

## Progression approach for image denoising

**Bilal Charmouti<sup>\*1</sup>, Ahmad Kadri Junoh<sup>2</sup>, Mohd Yusoff Mashor<sup>3</sup>, Najah Ghazali<sup>4</sup>,  
Mahyun Ab Wahab<sup>5</sup>, Wan Zuki Azman Wan Muhamad<sup>6</sup>,  
Zainab Yahya<sup>7</sup>, Abdesselam Beroual<sup>8</sup>**

<sup>1,2,4,6,7</sup>Institute of Engineering Mathematics, Universiti Malaysia Perlis (UniMAP), Perlis, Malaysia

<sup>3</sup>School of Mechatronic, Universiti Malaysia Perlis (UniMAP), Perlis, Malaysia

<sup>5</sup>School of Environmental Engineering, Universiti Malaysia Perlis (UniMAP), Perlis, Malaysia

<sup>8</sup>Department of Computer Science, International Islamic University Malaysia, Malaysia

\*Corresponding author, e-mail: dybilel@yahoo.fr

### Abstract

*Removing noise from the image by retaining the details and features of this treated image remains a standing challenge for the researchers in this field. Therefore, this study is carried out to propose and implement a new denoising technique for removing impulse noise from the digital image, using a new way. This technique permits the narrowing of the gap between the original and the restored images, visually and quantitatively by adopting the mathematical concept "arithmetic progression". Through this paper, this concept is integrated into the image denoising, due to its ability in modelling the variation of pixels' intensity in the image. The principle of the proposed denoising technique relies on the precision, where it keeps the uncorrupted pixels by using effective noise detection and converts the corrupted pixels by replacing them with other closest pixels from the original image at lower cost and with more simplicity.*

**Keywords:** arithmetic progression, denoising technique, image, image processing, impulse noise, noise

**Copyright © 2019 Universitas Ahmad Dahlan. All rights reserved.**

### 1. Introduction

Image is considered as a powerful platform to carry and to transmit information between people, where it is very important in a lot of fields such as biology, astronomy, industrial, medical and surveillance [1]. Thus, it attracts the attention of a lot of researchers in restoring the unknown original image from the degraded image caused by any factors that may degrade or reduce the image quality (e.g. blur). One of the factors is the noise that may be introduced in the image with many forms (additive, multiplicative and impulsive), through one of these phases: image acquisition, transmission or storage [2]. The noise is a parasitic or weird information that affects the visual aspect of the image by changing the pixel values and it makes the other subsequent image processing such as segmentation, compression, analyses, extraction of information, classification and etc more difficult. The image is corrupted due to the fact that there are various types of noise such as the Gaussian noise, Poisson noise, Speckle noise, Salt and Pepper noise and many more fundamental noise types in the case of digital images [3].

It should be noted that the image restoration concept varies according to the degradation factor where it might be deblurring [4], inpainting [5] and etc. In the case of degradation caused by the noise, the image restoration can also be called as image denoising, noise removal, or noise reduction, where it was firstly introduced by Wiener and Kolmogorov in the 1940s [2]. Hence, Denoising image is a critical and primary phase in the image processing (preprocessing phase), aiming to remove or reduce the noise from the noisy image by preserving the image features, using the various techniques (filters). The observed image (e.g. photograph, chart) was initially digitized and stored in the digital memory as a matrix of binary numbers, where this digital image can be processed [6].

The function principle of the spatial domain filtering is to replace the corrupted pixel (noise) by another value from its neighbors in the noisy image, such as the median, the mean and so on. The manner of selecting this value, is one of the most important points which affect the effectiveness and efficiency of the image denoising method. This study aims to treat this by proposing a new technique for detecting and replacing the corrupted pixel, by integrating the mathematical concept " Arithmetic Progression " in image denoising. This is in order to get

the best similarity with the original image. The performance of the denoising techniques varies from one filter to another filter, and it can be evaluated according to various criteria, which are:

- The noised image (input): the type of noise (additive, multiplicative, impulse noise, mixed noise), level of noise (high, low), structure of image (texture, smooth, edge), pixel intensity.
- Filter (denoising tool): computational cost (acceptable, high), filter implementation (simple, complex).
- Restored image (output): measurement of noise (SNR, MSE, PSNR, ect.), visual quality (blur, artifacts, information loss and etc).

in brief, this section provides the necessary information restricted in the primary phase of image processing (image pre-processing), which is called Image denoising.

## 2. Literature Review

### 2.1. Impulse Noise

In case of the grayscale image, the impulse noise may be represented by random values (RV) of pixels (value between 0 to 255) in the corrupted image, or by fixed values (FV) also called "salt & pepper" noise produced by random partial distribution of white pixels (value 255) and black pixels (value 0) into the image [7], as shown in Figure 1, unlike gaussian noise with the entire distribution (all image pixels) [8]. The image  $O(i, j)$  corrupted by RV and FV noise is described by  $N_{RV}(i, j)$  (2) and  $N_{FV}(i, j)$  (1), respectively [9].

$$N_{RV}(i, j) = \begin{cases} n(i, j) \in [0, 255], & \text{with probability } p \\ O(i, j), & \text{with probability } 1 - p \end{cases} \quad (1)$$

$$N_{FV}(i, j) = \begin{cases} 0 \text{ or } 255, & \text{with probability } p \\ O(i, j), & \text{with probability } 1 - p \end{cases} \quad (2)$$

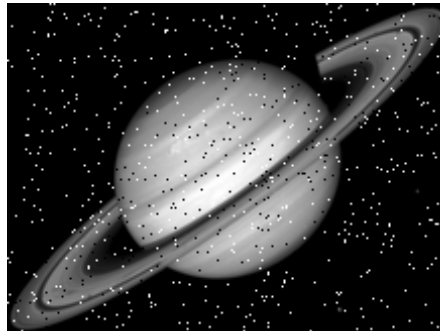


Figure 1. Image with salt and pepper noise

## 2.2. Efficient Methods for Removing Impulse Noise from Image

### 2.2.1. Median-related Filters

Median filter [2] belongs to the family of non-linear filters, it is a simple filter [8], which is based on the rank ordering of pixel values from the processed area. The corrupted pixel is replaced by one (median) taken from all pixels in the analysed window centered on that pixel, instead of the mean value which is derived from a calculated value, and this is an advantage for the median filter [10]. The median filter is robust to different types of noise, where it yields great results with impulse noise [11] and outperforms the linear filter in preserving image edges [12]. Nevertheless, it shows a limitation in case of high density of noise by removing some important information from the image [8].

In order to exceed this limitation, several extension techniques (derived from MF) have been proposed such as: Weighted median filter (WMF) [13] which attaches higher weights (coefficients) to the pixels that are closer to the central pixel, knowing that in the case of MF

the weights are equal. Meanwhile, in the case that this additional weight goes only to the central pixel of treated window, the filter will be called the center weighted median filters (CWMF) [14], directional weighted median filter (DWMF) [15], switching Median Filter [2], recursive weighted median filter (RWMF) [4] and others [16].

### 2.2.2. Adaptive Filtering

Over many years the adaptive filtering techniques take a significant part of attention and usage [17] that carry great importance in many applications of signal processing. As a statistical approach, this type of algorithms changes its characteristics automatically and recursively to be adapted to the statistical parameters of the treating signal with no required prior information, in order to optimise the inner coefficients of the filter [18].

The operation mechanism of the adaptive filter is illustrated in Figure 2, which aims to minimize the error  $e$  produced by the subtraction of  $\hat{u}$  (output of the flexible algorithm) from the original image  $u$  (free of noise) as reference. Then, this error is used with the noisy image  $v$  (input of algorithm) to update the filter parameters, with attention to the minimised criteria in order to achieve an optimal algorithm at the end of the process [17]. There are a lot of basic filtering techniques adopted for this concept to be an adaptive technique such as median filter, bilateral filter, Wiener filter, fuzzy filter, morphological filter, and so on. Some of them are presented in [7, 9].

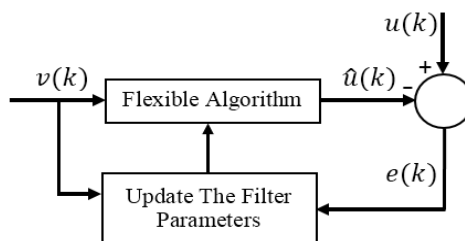


Figure 2. Adaptive filter

#### a. Adaptive Median Filter (AMF)

The adaptive median filter was being used more familiarly than the classic median filter since it is more effective compared to this last. The AMF works on the detection of the corrupted pixel compared to its neighborhood in the treated window to be labeled as a noisy pixel; the size of this window may be varied according to the comparison criteria. Then, this labeled pixel is replaced by the median pixel of the tested neighbourhood [19]. AMF gives a much better result in removing the salt and pepper noise, compared with other median filter types, whether in the visual quality or Noise ratio criteria [16].

#### b. Adaptive Weighted Median Filter (AWMF)

The AWMF is considered as an advanced technique compared with the classic weighted median filtering. AWMF is the process by which the weighted median filter has been applied adaptively to the noisy image, by way of adjusting the filter parameters and those weight coefficients by the local statistics of the treated area. The AWMF gives the possibility of removing noise by preserving edges and image features [12].

### 2.2.3. Fuzzy Filtering

The concept of the fuzzy set theory, had been used the first time for image processing in [20]. Then, several filtering techniques has either been generated or developed gradually from this logic in these last decades [21]. These fuzzy techniques treat the variety in the noisy data when it comes from ambiguity instead of randomness [21].

Fuzzy image processing holds three main phases [22], the first one called "fuzzification", that transforms the input data to the membership plane to deal with membership values, while the second and the most crucial one is the modification of these membership values using suitable fuzzy techniques, and the last one is the "defuzzification", performed to get the output data in the original plane. Fuzzy logic is characterized by the inherent uncertainty,

that makes it more fit to the image corrupted by the impulse noise [8]. Therefore, It deals much more with the median filters [23], as long as it is a good tool for noise detection. The fuzzy filter is based on two main features; the first one is to estimate a "fuzzy derivative" to be soft with the fine details of image such as the edge, according to the local variation of image; the second, is that the membership functions are adaptive to the noise level to yield "fuzzy smoothing" [24].

There are several types of fuzzy filters that have been introduced by researchers such as, the well-known, FIRE filters [25]. The adaptive weighted fuzzy mean filter (AWFM) [26], adaptive fuzzy switching filter (AFSF) [27], the iterative fuzzy control based filter from [22], the fuzzy bilateral filtering (FBF) [5], the fuzzy random impulse noise reduction method (FRINRM) [28], the fuzzy similarity filter (FSB) [6], A Fuzzy Noise Reduction Method for Color Images [28] and others in [7, 9, 16].

### 2.3. Evaluation Parameters of Image Quality

The image quality is one of the evaluation criteria of the denoising techniques performance (cited in the last section), hence the question posed is how to assess this quality. The way by which the image quality is evaluated (evaluation criteria) may split into two ways the first one is the visual evaluation determined by the observer, where the human judgment is interested in the image components appearance, whether or not it contains any degradation factors such as artifacts, discontinuities and blur [29]. The second one is the quantitative evaluation by using the measurement parameters, among them and the most used are: Signal to Noise Ratio (SNR), which measures the amount of noise  $n$  in the noisy image  $I(i, j)$  using the standard deviation of the noise  $\sigma(n)$  and image  $\sigma(I)$  ( $\sigma(I) = 60$  indicating good image quality) [30], as given by (3); Mean Squared Error (MSE), that measures the dissimilarity between the restored image  $\hat{I}(i, j)$  and the original one  $I(i, j)$  as shown in (4), thus whenever the MSE is lower, the image denoising will achieve more success [31]; Peak Signal to Noise Ratio (PSNR) [32], a well known parameter which has an inverse relationship with MSE, as denoted in (5).

It is not necessary to be an entailment relation between the visual and quantitative assessments, because sometimes an image, even with high PSNR or low MSE, does not appear to be clean.

$$SNR = \frac{\sigma(I)}{\sigma(n)} \quad (3)$$

$$MSE = \frac{1}{N} \sum_{i,j} (I[i, j] - \hat{I}[i, j])^2 \quad (4)$$

$$PSNR = 10 \log_{10} \left( \frac{255^2}{MSE} \right) \quad (5)$$

$N$ : The number of pixels in image.

### 2.4. Arithmetic Progression (AP)

A sequence is a set of numbers written in a particular order, such as these numbers which are given by the form 2 4 6 8 10 12 . . . An arithmetic progression (AP) is a particular case of sequence where any term in this sequence is obtained by adding a constant value to the previous term, where the constant is called the common difference  $CD$ . For example, the terms of an arithmetic progression started by the term  $a$ , are as follows:  $a, a + CD, a + 2CD, a + 3CD, \dots$

#### 2.4.1. Finding a Missing (Unknown) Term of an Arithmetic Progression

The general term of an arithmetic progression (AP), [33] can be presented by the (6). Thus, to find the missing (desired) term of an arithmetic progression (AP) we simply use the following formula:

$$AP_n = a + CD(n - 1) \quad (6)$$

where,  $n$  is the number of desired terms in this sequence, the common difference of AP and  $a$  is the first term.

### 2.4.2. Arithmetic Progression in Image Processing

Many aspects and situations of lives can be modeled by the arithmetic progression, and a lot of phenomenon can be interpreted to a sequence of terms. We just need to know how can use it in real life and how can exploit it to have various viewpoint about how things occur in the lifetime. Concerning the image processing, the arithmetic progression has a portion in treating the image. For example, ordering the pixels of the image based on the arithmetic sequence as done in [34]. Furthermore, in [35], the arithmetic progression is used for image watermarking.

### 3. Research Method

The proposed methodology of this denoising method introduces in details the several steps that are adopted to restore perfectly the original image from the noisy image in order to solve the problem of noise which reduces and threatens the image quality, using a simple mathematical concept called "Arithmetic Progression (AP)". Figure 3 describes the flow chart of those steps which are in all, three steps-the first step is to create the noisy image by adding noise to the original or noise free image (input image), then this noisy image is passed to the second step which is the filtering phase to remove noise from the image and it is also divided into two sub-phases. The first one is the noise detection, where the corrupted pixels should be distinguished from the uncorrupted pixels, and then these corrupted pixels will be replaced by the correct values in the second sub-phase (pixel restoration), where the uncorrupted pixels rest intangible. The extraction of the alternate value of each pixel (correct value) will be done to the noisy image. These former steps (filtering) yield the estimation (restoration) of the original image (output image). Finally, in the last step, MSE and PSNR are calculated using the input and output images, in order to evaluate the performance of proposed method by comparing these last values with other values related to other denoising methods.

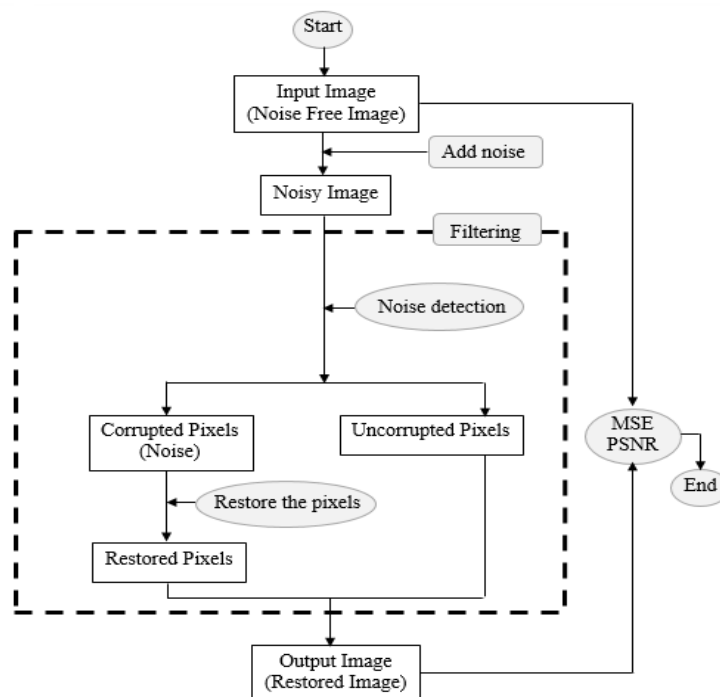


Figure 3. Methodology flowchart

### 3.1. Add Noise

This phase is concerned with creating the noisy image which is then used in the denoising technique. This noisy image is obtained by adding noise to the original image (noise free image) according to two parameters linked with the noise which are the type and

the intensity of noise. In the case of impulse noise, a number of pixels in the original image are chosen randomly to be changed by random values of pixels (values ranging from 0 to 255). That action yields a noisy image, where the intensity of noise is controllable and it is represented by the percentage of these random values (noise) in the noisy image. As mentioned previously, the impulse noise is the type of noise chosen to be treated in this study, due to its structure that makes the adoption of the proposed mathematical concept "arithmetic progression" more able in successfully solving this kind of problems.

### 3.2. Filtering

This step is the opposite of the first one. The main concern of the filtering phase is to detect the corrupted pixels (noise) and then change its value by the correct one, which is the nearest value to the original. This enables the removal of the noise by preserving other image features. The denoising method (filtering) or restoration's operation in the noisy image comes from the uncorrupted pixels [36]. Thus, it is very important to observe (model) the connection between these noise-free pixels, and how the pixel intensity changes through the image. This gives the opportunity to treat the two main points in this phase (detection and suppression of noise) because any pixel that does not subject to this status (model) in the image, can be considered as a noise, and should replace its value by another value derived from other uncorrupted pixels and subject to the intensity behavior. Finally, the obtained restored image carries the same features of the original image.

The pixels are located in different areas in the image with a particular order due to the intensity behavior. The intensity of the pixel varies in the image gradually (gradient color) or constantly (one color). These intensity values may be modeled as terms of an arithmetic progression with a common difference  $CD$ : take a constant value different from 0 in the case of gradient color and it is equal to 0 in the case of one color, as it is shown in Figure 4.

The proposed denoising method belongs to the spatial domain filtering and it exploits this existing characteristic in the image illustrated in Figure 5, where each pixel belongs to one or many sequences of pixel values. Then, this characteristic can be modeled using a mathematical concept called "arithmetic progression" to get the alternate pixel of the corrupted one, which should belong to this sequence.

The Figure 5 illustrates how the intensity of pixels changes in the image without noise, where each pixel that belongs to one or more particular orders (sequences) of integer numbers go from 0 to 255 (0,1,2,...,255), for example the pixel with value 42 belongs to many sequences (arithmetic progression) which are indicated in the figure by blue arrows, where each sequence is characterized by its particular common difference  $CD$ . As it mentioned in the previous chapter, the arithmetic progression is characterised by a term and a constant  $CD$ , while in the real image the difference between pixels in a sequence of pixels is not usually constant in most case, therefore to model these sequence as an arithmetic progression we need to estimate a common difference  $CD$  from these differences by making its average as the estimator of  $CD$ . For example, in the case of a sequence of pixels with the following values and differences ( $D_1, D_2, D_3, D_4$ ):

$$\begin{array}{ccccccc} 44 & & 50 & & 57 & & 65 & & 69 \\ & \underbrace{\hspace{1.5cm}}_{D_1} & & \underbrace{\hspace{1.5cm}}_{D_2} & & \underbrace{\hspace{1.5cm}}_{D_3} & & \underbrace{\hspace{1.5cm}}_{D_4} & \end{array}$$

where:

$$D_1 = 50 - 44 = 6 \quad (7)$$

$$D_2 = 57 - 50 = 7 \quad (8)$$

$$D_3 = 65 - 57 = 8 \quad (9)$$

$$D_4 = 69 - 65 = 4 \quad (10)$$

The estimator  $\widehat{CD}$  of  $CD$  is calculated from (7), (8), (9), (10) by the average of  $D_1, D_2, D_3, D_4$ .

$$\widehat{CD} = (D_1 + D_2 + D_3 + D_4) / 4 \approx 6 \tag{11}$$

in this case, the pixel can be considered as a term of arithmetic progression, and to restore its value we apply the rules of calculation of unknown AP's term to get the most correct value for that pixel. According to that, this mathematical concept (arithmetic progression) can be used as a tool for removing the noise and restoring the degraded image.

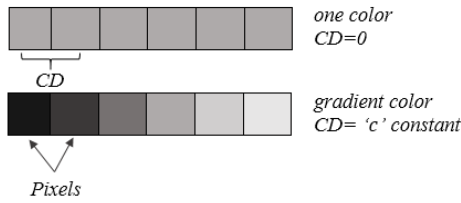
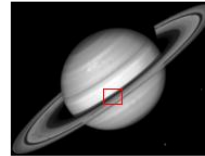


Figure 4. Intensity variation in image



238	235	235	231	195	131	88	69	50
236	234	224	175	113	83	65	44	47
235	212	152	99	76	57	42	54	81
196	131	89	71	50	42	64	90	107
113	82	63	44	48	73	97	113	123
77	55	42	56	83	103	116	125	132
48	43	65	91	108	120	129	134	141
50	77	98	113	123	130	137	145	157
85	105	117	124	131	139	149	159	154
110	119	126	134	142	155	159	150	140

Figure 5. The appearance of pixels values in noise-free image

### 3.2.1. Noise Detection

The efficiency of the proposed denoising method relies on two crucial phases, one of them is the detection noise which aims to make the treatment operation (the change) confined only to the corrupted pixels. Thus, this section shows how to distinguish between the corrupted (noise) and uncorrupted pixels. Usually as in the case of other image filtering methods which adopt the noise detection phase in its processing, the filter is first looking for the corrupted pixel (detection) and then looking for the witch value (alternate pixel) that should replace this pixel, in order to treat only the corrupted pixels and keep others with the same values (objective of noise detection phase). In our approach we seek to achieve that objective based on the AP, where the pixel which does not belong to any sequences of numbers with a determined common difference  $CD$ , will be considered as a noise (corrupted pixel), as it is clearly seen in Figure 6.

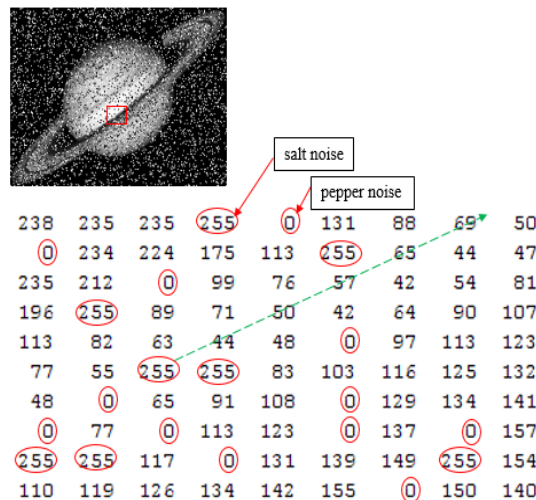


Figure 6. The appearance of pixels values in noisy Image (salt & pepper)

### 3.2.2. Restoring the Pixels (Noise Suppression)

This step is the second crucial phase in the filtering process. Based on the previous sub-phase, every pixel which is considered as noise will be replaced by the correct extracted value and keeps other pixels with the same values to obtain the desired image. Thus, the question here is how to obtain this correct value, and to answer this question we use as an example, Figure 5, which denotes the noise-free image, and Figure 6 below, which illustrates the distribution of corrupted pixels values (assigned by red circles) in the noisy image of the same original image in Figure 5.

It is clearly seen that, from the first case (noise-free image) to the second one (noisy image), the pixel with the value 42 assigned by green circle in Figure 5, was changed to be on the value 255 (salt noise) assigned by red circle in Figure 6. In order to restore the missing (original) value of the corrupted pixel, this last is taken as an unknown term of one of the existing arithmetic progressions surrounding this pixel.

In this case the corrupted pixel is surrounded by two sequences, the first one is indicated by a green arrow and denoted by the terms: 44 50 57 65 69. The second sequence is indicated by a blue arrow and denoted by the terms: 91 123 139. By comparing these sequences, we see that the first one contains a higher number of terms, and with differences closer to each other. Thus, the first sequence is more able to be modeled as an arithmetic progression. The common difference  $CD$  is estimated using (11). With regard to the treated image in Figure 6, the corrupted pixel  $P$  by the value "255" is considered as an unknown term of this AP, and it can be restored using its neighbor "44" by the next (12).

$$\boxed{255 \quad 44 \quad 50 \quad 57 \quad 65 \quad 69}$$

$$P = 44 - 6 = 38 \quad (12)$$

Referring to the original image in Figure 5, the original value of the pixel  $P$  is "42", and the obtained value by the proposed method is "38", while in the case that we use for example the median filter, the restored pixel takes the value "65". As it is, the proposed method gives a restored value which is closer to the original, compared to the median filtering.

### 3.3. Evaluation of Filtering Approach

In this last step, the input (noisy image) and output (restored image) of the proposed filter are used to calculate the evaluation parameters which are MSE and PSNR for comparison with other denoising methods, in order to evaluate the performance of this filter.

## 4. Results and Analysis

In this study another way is taken to develop a new denoising method that aims to remove the impulse noise from image by preserving the fine details of image features such as the edges, and avoiding other effects of filtering, for example the blur. This denoising method treats the impulse noise and it is expected to give good quality of restoration whether visually or computational, whereas it is also expected to be extended in the future to another type of noise such as additive or multiplicative noise.

In order to test the performance of the proposed denoising technique, eight images as sampling have been used and they are presented in Figure 7 (Boats, Peppers, House, Mandrill). The result of those filtering methods is indicated visually (pictures) in Figure 8 and quantitatively PSNR in Table 1, with the proposed technique, in several values of noise amount assigned by percentage.

The primary implementation of this filter to remove the salt and pepper noise, to give acceptable results compared to some methods which are considered as efficient methods for removing impulse noise from images: Standard median filter (SMF), weighted median filter (WMF), directional weighted median filter (DWMF). The results presented in Figure 8 and the Table 1, illustrate that the proposed denoising technique gives an acceptable performance compared to the existing methods whether visually or quantitatively with PSNR. These results illustrate that the proposed method has succeeded to get a closer restored image to the original image, by way of replacing the corrupted pixel by the most correct one.



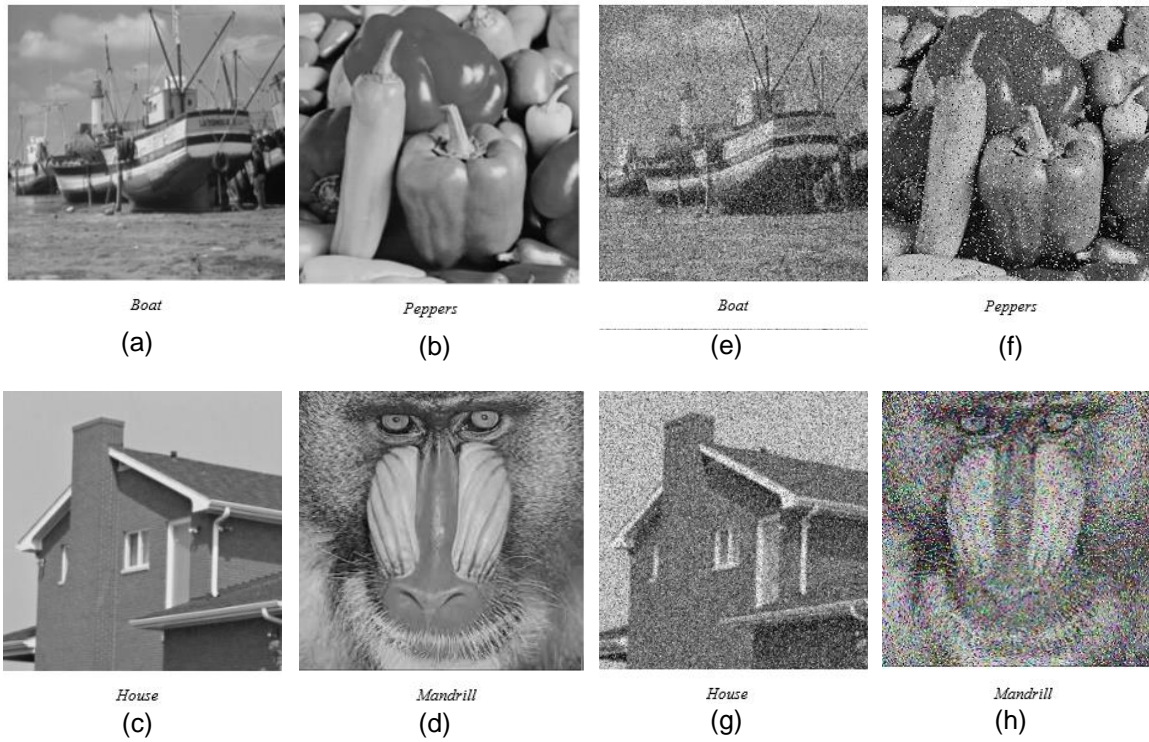


Figure 7. The sampling images (a-d) without noise (original images) and (e-h) with 20% salt and pepper noise

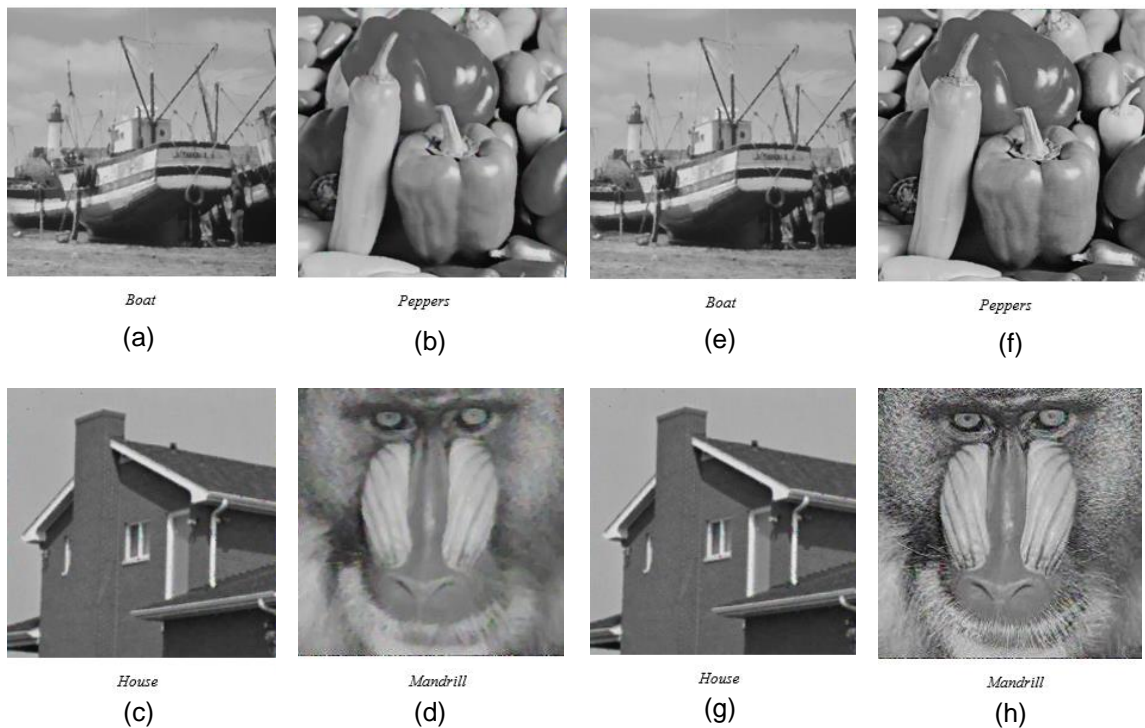


Figure 8. Comparison of restoration results of the SMF with the proposed technique for images corrupted by fixed-valued impulse noise (a-d) filtered with SMF (e-h) filtered with the proposed method

Table 1. Comparison of Restoration Results in PSNR for Images Corrupted by Fixed-valued Impulse Noise (Salt & Pepper)

Images\Methods		SMF	WMF	DWMF	Proposed
Boats	20%	33.18	35.55	37.11	38.01
	60%	22.40	24.74	26.34	27.95
Peppers	20%	35.25	37.12	39.11	39.82
	60%	25.45	27.20	30.34	31.55
House	20%	33.66	35.85	37.24	38.91
	60%	22.13	24.53	27.54	28.42
Mandrill	20%	31.21	34.32	36.11	37.63
	60%	22.64	25.76	26.34	27.98

## 5. Conclusion

This work is carried out to treat the problem of impulse noise in the image, which threatens its visual quality and make other image processing such as segmentation and compression more difficult. The chosen way for that is the spatial filtering in which the process is simple and deals directly with the pixels as done in the median and mean filters. The adopted mathematical concept in this study is the Arithmetic Progression, which allows the modeling of the behaviour or variation of pixels intensity in the image with high precision. In order to derive new values for the corrupted pixels that are subject to the same features and give a perfect restoration of the image by preserving the fine details (originality of image). Finally, this denoising method is proven as an efficient way to restore the image compared to the current familiar filters in the aspect of evaluation criteria.

Besides what this denoising method provides as an extension of the range of solutions against this problem of noise in the image, it also holds limitations about the things that should be exposed. One limitation is that the study is constrained by the number of sampling images which total four images (Boats, Peppers, House, Mandrill). When the number of sampling images is higher, the method will prove its efficiency in restoring all image models. Another limitation is about the performance of the filter which decreases with the rise of the noise's amount in the noisy image, while this drop should be very slow to preserve the stability in the filtering performance. Furthermore, the study proposes a denoising technique confined in removing one type of noise which is impulse noise. This is when a great number of images used in many fields in our lives are corrupted by the two other types of noise (additive and multiplicative), for example the medical image that is usually corrupted by the speckle noise.

In this denoising technique, there are four points which are briefly mentioned in this section and can be used to develop this method due to the influence on the filtering performance. The first point is the number of arithmetic progression terms, chosen to determine this sequence. The second is the manner of estimating the common difference,  $CD$  of the AP. In this study,  $CD$  is estimated by the average of the differences between the pixels of this AP, where it is possible to use other manners. The choice of the most suitable sequence is selected to restore the corrupted pixel. The last point is the characteristics of the AP (convergence of a sequence) which can be used to select the appropriate AP and the most correct pixel value to restore the corrupted one.

## Acknowledgement

This work was financially supported by Ministry of Education Malaysia (MOE) under Fundamental Research Grant Scheme (FRGS) ((Ref: FRGS/1/2019/STG06/UNIMAP/02/3).

## References

- [1] Kumar A, Shaik F. Image processing in diabetic related causes. *Springer*. 2015.
- [2] Li X. Image restoration: Fundamentals and advances: CRC Press. 2012.
- [3] Boyat AK, Joshi BK. A review paper: noise models in digital image processing. arXiv preprint 2015. arXiv:1505.03489.
- [4] Kundur D, Hatzinakos D. Blind image deconvolution. *IEEE signal processing magazine*. 1996; 13(3): 43-64.
- [5] Li X. Image recovery via hybrid sparse representations: A deterministic annealing approach. *IEEE Journal of Selected Topics in Signal Processing*. 2011; 5(5): 953-962.

- [6] Jain AK. Fundamentals of digital image processing: Prentice-Hall, Inc. 1989.
- [7] Habib M, Hussain A, Choi TS. Adaptive threshold based fuzzy directional filter design using background information. *Applied soft computing*. 2015; 29: 471-478.
- [8] Liu L, Chen CP, Zhou Y, You X. A new weighted mean filter with a two-phase detector for removing impulse noise. *Information Sciences*. 2015; 315: 1-16.
- [9] Habib M, Hussain A, Rasheed S, Ali, M. Adaptive fuzzy inference system based directional median filter for impulse noise removal. *AEU-International Journal of Electronics and Communications*. 2016. 70(5): 689-697.
- [10] Kumar S, Kumar P, Gupta M, Nagawat AK. Performance comparison of median and wiener filter in image de-noising. *International Journal of Computer Applications*. 2010; 12: 0975–8887.
- [11] Rani R, Singh S, Malik A. Image denoising using hybrid filter. *International Journal of Innovative Technology and Exploring Engineering (IJITEE)*. 2012; 1(1).
- [12] Loupas T, McDicken W, Allan P. An adaptive weighted median filter for speckle suppression in medical ultrasonic images. *IEEE transactions on circuits and systems*. 1989; 36(1): 129-135.
- [13] Brownrigg D. The weighted median filter. *Communications of the ACM*. 1984; 27(8): 807-818.
- [14] Ko SJ, Lee YH. Center weighted median filters and their applications to image enhancement. *IEEE transactions on circuits and systems*. 1991; 38(9): 984-993.
- [15] Dong Y, Xu S. A new directional weighted median filter for removal of random-valued impulse noise. *IEEE Signal Processing Letters*. 2007; 14(3): 193-196.
- [16] Kartik S, Anay G, Amitabha C. Reduction of Salt and Pepper Noises from a Degraded Image Based on Fuzzy Techniques. *Indian Journal of Science and Technology*. 2016; 9(43): 1-10.
- [17] Bellanger, M. Adaptive digital filters: CRC Press. 2001.
- [18] Diniz PS. Adaptive filtering: Springer. 1997.
- [19] Sravani, B., & Rao, M. N. *Removing of high-density salt and pepper noise using fuzzy median filter*. Paper presented at the High-Performance Computing and Applications (ICHPCA), International Conference on 2014. 2014.
- [20] Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8(3): 338-353.
- [21] Kerre EE, Nachtegaal M. Fuzzy techniques in image processing. Physica springer. 2013; 52.
- [22] Farbiz F, Menhaj MB. A fuzzy logic control-based approach for image filtering. Fuzzy techniques in image processing. Springer. 2000: 194-221.
- [23] Mahesh T, Prabhajan S, Vinayababu M. Noise Reduction by Using Fuzzy Image Filtering. *Journal of Theoretical & Applied Information Technology*. 2010; 15(2): 115-120.
- [24] Rao D, Panduranga PP. *A survey on image enhancement techniques: classical spatial filter, neural network, cellular neural network, and fuzzy filter*. 2006 IEEE International Conference on Industrial Technology, Mumbai. 2006: 2821-2826.
- [25] Russo F, Ramponi G. *Combined FIRE filters for image enhancement*. Paper presented at the Fuzzy Systems. Proceedings of 1994 IEEE 3<sup>rd</sup> International Fuzzy Systems Conference. Orlando, FL. 1994; 1: 260-264.
- [26] Lee CS, Kuo YH. Adaptive fuzzy filter and its application to image enhancement Fuzzy techniques in image processing. Springer. 2000: 172-193.
- [27] Xu H, Zhu G, Peng H, Wang D. Adaptive fuzzy switching filter for images corrupted by impulse noise. *Pattern Recognition Letters*. 2004; 25(15):1657-1663.
- [28] Schulte S, De Witte V, Kerre EE. A fuzzy noise reduction method for color images. *IEEE Transactions on image Processing*. 2007; 16(5): 1425-1436.
- [29] Arce GR, Paredes JL. Recursive weighted median filters admitting negative weights and their optimization. *IEEE Transactions on Signal Processing*. 2000; 48(3): 768-779.
- [30] Buades A, Coll B, Morel JM. A review of image denoising algorithms, with a new one. *Multiscale Modeling & Simulation*. 2005; 4(2): 490-530.
- [31] Quan J. Image Denoising of Gaussian and Poisson Noise Based on Wavelet Thresholding. University of Cincinnati. 2013.
- [32] Sakrison D. On the role of the observer and a distortion measure in image transmission. *IEEE Transactions on Communications*. 1977; 25(11): 1251-1267.
- [33] Polyanin AD, Manzhirov AV. Handbook of mathematics for engineers and scientists: CRC Press. 2006.
- [34] Anderson PG. Linear pixel shuffling for image processing: an introduction. *Journal of Electronic Imaging*. 1993; 2(2): 147-155.
- [35] Malonia M, Agarwal SK. *Digital Image Watermarking using Discrete Wavelet Transform and Arithmetic Progression Technique*. Paper presented at the Electrical, Electronics and Computer Science (SCEECs), IEEE Students' Conference on. 2016.
- [36] Liu L, Chen L, Chen CP, Tang YY, Pun, C. M. Weighted joint sparse representation for removing mixed noise in image. *IEEE transactions on cybernetics*. 2016.