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Internet Pricing on Bandwidth Function Diminished with Increasing Bandwidth Utility Function

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Abstract

In this paper we analyze the internet pricing schemes based on bandwidth function diminished with increasing bandwidth utility function with 3 pricing strategies for homogeneous and heterogeneous consumer. The new proposed pricing schemes with this utility function will give the information to the internet service providers (ISP) in maximizing profits and provide better service quality for users. The Models on every type of consumer are applied to the data traffic in Palembang local server. LINGO 11.0 is used to compute the nonlinear programming problem to get the optimal solution. The results showed that for each case based on 3-pricing scheme, ISPs get better profit by choosing all three schemes in consumers type of homogenous case while for heterogeneous cases on willingness to pay and based on demand of the consumers*, ISPs can select flat fee scheme to gain higher profit rather than those two other schemes.*

Keywords*: utility functions, the function of diminished bandwidth with increasing bandwidth, pricing schemes, consumer homogeneous, heterogeneous consumers*

1. Introduction

Internet has an important role in the economy and education around the world. The Internet is a multimedia library, because it has a lot of complete information. Complete information and quickly make consumers interested in becoming a consumer internet services. Consumers who make a lot of Internet Service Providers (ISPs) compete to provide services of the highest quality (Quality of Service) and the optimal prices for consumers [1]-[3]. In addition in maintaining the quality of service and optimal prices for consumers, Internet Service Provider (ISP) should also consider profits.

The research on differentiated network in general network architecture with quality of service are due to [4]-[7] which then are improved by [8] in multi QoS networks and [9] in multi service networks. In particular, the recent research focus on wireless mesh QoS network architecture are due to [10],[11] that mainly discuss the advanced technology in communication network.

There are some assumptions for utility function to be applied in the model but the researchers usually use the bandwidth function with fixed loss and delay and follow the rules that marginal utility as bandwidth function diminishing with increasing bandwidth [4-8]. The other reason dealing with the choices of utility function is that the utility function should be differentiable and easily to be analyzed the homogeneity and heterogeneity that impacts the choice of pricing structure for the companies. Kelly [12]-[14] also contends that the utility function also can be assumed to be increasing function, strictly concave and continuously differentiable.

In [15], the finding of internet charging is based on analytical steps and on Cobb-Douglass utility function. Other useful utility functions are provided, but only a few were discussed. Sometimes, it is more likely have a good advantage if dealing with finding the solution numerically rather than analytically, if involving many variables and parameters.

So, we provide to search the optimal solutions numerically for three internet pricing schemes which are flat fee, usage-based, and two-part tariff for homogeneous and heterogeneous consumers based on function of bandwidth diminished with increasing bandwidth using LINGO 11.0 [16].

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2. Research Method

In this paper, the internet pricing schemes will be completed by the program LINGO 11.0 to obtain the optimal solution. The solution obtained will help determine the optimal price on the flat fee, usage-based, and two-part tariff for internet pricing schemes.

3. Model

The general form of utility function based on the Function of Diminished Bandwidth with Increasing Bandwidth: $U_{kj} = U_{0j} + W_j \ \mathrm{In} \frac{X_{kj}}{L_{mj}}$

j class is divided into classes during peak hour (X) and off-peak hour (Y) to obtain:

$$U_{kx} = U_{0x} + W_x \mathrm{In} \frac{X_{kx}}{L_{mx}} \quad (1)$$

$$U_{ky} = U_{0y} + W_y \mathrm{In} \frac{X_{ky}}{L_{my}} \quad (2)$$

where

$$\begin{aligned} &U_0 = U_{0x} + U_{0y} \\ &W_x = a \ and \ W_y = b \\ &X_{kx} = X \ and \ X_{ky} = Y \\ &L_{mx} = X_m \ and \ L_{my} = Y_m \end{aligned} \quad (3)$$

Then, it will be

$$U(x,y) = U_{0x} + W_x \mathrm{In} \frac{X_{kx}}{L_{mx}} + U_{0y} + W_y \mathrm{In} \frac{X_{ky}}{L_{my}} \quad (4)$$

$$U(x,y) = U_0 + a \ \mathrm{In} \frac{X}{X_m} + b \ \mathrm{In} \frac{Y}{Y_m} \quad (5)$$

Then, Eq(5) becomes

$$U(x,y) = U_0 + a \ \mathrm{In} \frac{X+1}{X_m+1} + b \ \mathrm{In} \frac{Y+1}{Y_m+1} \quad (6)$$

This change was made to simplify the calculation when the minimum consumption level X_m dan Y_m respectively, as well as the level of consumption during peak hours and off-peak hours X dan Y can produce a minimum value of 0 than creating negative value.

For the case of homogeneous consumers,

$$\mathrm{Max} \ U_0 + a \ \mathrm{In} \frac{X+1}{X_m+1} + b \ \mathrm{In} \frac{Y+1}{Y_m+1} - P_x X - P_y Y - PZ \quad (7)$$

Subject to

$$X \leq \bar{X} Z \quad (8)$$

$$Y \leq \bar{Y} Z \quad (9)$$

$$U_0 + a \ \mathrm{In} \frac{X+1}{X_m+1} + b \ \mathrm{In} \frac{Y+1}{Y_m+1} - P_x X - P_y Y - PZ \geq 0 \quad (10)$$

$$Z = 0 \text{ or } 1 \quad (11)$$

For the case of heterogeneous upper and lower class consumers, suppose that there are m consumers upper class (i = 1) and n lower class consumers (i = 2). It is assumed that each of these heterogeneous consumers have a limit on the same $\bar{X}$ the level of consumption during peak hours and $\bar{Y}$ the level of consumption during off-peak hours $a_1 > a_2$ dan $b_1 > b_2$.

Consumer Optimization Problems:

$$\text{Max}_{X_i,Y_i,Z_i} U_0 + a \ \text{In} \frac{X+1}{X_m+1} + b \ \text{In} \frac{Y+1}{Y_m+1} - P_x X_i - P_y Y_i \geq 0 \quad (12)$$

Subject to

$$X_i \leq \bar{X}_i Z_i \quad (13)$$

$$Y_i \leq \bar{Y}_i Z_i \quad (14)$$

$$U_0 + a \ \text{In} \frac{X+1}{X_m+1} + b \ \text{In} \frac{Y+1}{Y_m+1} - P_x X_i - P_y Y_i \geq 0 \quad (15)$$

$$Z_i = 0 \text{ or } 1 \quad (16)$$

As for the case of heterogeneous high level of usage and low usage level consumers, suppose assumed two types of consumers, consumer consumption level is high (i = 1) with a maximum consumption rate of $\bar{X}_\iota$ and $\bar{Y}_1$ and consumer usage rate is low (i = 2) with a maximum consumption rate of $\overline{X_2}$ dan $\bar{Y}_2$. There are *m* consumers of type 1 and type 2 *n* consumers with $a_1 = a_2 = a$ dan $b_1 = b_2 = b$.

4. Result and Analysis

Pricing schemes internet problems solved using the same model by [17] with the parameter values used are in Table 1-3 below.

Table 1. Parameter Values Used in Case 1-3

Parameter	Case 1	Case 2	Case 3
a	4	4	4
b	3	3	3
x	2656.17	2656.17	2656.17
y	5748.88	5748.88	5748.88
X_m	20.89	20.89	20.89
Y_m	49.43	49.43	49.43
P_x	0	0.2×10^{-1}	0.2×10^{-1}
P_y	0	0	0
P	58.5	0	1.8
Z	1	1	1
U_0	1	1	1

Table 2. Parameter Values Used in Case 4-6

Parameter	Case 4	Case 5	Case 6
X_1	2656.17	283.8350	282.5491
Y_1	5748.88	212.6416	211.6775
X_2	2314.40	69.21977	69.14995
Y_2	2406.87	45.82485	45.77830
X_m	20.89	20.89	20.89
Y_m	49.43	49.43	49.43
P_x	0	0.1	0.1
P_y	0	0.1	0.1
P	49.1	0	0.1
Z_1	1	1	1
Z_2	1	1	1
U_{01}	1	1	1
U_{02}	1	1	1

Table 3. Parameter Values Used in Case 7-9

Parameter	Case 7	Case 8	Case 9
X_1	2656.17	222.93	221.61
Y_1	5748.8	148.30	147.42
X_2	2314.4	69.937	69.847
Y_2	2406.8	46.303	46.243
X_m	20.89	20.89	20.89
Y_m	49.43	49.43	49.43
P_x	0	0	0.1
P_y	0	0.1	0.1
P	49.16	0	0.1
Z_1	1	1	1
Z_2	1	1	1
U_{01}	1	1	1
U_{02}	1	1	1

The values of the parameters are substituted into the model, then we have:

Case 1: For flat fee pricing schemes, set $P_x = 0, P_y = 0, \text{and } P > 0$ meaning that the prices used by the service provider has no effect on the time of use.

Case 2: For usage-based pricing scheme by setting $P_x > 0, P_y > 0, \text{and } P = 0$, meaning that service providers deliver differentiated prices, the price of consumption during peak hours and at off-peak hours.

Case 3: For pricing scheme with a two-part tariff, set $P_x > 0, P_y > 0, \text{and } P = 0$ means that service providers deliver differentiated price, i.e. the price of consumption during peak hours and off-peak hours.

Case 4: For pricing scheme by setting a flat fee, then $P_x = 0, P_y = 0, \text{and } P > 0$, meaning that the prices used by the service provider has no effect on the time of use, then consumers will choose the maximum consumption rate $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y}, \text{and } Y_2 = \bar{Y}$.

Case 5: For usage-based pricing scheme by setting $P_x > 0, P_y > 0, \text{and } P = 0$, then a maximum consumption rate $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y}, \text{and } Y_2 = \bar{Y}$. Then consumers will choose the maximum consumption rate $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y}, \text{and } Y_2 = \bar{Y}$.

Case 6: For two-part tariff pricing scheme, set $P_x > 0, P_y > 0, \text{and } P = 0$, with a maximum consumption rate $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y}, \text{and } Y_2 = \bar{Y}$. Then consumers will choose the maximum consumption rate $X_1 = \bar{X}, X_2 = \bar{X}, Y_1 = \bar{Y}, \text{and } Y_2 = \bar{Y}$.

Case 7: For flat fee pricing schemes set $P_x = 0, P_y = 0, \text{and } P > 0$, by choosing the level of consumption $X_1 = \overline{X_1}, Y_1 = \overline{Y_1}, \text{ atau } X_2 = \overline{X_2}, Y_2 = \overline{Y_2}$.

Case 8: For usage-based pricing scheme, set $P_x > 0, P_y > 0, \text{and } P = 0$, by choosing the level of consumption $X_1 = \overline{X_1}, Y_1 = \overline{Y_1}, \text{ atau } X_2 = \overline{X_2}, Y_2 = \overline{Y_2}$.

Case 9: For Pricing scheme with a two-part tariff, set $P_x > 0, P_y > 0, \text{and } P = 0$, by choosing the level of consumption $X_1 = \overline{X_1}, Y_1 = \overline{Y_1}, \text{ atau } X_2 = \overline{X_2}, Y_2 = \overline{Y_2}$.

Table 4 below explains the data usage for peak and off-peak hours served by local server.

Table 4. Data Usage for Peak Hours and Off-Peak Hours

	***Mail* (*byte*)**	***Mail* (*kbps*)**
$\bar{X} = \overline{X_1}$	2719914.01	2656.17
$\overline{X_2}$	2369946.51	2314.40
X_m	21388.28	20.89
$\bar{Y} = \overline{Y_1}$	5886849.92	5748.88
$\overline{Y_2}$	2464637.66	2406.87
Y_m	50619.47	49.43

where
1. $\bar{X}$ or $\overline{X_1}$ is the maximum possible level of consumption during peak hours both in units of kilo bytes per second.
2. $\overline{X_2}$ is the maximum possible level of consumption during off-peak hours in units of kilo bytes per second.
3. X_m is the most low level of consumption during peak hours in units of kilo bytes per second.
4. $\bar{Y}$ or $\overline{Y_1}$ is the maximum possible level of consumption both during peak hours in units of kilo bytes per second.
5. $\overline{Y_2}$ maximum possible level of consumption during peak hours in units of kilo bytes per second.
6. Y_m is the most low level of consumption during off-peak hours in units of kilo bytes per second.

Table 5. describe the optimal solution of using utility function of the function of bandwidth diminished with increasing bandwidth.

Table 5. Solution for Utility Functions of the Function Bandwidth Diminished with Increasing Bandwidth

	Case								
	1	2	3	4	5	6	7	8	9
Profit	58.511	58.511	58.511	245.815	171.952	171.754	204.46	134.621	134.428

We can see from Table 5 that in homogenous case, we obtain the same maximum profit for all case of flat fee, usage based and two part tariff schemes. In other case, when we deal with heterogeneous high end and low end user consumers, the maximum profit is achieved when we apply the usage based. The last case when dealing with high and low demand users, again, the usage based yield the maximum profit.

If we compare the result in [18],[19], we have slightly difference. If using the modified Cobb-Douglass utility function, the maximum profit achieved when we apply the flat fee and two part tariff schemes for homogenous case. For heterogeneous case, maximum profit occurs when we apply the flat fee and two part tariff schemes. In our utility function, the three schemes yield the same profit in homogeneous case, while in heterogeneous case we obtain higher profit if we apply usage based. The advantages of using the utility function we choose that the provider has other choices in applying pricing schemes that attract the customer to join the schemes.

5. Conclusion

According to above result we can conclude that if ISP intends to obtain maximum profit, ISP can choose all three schemes if dealing with homogenous case. For heterogeneous case based on willingness to pay and based on demand of the consumers, ISP can adopt flat fee to gain maximum profit. For further research, we can consider other utility functions that fit with ISP choices to maximum their benefit.

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