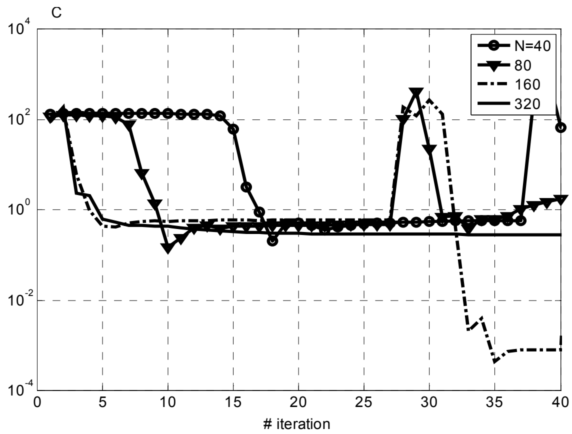

Figure 7. Convergence of the reconstructing process of three-layer dielectric.

4.2. Three-layered structures

The layers under consideration consist of 10 mm dielectric (ε_r = 2), 8 mm air and 8 mm dielectric (ε_r = 2). In front and back side there is a 2mm-air layer. In the reconstructing process the overall region observed is 30 mm width.

The reconstruction process uses 160 frequency points in intervall 9 GHz to 10 GHz. Coarse discretizations (N=40 and N=80) are not able to reconstruct the profile, the end cost is about 0.6 even after a restart. A discretization of N=160, leads to better reconstructed profile (dashed line curve). In intervall z = 0 to 2 mm the permittivity is about 1.0, between 2 mm to 12 mm, the permittivity reconstructed is between 1.9 to 2.15 from the exact value of 2.0. Between 12 mm and 20 mm, the permittivity reconstructed is almost exact, in intervall 20 mm to 28 mm the optimization gives the permittivity value of about 1.8 (exact value is 2.0), and in the last intervall 28 mm to 30 mm, almost the exact value of 1.0 can be reconstructed. The cost of this value combination is in about 8.0×10^{-4}. This is obtained after setting the initial contrast for second time.

A finer discretization of N=320 yields worse result, with cost of about 0.3. Although the profile can be reconstructed well. A reason could be, we have more unknowns than the number of frequency points. So that, the choice N=160 is the optimal solution.

5. Conclusion

The method combining the integral equation method based on the used of Green's function and the multiplicative regularization is able to reconstruct several dielectric profile with different thickness and number of layers. Layers with very thin thickness is more challenging to reconstruct, more discretization elements are needed to get good result. Structures with several layers including thin layer are more challenging. There is a need to use more frequency points for better reconstruction.

There are some open questions; how the line search gives impact to the quality of the result. The choice of the initial value for the contrast can indeed have effects to the result, so that we must try to start the optimization process with different starting values. Another way could be the use of global optimization method.

Acknowledgments

The authors would like to thanks the Directorate General of Higher Education of the Ministry of Education and Culture of the Republic of Indonesia for granting the research under the contract 005/K3.KU/2012, date 01/03/2012

References

[1] Abubakar A,Habashy TM, Druskin VL, Knizhnerman L, Alumbaugh D. 2.5D forward and inverse modeling for interpreting low-frequency electromagnetic measurements. *Geophysics*. 2008; 73(4): F165–F177.
[2] Zhang W, Hoorfar A, Li L. Through-the-wall target localization with time reversal music method, *Progress in Electromagnetics Research*. 2010; 106: 75-89.
[3] Meaney P,Fanning M, Li D, Poplack S, Paulsen K. A clinical prototype for active microwave imaging of the breast," *IEEE Transactions on Microwave Theory and Techniques*. 2000; 48(11): 1841-1853.
[4] Balanis C. *Advanced engineering electromagnetics*. 2nd ed. Wiley, New York, 2012.
[5] Alaydrus M. Hybrid methods in designing Sierpinksi gasket antennas.*Telkomnika*. 2010; 8(3): 225-234.
[6] Alaydrus M. *Determining the conductivity and relative permittivity of fluids in microwave frequencies using waveguide structures*. Int. Conf. on Instrumentation, Communication, Information Techn. and Biomedical Eng. (ICICI-BME). Bandung. 2009.
[7] Musa S, Sadiku MNO. Finite element approach for coupled striplines embedded in dielectric material.*Telkomnika*. 2013; 11(1): 47-54.
[8] Groetsch CW. Integral equations of the first kind, inverse problems and regularization: a crash course, *Journal of Physics: Conference series*. 2007; 73. 012001, doi:10.1088/1742-6596/73/1/012001.
[9] Aster RC, Borschers B, Thurber CH. *Parameter Estimation and Inverse Problems*. 2nd. Ed., Burlington: Elsevier. 2012.
[10] hHea JD, Van Veen BD, Hagness SC. A TSVD analysis of microwave inverse scattering for breast imaging, *IEEE Trans. Biomedical Eng*. 2012; 59(4): 936-945.
[11] Charbonnier P, Blanc-Feraud L, Aubert G, Barlaud M. Deterministic edge-preserving regularization in computed imaging. *IEEE Trans. Image Processing*. 1997; 6(2): 298–311.
[12] Gilmore C, Mojabi P, LoVetri J. Comparison of an enhanced distorted born iterative method and the multiplicative-regularized contrast source inversion method. *IEEE Trans. Antennas Propag.* 2009; 57(8): 2341–2251.
[13] Kilic E, Siart U, Eibert T. Regularized 1-D dielectric profile inversion in a uniform metallic waveguide by measurement and simulation. *IEEE Trans. On Microwave Theory and Techniques*. 2012; 60(5):1437–1443.
[14] Aly OAM, Omar AS. Reconstructing stratified permittivity profiles using super-resolution techniques. *IEEE Trans. On Microwave Theory and Techniques*. 2006; 54(1):492–498.

[15] Brovko A, Murphy E, Yakovlev V. Waveguide microwave imaging: Neural network reconstruction of functional 2-D permittivity profiles. *IEEE Trans. Microw. Theory Tech.* 2009; 57(2): 406–414.
[16] Requena-Perez M, Albero-Ortiz A, Monzo-Cabrera J, Diaz-Morcillo A. Combined use of genetic algorithms and gradient descent optmization methods for accurate inverse permittivity. *IEEE Trans. Microw. Theory Tech.* 2006; 54(2): 615–624.
[17] Chong E, Zak S. *An Introduction to Optimization*. 4th ed.,Wiley, New York, 2013.
[18] Gilmore C, LoVetri L. Enhancement of microwave tomography through the use of electrically conducting enclosures, *Inverse Problems*. 2008; 24: 035008 (21).