A novel delay dictionary design for compressive sensing-based time varying channel estimation in OFDM systems

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Article Info	ABSTRACT
	Compressive sensing (CS) is a new attractive technique adopted for linear time varying channel estimation. orthogonal frequency division multiplexing (OFDM) was proposed to be used in 4G and 5G which supports high data rate requirements. Different pilot aided channel estimation techniques were proposed to better track the channel conditions, which consumes bandwidth, thus, considerable data rate reduced. In order to estimate the channel with minimum number of pilots, compressive sensing CS was proposed to efficiently estimate the channel variations. In this paper, a novel delay dictionary-based CS was designed and simulated to estimate the linear time varying (LTV) channel. The proposed dictionary shows the suitability of estimating the channel impulse response (CIR) with low to moderate Doppler frequency shifts with acceptable bit error rate (BER) performance. *This is an open access article under the CC BY-SA license.*

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1. INTRODUCTION

The performance of high data rate transmissions over wireless fading channels severely degraded due to the multi path effects which causes inter symbol interference (ISI). In order to combat the fading effects, OFDM has been widely adopted to wireless transmission [1-8]. The multipath propagation causes a time varying channel state information (CSI), which needed to be predicated or estimated using channel estimation techniques in order to recover the transmitted signal. Pilot aided channel estimation is the widely used technique begins from the traditional techniques such as, least square (LS) and linear minimum mean square (LMMS), ending with many recent ones used to improve the estimation performance [2, 9].

Compressive sensing (CS) is one of the recent techniques adopted for channel estimation in OFDM systems by exploiting channel sparsely representation with dictionary basis [10]. Several approaches have been employed to construct matrices in order to represent the channel in a sparse manner such as, discrete Fourier transform (DFT) and random dictionaries. These approaches do not consider the time variation property of the channel since the time varying channel parameters aren't taken into account [11-14].

The main contribution of this paper is, the design of a novel delay dictionary-based CS technique to overcome the problem of the dictionary proposed by [15] in estimating LTV channel in the presence of Doppler frequency shifts. In [15], a sample spaced delay dictionary was proposed to recover the CSI using CS in multiple input multiple output (MIMO)-OFDM system. The concept of the research based on estimate the channel coefficients for a time varying channel considering the useful OFDM symbol duration regarding the guard band, and tacking the delay profile into account. Considering channel delay parameters, the dictionary proposed by [15] improves its ability to recover the CSI even when number of pilots reduced, while it fails

in estimating LTV channel coefficients in the presence of Doppler effect since it exploits the time variability characteristic of the channel. Which leads to a conclusion that, the dictionary proposed by [15] can't be applied for channel estimation in LTV channels since it doesn't sense the Doppler effect of the channel.

The rest of this paper is organized as follows, in section 2, a brief introduction to CS theory introduced with the required analysis of LTV channel and the proposed system model for sparse channel estimation. Simulation tests, required system parameters, and test results are introduced in section 3. Finally, the main concluded remarks and future work are listed in section 4.

2. LTV CHANNEL ANALYSIS AND ESTIMATION BASED CS THEORY
2.1. Compressive sensing

Since the idea behind signal sparsity appears, many publications of sparse signal representations and compressive sensing introduced especially in signal processing community [16]. With compressive sensing, a real finite signal $x \in R^M$, can be expressed in an orthonormal basis;

$$x = \sum_1^M \psi_i \theta_i \tag{1}$$

where $\psi = [\psi_1 \psi_2 \dots \psi_M]$ represents the orthonormal basis, and $\theta = [\theta_1 \theta_2 \dots \theta_M]$ is the sparse vector where the number of non-zero elements *(K<<M)* much smaller than the number of zero elements and named as a K-sparse vector. Using matrix notations, $x = \psi\theta$, where ψ of size M x M [17]. Consider a classical linear measurement model where $y = \phi x = \phi\psi\theta$. Where θ represent the k-sparse vector of size M x 1 to be estimated using the effective measurement matrix $\phi\psi$, where the measurement matrix ϕ is of size N x M, and y is the measurement vector of size N x 1. Hence, each observation of y vector represents the projection of vector x on a row of the sensing matrix ϕ as described in Figure 1 [18].

From the mathematical expression of CS in the Figure 1, it is clear that a non-linear system of equations must be solved to recover the sparse vector θ, where the number of observations N is much less than number of unknowns M. Since ϕ matrix projecting the vector x, low value of incoherence is required to insure mutually independent matrices and hence better CS performance. The maximum value amongst inner product of the Orthonormal basis and the orthonormal measurement matrix defined as incoherence. Therefore, to recover the sparse vector correctly from $y = \phi x$, the sensing matrix ϕ should be designed carefully [19, 20].
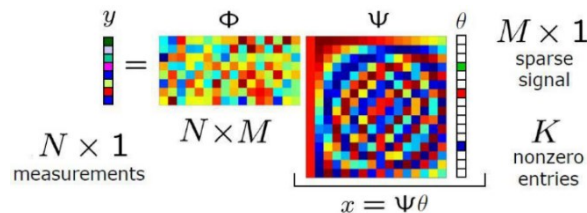

Figure 1. CS mathematical representation

2.2. System model and LTV channel

In this paper, the OFDM system of Figure 2 is considered. At the transmitter T_x side of this system, a stream of symbols x[k] (data d[k] and pilots p[k]) are mapped using binary phase shift keying (BPSK), where x[k] split into data blocks after serial to parallel conversion. Each of these blocks represent OFDM block contain data and pilot symbols. The length of each OFDM block is *N* subcarriers. A cyclic prefix (CP) of length (L_{cp}) is prepended to each OFDM block to prevent adjacent interference and considered as a guard band (G_i). After CP insertion, the OFDM block transmitted over an LTV channel which is a multipath propagation channel. In the proposed work, the LTV channel has been assumed to have a finite impulse response with *L* paths. The transmitter and receiver are assumed synchronized in both time and carrier frequency.

The multipath fading channel response is expressed as follows [21, 22];

$$h(t) = \sum_{i=0}^{L-1} a_i * \delta(t - \tau_i) \tag{2}$$

where, the ith path of wireless environment is characterized by a propagation delay (τ_i) and attenuation (a_i). The received baseband signal r(t) is modeled by two components, amplitude and phase, which can be expressed as;

$$r(t) = \sum_{i=0}^{L-1} a_i * \delta(t - \tau_i) * e^{-j2\pi f_c \tau_i} \tag{3}$$

where, $e^{-j2\pi f_c \tau_i}$ is the complex phase factor, and for a narrow band transmission; $\delta(t - \tau_i) = \delta(t)$ [23]. Hence;

$$h(t) = \sum_{i=0}^{L-1} a_i\, e^{-j2\pi f_c \tau_i} \tag{4}$$

The time varying property of the channel implies that channel coefficients changed over time. This changing is related to the change in the frequency of the received signal, which related to the relative movement between the transmitter and the receiver, Hence, the corresponding channel delay (τ_i) is changing [21, 22];

$$\tau_i(t) = \tau_i - \frac{v \cos\theta t}{c} \tag{5}$$

where $\tau_i(t)$ is a function of distance, and hence;

$$h(t) = \sum_{i=0}^{L-1} a_i\, e^{-j2\pi f_c * [\tau_i - \frac{v\cos\theta t}{c}]} \tag{6}$$

$$h(t) = \sum_{i=0}^{L-1} a_i\, e^{-j2\pi f_c \tau_i}\; e^{j2\pi f_c \frac{v\cos\theta t}{c}} \tag{7}$$

where, $f_c \frac{v\cos\theta}{c}$ is the Doppler frequency $f_d = f_{dmax}\cos\theta$, where $f_{dmax} = f_c \frac{v}{c}$ is the maximum Doppler shift (f_{dmax}). Assuming that the movement of the mobile system is uniformly distributed from $0 \le \theta \le \pi\ rad$, and θ is normalized. Thus, the channel impulse response is;

$$h(t) = \sum_{i=0}^{L-1} a_i\, e^{-j2\pi f_c \tau_i}\; e^{j2\pi f_d t} \tag{8}$$

Since this component ($e^{j2\pi f_d t}$) is a function of time, as a result, the channel coefficients $h(t)$ are time varying. Such a time varying channel is known as a time selective channel. How fast or slow the channel changes depends on the channel coherence time (T_c) where the channel is approximately constant during T_c [21].

At the other hand, in order to estimate the time variant channel coefficients using CS technique, the sensing matrix should be designed with atoms related to the two effecting parameters ($\tau_i\ and\ f_d$) of the LTV channel of (8). This will lead to compute the rate of change of the wireless channel by analyzing the correlation between channel coefficients. Assume that $a_i(t)$ is the channel coefficient at the ith path at time t;

$$a_i(t) = a_i\, e^{-j2\pi f \tau_i}\, e^{j2\pi f_d t} \tag{9}$$

Thus, to compute the correlation between $a_i(t)$ and $a_i(t + \Delta t)$, the expectation of $a_i(t)$ and $a_i(t + \Delta t)$, $E\{a_i(t)\, a_i(t + \Delta t)\}$ should be computed [24];

$$a_i(t) = a_i\, e^{-j2\pi f \tau_i}\, e^{j2\pi f_d t} \tag{10}$$

$$a_i(t + \Delta t) = a_i\, e^{-j2\pi f \tau_i}\, e^{j2\pi f_d (t + \Delta t)} \tag{11}$$

thus;

$$\Psi(\Delta t) = E\{|a_i|^2 * e^{j2\pi f_d \Delta t}\} \tag{12}$$

where $\Psi(\Delta t)$ refer to the correlation function between $a_i(t)$ and $a_i(t + \Delta t)$. Let $|a_i|^2$ normalized to be 1, thus,

$$\Psi(\Delta t) = E\{e^{j2\pi f_d \Delta t}\} \tag{13}$$

which summarized as;

$$\Psi(\Delta t) = J_0(2\pi f_d \Delta t) \tag{14}$$

where f_d represents the maximum Doppler frequency and J_0 is the Bessel function of 0th order. Finally, the autocorrelation function $\Psi(\Delta t)$ of LTV channel can be expressed in terms of coherence time [25];

$$\Psi(\Delta t) = J_0(\frac{\pi}{2} \cdot \frac{\Delta t}{T_c})$$

(15)

where; $\Delta t = xT_c, x = 1,2,3 \dots \dots,$ and, $T_c = \frac{1}{4f_d}$.
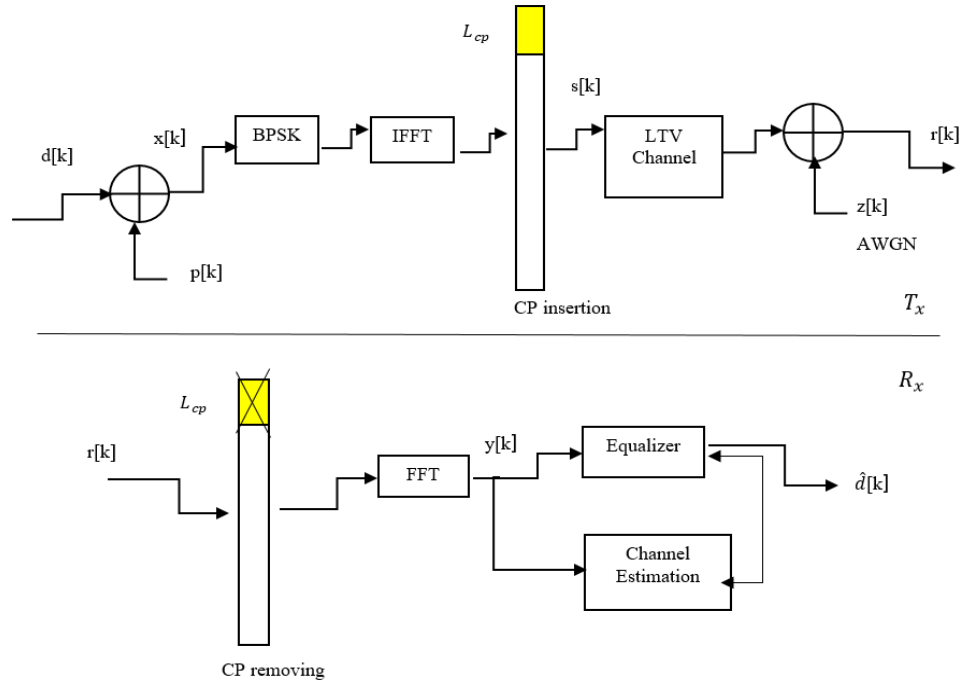

Figure 2. OFDM System Model

2.3. Sparse channel estimation

       Since CS has gained a much popularity in communications, recently, it is one of the small numbers of strong paths used for channel estimation. It assumes that sparse signals can be approximated with a small number of measurements compared to the large number required with Shannon-Nyquist rate [1]. Hence, to estimate the channel vector $h \in C^{N \times 1}$ from y measurements, a CS problem of section (2.1) should be solved, where y is expressed as follows;

$$y = Ah + n$$

(16)

n: AWGN noise with zero mean and variance $\sigma^2{}_n = \frac{N_0}{2}$

A: The sensing matrix

       The sparse representation of data in terms of atoms is the main objective of the dictionary design, which later used to reconstruct the sparse signal, where h assumed to be K-sparse CSI and its energy uniformly distributed among a small number of taps without any prior knowledge of their location, which must be estimated with effective sensing. It is clear from (15) that channel coefficients are changed with respect to the channel coherence time. Therefore, in this paper, the dictionary matrix is designed in a manner in which the two delay parameters of the autocorrelation function are taken into account. The equispaced pilot subcarriers p[k] are embedded within the data subcarriers d[k] of the OFDM system of Figure 2, where the number of training pilots is N_p, and Δt is assumed to be a taped delay profile along the OFDM symbol. Where;

$$\Delta t = \left[0, i \times \frac{\alpha}{N}, \alpha\right] \quad i = 1, 2, 3, \dots, N$$

(17)

α represent the minimum channel tap spacing which equals to $(G_i \times T_s - \frac{G_i \times T_s}{T_c \times N})$, and G_i is the guard interval which assumed to be the CP appended to each OFDM symbol in order to mitigate the ICI, and T_s is the OFDM sample time. An N x N dictionary matrix is constructed with atoms related to each subcarrier position ℓ_i along the OFDM block length T_{Symbol}, and multiplied by the taped delay atoms τ_i of Δt. $T_{Symbol} = (N + G_i) T_s$, is the OFDM symbol time including G_i. Therefore, the dictionary $D_{N \times N}$ is represented as follows;

$$D = \begin{bmatrix} e^{-\frac{j2\pi \ell_1 \tau_1}{T_{Symbol}}} & \cdots & e^{-\frac{j2\pi \ell_1 \tau_N}{T_{Symbol}}} \\ \vdots & \ddots & \vdots \\ e^{-\frac{j2\pi \ell_N \tau_1}{T_{Symbol}}} & \cdots & e^{-\frac{j2\pi \ell_N \tau_N}{T_{Symbol}}} \end{bmatrix}_{N \times N} \tag{18}$$

where the rows of dictionary matrix D are refer to the subcarriers positions along the OFDM symbol, while columns are refer to the delay vector of each subcarrier. Regarding sensing matrix. A construction, an N_p rows are selected from D related to pilot locations, and multiplied by $N_p \times N$ matrix of pilot data using dot product multiplication;

$$A = \begin{bmatrix} e^{\frac{j2\pi \ell_1 \tau_1}{T_{Symbol}}} & \cdots & e^{-\frac{j2\pi \ell_1 \tau_N}{T_{Symbol}}} \\ \vdots & \ddots & \vdots \\ e^{-\frac{j2\pi \ell_{N_p} \tau_1}{T_{Symbol}}} & \cdots & e^{\frac{j2\pi \ell_N \tau_N}{T_{Symbol}}} \end{bmatrix}_{N_p \times N} . \begin{bmatrix} e^{-j\pi P_1} & \cdots & e^{-j\pi P_{1(N)}} \\ \vdots & \ddots & \vdots \\ e^{-j\pi P_{N_p}} & \cdots & e^{-j\pi P_{N_p(N)}} \end{bmatrix}_{N_p \times N} \tag{19}$$

P_i is the pilot symbol used for estimation considering equally likely symbols of [0, 1], and $i = 1,2,3, \ldots , N_p$. Different sparse signal recovery algorithms can be applied to solve the CS problem in a number of iterations to minimize CS error with respect to D. Once the channel dominant taps estimated (i.e. recovering of the sparse vector θ), the whole CIR is built at all locations simply from $\hat{h} = D \times \theta$, and the equalization process implemented.

3. SIMULATION TEST AND RESULTS

In this paper, the OFDM system of Figure 2 compares the test results of both least square (LS) and basis pursuit (BP) based channel estimation techniques. Different OFDM system parameters are listed in Table 1. In addition to AWGN noise, a 6 tap LTV channel is considered as a Rayleigh fading channel with paths delays and power vectors standardized by ITU channel model of Table 2. Basis pursuit (BP) algorithm is used to solve the convex optimization problem with MatLab for sparse signal recovery, where it uses the l_1_norm to regularize the problem [9]. The performance test of OFDM system was shown in the form of bit error rate (BER) versus signal to noise ratio (SNR), where SNR is determined by the corresponding $(^{E_b}/_{N_o})$ in dB.

Table 1. OFDM system parameters

Parameter	Value
Number of transmitted bits	64000, 128000, 256000
Modulation	BPSK
Sampling Time (T_s)	1 μsec
OFDM Subcarriers	64, 128, 256
Number of pilots	16, 13
Cyclic prefix length (L_{cp})	16
Maximum Doppler shift (f_d)Hz	0, 10, 20, 40

In Figure 3, the BER performance of OFDM system over a multipath indoor channel environment using LS and BP is presented. In addition to compare the estimation performance of BP over LS algorithm, the purpose of this test is to show the ability of the proposed dictionary to recover the CSI with different delay parameters for both indoor and outdoor environments. In the present test, BP performance outperforms LS technique by about 4.5 dB at a BER of 10^{-3} with 16 pilots out of 64 subcarriers and zero Doppler frequency. In Figure 4, LS and BP algorithms are tested over a multipath outdoor channel environment of Table 2. In this test, the OFDM block containing 64 subcarriers and the number of pilot subcarriers used is 16. Different Doppler shifts are considered in order to test the recovering ability of the proposed dictionary in the presence of Doppler effects.
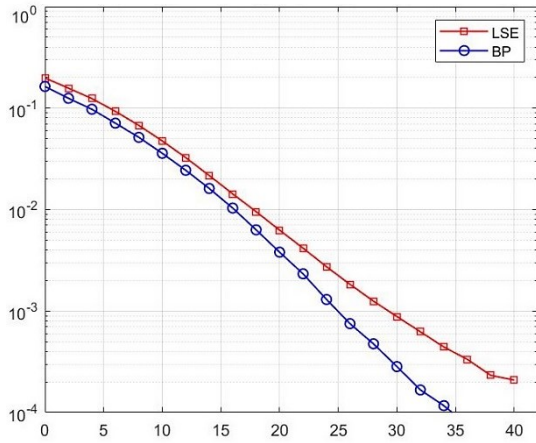
Figure 3. BER performance with
$f_d = 0\,H_z, N_p = 16, and\ N = 64$

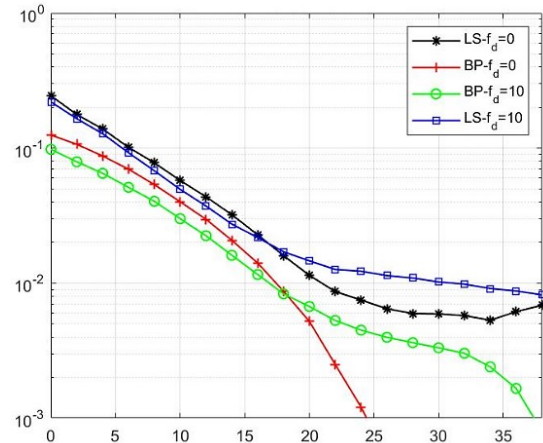Figure 4. BER performance with
$N_p = 16, and\ N = 64$

Table 2. ITU Channel Models [26]

Indoor		Outdoor	
Delay (ns)	Power (dB)	Delay (ns)	Power (dB)
0	0	0	0
50	-3	310	-1.5
110	-10	710	-9.0
170	-18	1090	-10.0
290	-26	1730	-15.0
310	-32	2510	-20.0

As could be noticed, CS based channel estimation algorithm improves the estimation performance as compared to LS algorithm even with Doppler effect. By comparing BP performance for both $f_d = 0\,Hz$, and $f_d = 10\,Hz$, it is clear that as f_d increased to 10 Hz, the performance test degraded by about 15 dB at a BER of 10^{-3}. At the other hand, BP performance degraded when f_d increased more than 10 Hz and become worse than LS unless the number of pilots used for estimation increased, which in turn improves LS performance. The same test was repeated with a lower number of pilots, where 13 pilots was inserted within the OFDM block at equally spaced locations instead of 16 pilots as shown in Figure 5. The test shows that the BP performance degraded but still much better than LS. This observation leads to the possibility of using reduced number of pilots for channel estimation without sacrificing the accuracy of channel estimation, when the rate of change of channel coefficients increasing according to Doppler shift effects.
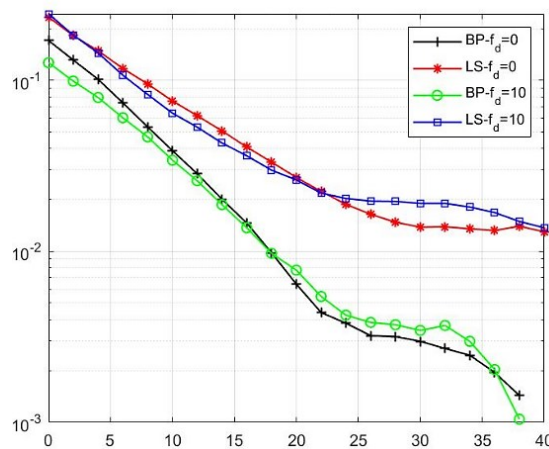

Figure 5. BER performance with $N_p = 13, and\ N = 64$

Another test considering different subcarrier numbers and Doppler shifts is shown in Figure 6. The test results proved that BP exceeded LS performance. But this superiority is still limited by the amount of Doppler shift and number of pilots, where it is degraded when f_d increased above than 20 H_z and 40 H_z for 128 and 256 OFDM subcarriers respectively with 16 pilots. This degradation is shown in Figure 7, where LS and BP algorithms are tested with, $f_d = 30\ H_z$ for $N = 128$, and $f_d = 50\ H_z$ for $N = 256$. Finally, it could be concluded that, as the amount of Doppler shift increase, the estimation performance degraded due to the Doppler effect on the channel. This degradation manifests itself when the number of subcarriers increased, where the subcarrier bandwidth will be decreased, so, it is more sensitive to the Doppler and requires an additional process to eliminate carrier frequency offset (CFO).
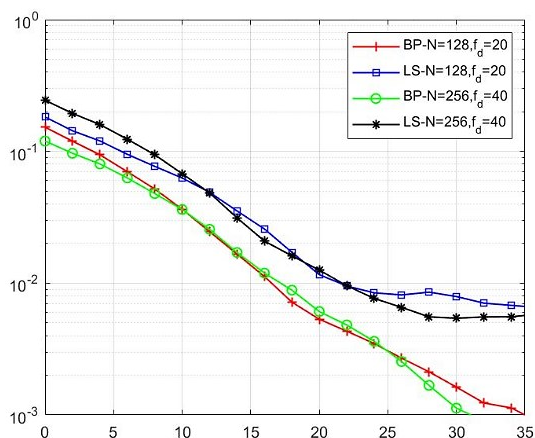

Figure 6. BER performance with
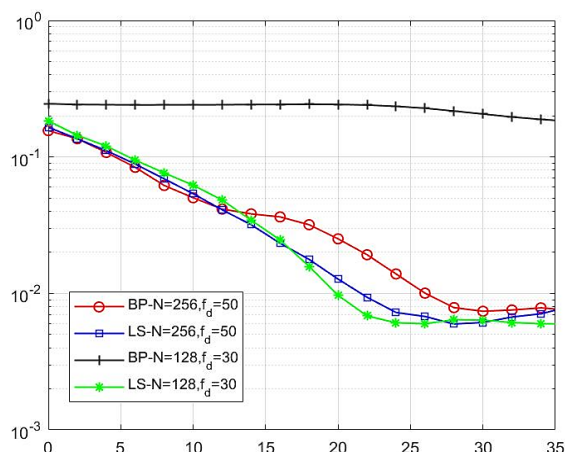$N_p = 16,\ N = 128\ and\ 256$

Figure 7. BER performance with $N_p = 16$

4. CONCLUSIONS AND FUTURE WORKS

In this paper, the proposed dictionary design was tested to achieving the desired results of BP based CS algorithm in estimating of CIR of a LTV channel. At the other hand, this performance is limited to the low to moderate Doppler frequency shifts. The future work may be carried to extend the current work to be used for estimation of LTV channel with high mobility or high Doppler frequency shifts.

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