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An overview of multi-filters for eliminating impulse noise for digital images

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ABSTRACT

An image through the digitization process is referred to as a digital image. The quality of the digital image may be degenerating due to interferences on the acquisition, transmission, extraction, etc. This attracted the attention of many researchers to study the causes of damage to the information in the image. In addition to finding cause of image damage, the researchers also looking for ways to overcome this problem. There are many filtering techniques that have been introduced to deal the damage to the information in the image. In addition to eliminating noise from the image, filtering techniques also aims to maintain the originality of the features in the image. Among the many research papers on image filtering there is a lack of review papers which are an important to facilitate researchers in understanding the differences in each filtering technique. Additionally, it helps researchers determine the direction of research conducted based on the results of previous research. Therefore, this paper presents a review of several filtering techniques that have been developed so far.

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1. INTRODUCTION

Image is a representation of the real state into a two-dimensional form. Currently, the image is one of the communication tools widely used by the general public and better known as the digital image. This is because it is easy to have a camera from our smartphone. In addition, the image itself is used in various areas such as medical, electrical, computer, mechatronics, etc. In this era of technology, the use of digital image is part of various aspects of life that cannot be separated. Images are also forms of communication where used to interact with others either directly or indirectly.

Digital images are the terms for images that have gone through digitization process. Digitization is the process of transforming landscapes or objects that can be viewed or perceived as electronic data that can be stored, transmitted, retrieved and/or returned via electronic equipment. Digital images have digital values called the picture elements or abbreviated to pixels. Pixels represent the values that represent the brightness of the colour inherent in digital images. There are three basic colours on digital images namely red, green, and blue. The three colours are used as the basis for image processing. Based on the pixel colours and the brightness of the pixels in the digital images there are three different types of images namely binary images, grayscale images, and full-colour or RGB images as shown in Figure 1 [1].

Image processing is the process of decomposing information that exists in the digital images. This can be done by transforming the digital images into numbers based on the brightness of the colour. Image processing is intended for obtaining better images or in order to obtain certain information contained in digital images. There are various types of image processing that have different functions and purposes such as image filtering, image clustering, image segmentation, image watermarking, edge detection, image recognition, etc.

Image filtering is an important part of image processing that works to improve damaged parts due to noise. Recently, a lot of researches in the filtering methods have been done and many new methods are being developed. The salt and pepper noise case is one of the cases of noise, with the number of studies has increasing rapidly. Among the research papers on image filtering, the number of review papers discussing about the existing filtering methods are limited. Since these review papers are important then they will help in facilitating researchers to understand the differences in each filtering methods. This paper presents a review of image filtering and several different filtering methods to facilitate researchers in this field.

(a) (b) (c)

Figure 1. Color coding of digital image: (a) binary image, (b) grayscale image, and (c) RGB image

2. NOISE ON THE IMAGE

Noise is a communication disruption that appears on the signal and/or image. The noise in the image is an alteration of information that in visual impairment of image quality and eliminates the important information that must exist in the image. Noise appears in the image during the acquisition, transmission, extraction, etc. It is clear that the origins of noise on the image are different, but the resulting effect on images are the same as losing edge sharpness in the images or even losing shape from the images itself [2, 3].

To recover the damaged information in the image, many researchers conducted research on the nature of the noise. In general, there are three types of noise that are often studied so far are impulse noise, additive noise, and multiplicative noise [4, 5]. The three types of noise have different alteration effects on digital images, depending on the amount of noise density and the colour intensity or brightness of the image. In the following subsection will be given explanations and examples of the three different types of noise.

2.1. Impulse noise

Images containing the impulse noise have pixel values between 0 and 255 or have a value of 0 or 255 randomly distributed on the image, while the remaining pixel values will remain. For the case of impulse noise with a fixed value is also known as salt and pepper noise. This noise occurs due to an interference in the data transmission process [6, 7]. This case occurs a lot in digital images as results of shooting in a digital camera. In general, impulse noise on the image $u(i,j)$ is denoted as follows:

$$c(i,j) = \begin{cases} c(i,j) \in \{0{,}255\}, & probability\ p \\ c(i,j) = u(i,j), & probability\ 1-p \end{cases} \quad (1)$$

2.2. Additive noise

Unlike impulse noise, additive noise spreads all over the image evenly, so that all pixel values in the image undergo alteration. There are a variety of additive noise, partly used extensively in research are Gaussian noise and Poisson noise. The appearance of this kind of noise in the image caused by the natural disruptions such as heat caused by the sun or other objects [1]. Similar to impulse noise, Gaussian noise can also occur in the image results of a digital camera. In general, the additive noise on the image $u(i,j)$ is denoted as follows:

$$c(i,j) = n(i,j) + u(i,j) \tag{2}$$

2.3. Multiplicative noise

Multiplicative noise is similar to additive noise in terms of the noise spreading. Additive noise can be expressed as a random value added to the pixel value of the image, whereas in multiplicative noise the random value is multiplied into the pixel value of the image. The case of the emergence of multiplicative noise can be occur in the radar catches [5, 8]. In addition, in the field of medical, images obtained from MRI, US, CT / CAT scan, etc. noisy images cases are also found. In general, the multiplicative noise on the image $u(i,j)$ is denoted as follows:

$$c(i,j) = n(i,j) \times u(i,j) \tag{3}$$

3. IMAGE DE-NOISING

It has previously been explained that the image contains a set of important information from an object in two-dimensional form. The information contained in the images may be corrupted so that the resulting images are damaged. The disruption in this image is called noise. In order to improve this condition, it is necessary to do the de-noising process of the image so that damaged information can be restored as the actual situation. Image de-noising is a pre-processing stage aimed at removing noise from the image. Image de-noising plays an important role in image processing, because the image processing requires good quality image and has the same information as the original condition. By doing image de-noising on corrupted images, it is hoped that new images can be obtained with better visuals. It aims to facilitate and maximize during image processing stages such as image segmentation, image enhancement, edge detection, classification, etc. [9-12].

Need to remember that image de-noising is the process of removing noise from the corrupted images, but in addition to eliminating noise we also need to maintain the undamaged image during the process of filtering. Over the years, research works have been done for image filtering in order to obtain the best image filters. This is not without reason, besides cleaning the image from the noise the main purpose of the image filter is to produce a recovery image with clear features such as texture, sharpness, and edges on the images. Therefore, research on image de-noising is constantly evolving in order to produce the best method. There are various filtering methods developed so far with different advantages and disadvantages. Research to develop the filtering methods continues in order to address the weaknesses of the previous methods. Filtering methods can be divided into two categories: spatial domain and transform domain filtering. This categorization is based on the platform used to perform the filtering process. Some filtering methods by category are presented in the Figure 2.

Before discussing further about the filtering method of spatial and transform domains, firstly the hybrid method will be explained. The hybrid method is a filtering technique built from combination of two or more filtering methods or several different mathematical concepts. Combining more than two filtering methods are expected to overcome the weaknesses of the filtering method used, thus maximizing the output. Lately the hybrid filtering method has been widely developed such as the combination of wavelet transform and total variation, a combination of bilateral filter and non-local mean filter, etc, since it works well and overcomes the weaknesses of the existing filtering methods [13-18].

The performance of filtering techniques can be measured qualitatively and quantitatively from the resulting recovery image. Qualitative assessment can be viewed directly from visual results of the recovery image. The quality of image filtering results can be assessed on several factors such as degradation of colour, texture, edge, and residual noise. While quantitative assessment is derived from the results of statistical calculations of the similarity level between the recovery image and reference image. There are three types of statistical calculations commonly used by researchers, namely mean squared error (MSE), peak signal to noise ratio (PSNR), and structural similarity index (SSIM). The mean squared error is used to calculate the level of dissimilarity between recovery image and reference image. The peak signal to noise ratio is more commonly used parameter in research which has an inverse relationship with mean squared error. The formulas for mean squared error and peak signal to noise ratio are denoted as follows [19]:

$$PSNR(dB) = 10 \cdot log_{10}(\frac{MAX}{MSE}) \quad (4)$$

where MAX is denoted as the largest value at gray-level of 255. While the MSE is denoted as follows:

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}|p(i,j) - \bar{p}(i,j)|^2 \quad (5)$$

where $p(i,j)$ and $\bar{p}(i,j)$ are the pixels value of the original image and recovery image. M and N are the size of the image.

The structural similarity index is a metric that demonstrates the degradation level of image quality resulting from processing by using a reference image. This means that the calculation of the structural similarity index is a process of measuring the visual similarity between the recovery image and reference image. The structural similarity index is well known in video processing, although it plays an important role in the stationary image field. The formula of the structural similarity index is denoted as follows [20]:

$$SSIM = \frac{(2\mu_x\mu_y + C_1) + (2\sigma_{xy} + C_2)}{(\mu_x^2\mu_y^2 + C_1) + (\sigma_x^2 + \sigma_y^2 + C_2)} \quad (6)$$

where μ_x, μ_y, σ_x and σ_y are the average intensities, standard deviation, and the covariance for the original image and recovery image.

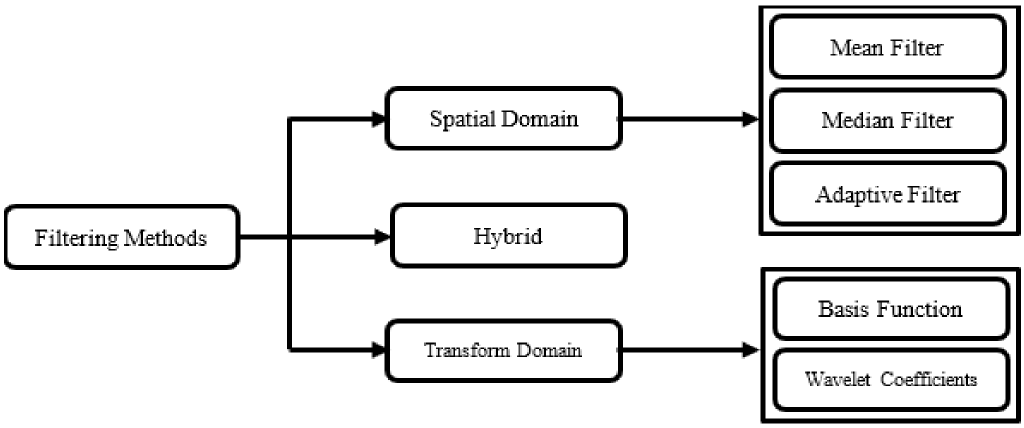

Figure 2. Image filtering methods

3.1. Transform domain

Transform domain filtering is the filtering method that treats the image into another form to collect the information contained on it. Images can be defined as two-dimensional signals for grayscale images and three-dimensional signals for RGB images. In domain transform filtering, images will be represented in another form for example the image can be transformed to the frequency domain before the filtering process done. Spatial domain filtering is widely used in terms of removing impulse noise in the images, while transform domain filtering is developed for additive and multiplicative noise cases. Several approaches to transform domain filtering have been developed to date to eliminate additive or multiplicative noise such as wavelets, wave atoms, curvelets, contourlets, wedgelets, and bandelets, which transform the image to the frequency domain [21-26].

3.2. Spatial domain

If transform domain filtering is done by changing the form of the image, then this differs from the filtering process in spatial domain. Spatial domain filtering is the filtering method that works directly on the pixels of images [27]. This method belongs to the traditional way of image de-noising. Although classified as traditional does not mean the filtering methods on spatial domain ceases to be developed. Basically, this filtering method can be divided into two types, namely linear filter and nonlinear filter. In linear filter, the images of the filtering result in a linear structural change with changes in the input images. The mean filter is an example of linear filtering that is commonly known and used. Furthermore, non-linear filters produce output that is not linear with changes in the input images. In addition, the median filter is a nonlinear filter that is still in used and developed.

3.2.1. Mean filter

Average filtering is a filtering technique by changing the pixel value with the average results in pixels at certain distance and colour intensities. Average filtering is classified as linear filtering. This filtering technique can be divided into two types, namely local mean filter and non-local mean filter. In the local mean filter, the process of determining the new pixel value is based on pixel values that are close to target pixels. Whereas the non-local mean filter is the development of the standard mean filter where pixel values are taken based on similarity levels that are centered on these pixels. Some types of average filters introduced include standard mean filters, Yaroslvsky filters, Gaussian filters, bilateral filters, and non-local mean filters [28-32].

In the case of impulse noise, especially salt and pepper, the mean filter has a poor performance due to a significant difference in pixel values. In 2014, the development of the mean filter was done by combining the concepts of adaptive filter and mean filter (AWMF). In this method, before the filtering process is performed, the size of the processing window is selected based on the minimum pixel value and the same maximum in the window [33]. The adaptive weighted mean filter has a good performance to remove low- and high-density noise. In the following year, a new weighted mean filter equipped with double noise detection was introduced. Noise detection in this method uses the concept of fuzzy logic and statistical concept in the detection process [34].

3.2.2. Median filter

The median filter is known as a simple and powerful filtering method that can be used for any types of noise. This filtering technique is included in non-linear filtering group, where filtering results are based on the ranking of ordered pixel values. As mentioned previously, the median filter can work well for various types of noise, for case of salt and pepper noise it can produce a better output compared to the mean filter where edges on images can be maintained. However, the median filter provides a weak filtering results for high-density noise cases.

In order to eliminate the disadvantages of the median filter, many researchers conducted research related to the median filters so far. [35], in his research proposed the development of median filter that is weighted median filter (WMF). The weighted median filter adds the coefficient as weight to the nearest pixels value to the target pixel for every image template. Furthermore, if the weight is centered in the target pixel of the image templates, then in this case the filter is called center weighted median filter (CWMF) [36]. The development of filtering methods is still ongoing, as some of the existing filtering methods have a drawback in the phase noise detections and pixel restoration. To overcome this problem, modification on the median filter is performed and various noise detection is introduced. Research on median filters continues to be reinforced and some use the weighted median filter (WMF) concept as the basis for these developments such as, directional weighted median filter (DWMF) and recursive weighted median filter (RWMF) in order to improve the efficiency of the filtering methods [37-40]. Directional weighted median filter (DWMF) can work well with the appearance of noise detection in the algorithm, however, for the case of high-density noise, the filtering results are blurry and leaving black and white mark. This indicates that the existing of noise detection in the directional weighted median filter (DWMF) does not work well in high density. In 2012, a modification of the directional weighted median filter was introduced. This method is based on the directional weighted median filter (DWMF) with much better filtering performance for high density noise [39].

In addition to focus solely on developing the median filter concept, hybridization techniques in the median filter are also developed to obtain better filtering methods. Adaptive filtering is a filtering technique commonly used in signal processing and incorporated into linear filtering method. Adaptive filtering is built with an algorithmic system that can adjust recursively the parameter changes to the target signal, so the optimum algorithm can be obtained [41-43]. Adaptive median filter and adaptive weighted median filter are the example of the hybridization results of the concepts between adaptive filter and median filter. This kind of filtering method provides better filtering results, visually and quantitatively, compared to the standard median filter [43, 44].

The concepts of fuzzy, adaptive filter, and median filter are widely developed and hybridized in order to build better and more effective filtering algorithms. The basic idea of adaptive median filter (AMF) is the addition of noise detection into the filtering algorithm. The noise detection uses the maximum and minimum colour intensity on the target pixel and tested neighbourhood pixels. The development of the adaptive median filter is an adaptive weighted median filter where the weighted median filter concept is applied to it. Furthermore, an example of fuzzy concept hybridization is an adaptive fuzzy filter algorithm (AFM). This method consists of noise removal using the concept of adaptive fuzzy and the median concept to determine the new pixel value [43, 44].

The other modification of median filter is switching median filter that selects the previous pixel value if the median value is noise [45, 46]. In 2010, the filtering method built on hybridization between the concepts of adaptive filter, fuzzy, and switching median filter was introduced. Switching median filter has advantage i.e. only choosing the uncorrupted pixel, which attracts many researchers to develop the method. By hybridizing the concept of fuzzy and switching median filter can improve the advantages of these concepts dan this method is known as noise adaptive fuzzy switching median filter (NAFSM) [47]. The filtering methods work well and provide visual results with clear edges, although the textures of the recovery images become blurry and leaving black and white dots in the case of high-density noise. In 2011, a new filtering method by combining the concept of median and mean filter by considering the number of noisy pixels in the tested neighbourhood, known as the modified decision based unsymmetric trimmed median filter (MDBUTMF), was introduced. This method was developed to overcome the drawback of decision-based algorithm (DBA) and decision based unsymmetric trimmed median filter (DBUTMF). Additionally, the modified decision based unsymmetric trimmed median filter provide similar filtering results with adaptive fuzzy noise switching median filters without black and white dots on the recovery image [48-50].

Recently, new filtering method know as based on pixel density (BPDF) was introduced. In this method all the noisy pixels are considered to be no noise when there is a pixel with value approaching the noisy pixel in the tested neighbourhood. If this condition is met, the target pixel value will be overwritten with the median value [51]. The recovery image from the based-on pixel density filter method has good textures, but the edges of the image become blurry. In addition, this method only works for low density noise and has poor performance for high density noise (cannot even be used). In the same year, a new filtering method was proposed by the same authors to overcome cases in high density noise, namely different applied median filters (DAMF) [52]. This filtering method uses the adaptive concept in determining the value of new pixels in corrupted pixels. After the adaptive windows that contain free-pixel noise is obtained, the median value is still used in the pixel restoration process. Implementing adaptive windows can improve the effectiveness of the pixel restoration process. Visually, the output images obtained from different applied median filters (DAMF) have clear textures and edges even in case of noise density above 70%.

In order to illustrate the performance of the mentioned filtering methods, experiments were conducted to evaluate the performance of the methods both statistically and visually. This experiment runs on Lena image with resolution of 512x512 corrupted by salt and pepper noise with density of 10% to 90%. Table 1 shows the PSNR of filtering results from different filtering methods namely median filter (MF), directional weighted median filter (DWMF), median filter switching (SMF), adaptive median filter (AMF), median noise adaptive fuzzy switching filter (NAFSM), modified decision based unsymmetric trimmed median filter (MDBUTMF), based on pixel density filter (BPDF), and different applied median filters (DAMF). In addition, Figure 3 shows the visual output of the eight methods for the case of noise with density of 80%.

Table 1. The results of PSNR calculation from the Lena image in case of noise density 10% to 90%

Filter	10%	20%	30%	40%	50%	60%	70%	80%	90%
MF	32.46	31.11	29.53	27.11	23.81	20.83	16.93	12.89	8.91
DWMF	37.53	35.58	33.38	30.55	26.17	18.68	12.68	8.70	6.37
SMF	37.17	35.35	33.44	31.68	29.41	27.61	25.28	22.98	19.58
AMF	37.45	35.71	33.42	31.86	30.29	28.66	26.82	24.74	20.27
NAFSM	38.68	35.63	33.61	32.31	31.09	29.77	28.54	27.06	23.56
MDBUTMF	38.46	34.55	33.57	32.31	31.07	29.91	28.83	27.41	23.41
BPDF	39.99	36.03	32.86	30.60	28.36	25.64	22.79	18.00	10.72
MDWF*	39.46	36.26	33.95	32.14	30.12	27.75	26.66	25.11	23.33
MDWMF**	41.45	38.22	35.97	34.07	32.69	31.21	29.72	27.94	25.50
TVWAF***	42.53	39.12	36.93	35.16	33.87	32.29	30.95	29.12	26.84
DAMF	42.92	39.15	36.79	34.76	33.12	31.72	30.17	28.41	25.86

*(Li et al, 2014)
**(Lu & Chou, 2012)
***(Lu et al, 2016)

Figure 3. The filtering results of Lena image corrupted by 80% salt and pepper noise; (a) free-noise image, (b) MF, (c) DWMF, (d) BPDF, (e) SMF, (f) AMF, (g) NAFSM, (h) MDBUTMF, (i) DAMF

4. CONCLUSION

This paper presents the review of several image filtering algorithms for consideration in conducting research related to this field. In this study several filtering techniques are described for impulse noise cases developed up to now. In general, filtering methods are divided into two types, namely spatial domain and domain transform. Filtering techniques in the case of impulse noise can be solved within the spatial domain, which is performed directly on the processed image pixels. The median filter is a very powerful filtering technique and widely used today, but this technique has a weakness in producing a blurry recovery image. To overcome this problem, many researchers combine one filter concept with another. Some of them add the concept of noise detection such as switching median filters, adaptive median filters, directional weighted median filters, adaptive fuzzy noise switching, median filters etc. These filtering techniques not only works on low density noise, but some may work very well even for noise with density above 60%. In addition to imparting filtering techniques for impulse noise cases, this paper also presents the commonly used image quality assessment, a peak signal to noise ratio and the structural similarity index, to estimate the efficiency of the existing techniques performance.

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REFERENCES

[1] Gonzalez R. and Woods R., “Digital Image Processing,” New Jersey: Pearson Prentice Hall, Upper Saddle River, 2008.
[2] Liu J., et al., “A weighted dictionary learning model for denoising images corrupted by mixed noise,” *IEEE T. Image Process*, vol. 22, no. 3, pp. 1108-1120, 2013.
[3] Zhu L., “Application of Stack Filter on Image Processing,” *Appl. Mech. Mater*, vol. 543-547, pp. 2163-2166, 2014.
[4] Saeedi J., et al., “A New Wavelet-Based Fuzzy Single and Multi-channel Image Denoising,” *Image Vision Comput*., vol. 28, no. 12, pp. 1611-1623, 2010.
[5] Schulte S., et al., “A Fuzzy Noise Reduction Method for Color Image,” *IEEE T. Image Process*, vol. 16, no. 5, pp. 1425-1436, 2007.
[6] Yan M., “Restoration of Images Corrupted by Impulse Noise and Mixed Gaussian Impulse Noise Using Blind Impainting,” *SIAM J. Imaging Sci*., vol. 6, no. 3, pp. 1227-1245, 2013.
[7] Baghaie A., “Markov Random Field Model-Based Salt and Pepper Noise Removal,” [Online], Available: https://arxiv.org/abs/1609.06341, 2016.
[8] Maity A., et al., “A Comparative Study on Approaches to Speckle Noise Reduction in Images,” *Proceeding IEEE Conference Computational Intelligence & Networks*, pp. 148-155, 2015.
[9] Lindenbaum M., et al., “On Gabor’s Constribution to Image Enhancement,” *Pattern Recogn.*, vol. 27, no 1, pp. 1-8, 1994.
[10] Chen L. C., et al., “Semantic Image Segmentation with Task-Specific Edge Detection Using CNNs and a Discriminatively Trained Domain Transform,” *Paper presented at IEEE Conference on Computer Vision and Pattern Recognition. (CVPR 2016)*, pp. 4545-4554, 2016.
[11] Malonia M. and Agarwal S. K., “Digital Image Watermarking using Discrete Wavelet Transform and Arithmetic Progression Technique,” *Paper presented at IEEE Students' Conference on the Electrical, Electronics and Computer Science (SCEECS)*, 2016.
[12] Liu X., et al., “Recent Progress in Semantic Image Segmentation,” *Artif. Intell.*, pp. 1-18, 2018.
[13] Durand S. and Froment J., “Reconstruction of Wavelet Coefficients Using Total Variation Minimization,” *SIAM J. Sci. Comput*., vol. 24, no. 5, pp. 1754-1767, 2001.
[14] Ding Y. and Selesnick I. W., “Artifact-free Wavelet Denoising: Non-convex Sparse Regularization, Convex Optimization,” *IEEE Signal Proc. Let*., vol. 22, no, 9, pp. 1364-1368, 2015.
[15] Lu X. G., et al., “An efficient nonconvex regularization for wavelet frame and total variation based image restoration,” *J. Comput. Appl. Math*., vol. 290, pp. 553-566, 2015.
[16] Elyasi I. and Pourmina M. A., “Reduction of Speckle Noise Ultrasound Images Based on TV Regularization and Modified Bayes Shrink Techniques,” *OPTIK*, vol. 127, no. 24, pp. 11732-11744, 2016.
[17] He W., et al., “Research on Mechanical Fault Diagnosis Scheme Based on Improved Wavelet Total Variation Denoising,” *Shock Vib*., vol. 2016, pp. 1-10, 2016.
[18] Zhou Y., et al., “An Image Denoising Algorithm for Mixed Noise Combining Nonlocal Means Filter and Sparse Representation Technique,” *J. Vis. Commun. Image R.*, vol. 41, pp. 74-86, 2016.
[19] Bovik A., “Handbook of Image and Video Processing,” New York: Academic Press, New York, 2000.
[20] Zhou W., et al., “Image Quality Assessment: From Error Visibility to Structural Similarity,” *IEEE T. Image Process*, vol. 13, no. 4, pp. 600-612, 2004.
[21] Donoho D. L., “Wedgelets: Nearly Minimax Estimation of Edges,” *Ann. Stat*., vol. 27, no. 3, pp. 859-897, 1999.
[22] Starck J. L., et al., “The Curvelet Transform for Image Denoising,” *IEEE T. Image Process*, vol. 11, no. 6, pp. 670-684, 2002.
[23] Do M. N. and Vetterli M., “The Contourlet Transform: An Efficient Directional Multiresolution Image Representation,” *IEEE T. Image Process*, vol. 14, no. 12, pp. 2091-2106, 2005.
[24] Le Pennec E. and Mallat S., “Sparse Geometric Image Representations with Bandelets,” *IEEE T. Image Process*, vol. 14, no. 4, pp. 423-438, 2005.
[25] Demanet L. and Ying L., “Wave atoms and sparsity of oscillatory patterns,” *Appl. Comput. Harmon. A.*, vol. 23, pp. 368-387, 2007.
[26] Mallat S., “A Wavelet Tour of Signal Processing: the sparse way,” New York, Academic press, 2008.
[27] Motwani M. C., et al., “Survey of image denoising techniques,” *Paper presented at the Proceedings of GSPX*, 2004.
[28] Yaroslavsky L. P., “Digital Picture Processing, volume 9 of Springer Series in Information Sciences,” Berlin, Springer-Verlag, 1985.
[29] Astola J. and Kuosmanen P., “Fundamentals of Nonlinear Digital Filtering,” USA, CRC Press, vol. 8, 1997.
[30] Tomasi C. and Manduchi R., “Bilateral Filtering for Gray and Color Images,” *Proceedings of the 1998 IEEE International Conference on Computer Visio*, 1998.
[31] Buades A., et al., “A Non-local Algorithm for Image Denoising,” *Paper presented at the IEEE Conference on Computer Society Computer Vision and Pattern Recognition, 2005. SVPR*, 2005.
[32] Li X., “Image Restoration: Fundamentals and Advances,” USA, CRC Press, 2012.
[33] Zhang P. and Li F., “A New Adaptive Weighted Mean Filter for Removing Salt-and-Pepper Noise,” *IEEE Signal Proc. Let*., vol. 21, no. 10, pp. 1280-1283, 2014.
[34] Liu L., et al., “A New Weighted Mean Filter with A Two-Phase Detector for Removing Impulse Noise,” *Inform. Sciences*, vol. 315, pp. 1-16, 2015.
[35] Brownrigg D., “The Weighted Median Filter,” *Commun. ACM*, vol. 27, no. 8, pp. 807-818, 1984.

[36] Ko S. J. and Lee Y. H., "Center Weighted Median Filters and Their Applications to Image Enhancement," *IEEE T. Circuits Syst.*, vol. 38, no. 9, pp. 984-993, 1991
[37] Arce G. R. and Paredes J. L., "Recursive weighted median filters admitting negative weights and their optimization," *IEEE T. Signal Proces*, vol. 48, no. 3, pp. 768-779, 2000.
[38] Dong Y. and Xu S., "A New Directional Weighted Median Filter for Removal of Random-Valued Impulse Noise," *IEEE T. Signal Proces*, vol. 14, no. 3, pp. 193-196, 2007.
[39] Lu C. and Chou T., "Denoising Of Salt-And-Pepper Noise Corrupted Image Using Modified Directional Weighted Median Filter," *Pattern Recog. Lett.*, vol. 33, no. 10, pp. 1287-1295, 2012.
[40] Li Z., et al., "Modified Directional Weighted Filter for Removal of Salt & Pepper Noise," *Pattern Recog. Lett.*, vol. 40, pp. 113-120, 2014.
[41] Bellanger M., "Adaptive Digital Filters," USA, CRC Press, 2001.
[42] Diniz P. S. R., "Adaptive filtering: Algorithms and Practical Implementation," New York, Springer-Verlag, 2008.
[43] Loupas T., et al., "An Adaptive Weighted Median Filter For Speckle Suppression in Medical Ultrasonic Images," *IEEE T. Circuits Syst.*, vol. 36, no. 1, pp. 129-135, 1989.
[44] Lei P., "Adaptive median filtering," *Paper presented at the Seminar Report, Machine Vision*, 2004.
[45] Wang Z. and Zhang D., "Progressive Swithcing Median Filter for the Removal of Impulse Noise from Highly Corrupted Images," *IEEE T. Circuit-II*, vol. 46, no. 1, pp. 78-80, 1999.
[46] Pushpavali R. and Sivaradje G., "Switching Median Filter for Image Enhancement," *International Journal of Scientific & Engineering Research*, vol. 3, no. 2, pp. 1-5, 2012.
[47] Toh K. K. V. and Isa N. A. M., "Noise Adaptive Fuzzy Switching Median Filter For Salt-And-Pepper Noise Reduction," *IEEE Signal Proc. Let.*, vol. 17, no. 3, pp. 281-284, 2010.
[48] Srinivasan K. S. and Ebenezer D., "A New Fast and Efficient Decision-Based Algorithm for Removal of High-Density Impulse Noises," *IEEE Signal Proc. Let*., vol. 14, no. 3, pp. 189-192, 2007.
[49] Esakkirajan S., et al., "Removal of High-Density Salt and Pepper Noise through Modifed Decision Based Unsymmetric Trimmed Median Filter," *IEEE Signal Proc. Let.*, vol. 18, no. 5, pp. 287-290, 2011.
[50] Vasanth K. and Senthilkumar V. J., "A Decision Based Unsymmetrical Trimmed Midpoint Algorithm for the Removal of High Density SPN," *Journal of Theoretical and Applied Information Technology*, vol. 42, pp. 245-252, 2012.
[51] Erkan U. and Gokrem L. A., "A New Method Based on Pixel Density in Salt and Pepper Noise Removal," *Turk. J. Electr. Eng. Co*., vol. 26, no. 1, pp. 162-171, 2018.
[52] Erkan U., et al., "Different Applied Median Filter in Salt and Pepper Noise," *Comput. Electr. Eng*., vol. 70, pp. 789-798, 2018.