TELKOMNIKA Telecommunication, Computing, Electronics and Control
Vol. 18, No. 2, April 2020, pp. 994~1000
ISSN: 1693-6930, accredited First Grade by Kemenristekdikti, Decree No: 21/E/KPT/2018
DOI: 10.12928/TELKOMNIKA.v18i2.14301

An optimal control for complete synchronization of 4D Rabinovich hyperchaotic systems

Shaymaa Y. Al-Hayali, Saad Fawzi Al-Azzawi
Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Iraq

Article Info

Article history:

Received Oct 8, 2019
Revised Jan 8, 2020
Accepted Feb 11, 2020

Keywords:

Active control strategy
Complete synchronization
Lyapunov second method
Nonlinear control strategy

ABSTRACT

This paper derives new results for the complete synchronization of 4D identical Rabinovich hyperchaotic systems by using two strategies: active and nonlinear control. Nonlinear control strategy is considered as one of the powerful tool for controlling the dynamical systems. The stabilization results of error dynamics systems are established based on Lyapunov second method. Control is designed via the relevant variables of drive and response systems. In comparison with previous strategies, the current controller (nonlinear control) focuses on convergence speed and the minimum limits of relevant variables. Better performance is to achieve full synchronization by designing the control with fewer terms. The proposed control has certain significance for reducing the time and complexity for strategy implementation.

Corresponding Author:

Saad Fawzi Al-Azzawi,
Department of Mathematics, College of Computer Science and Mathematics,
University of Mosul, Mosul, Iraq.
Email: saad_fawzi78@yahoo.com, saad_alazawi@uomosul.edu.iq

1. INTRODUCTION

After the pioneering work by Pecora and Carroll in 1990s [1], Chaos synchronization has attracted considerable attention due to its important applications in physical systems [2], biological systems [3], Encryption [4, 5] and secure communications [6], etc. The synchronization means making a system called a response system to follow another system which is called a drive system. Now days, enormous synchronization phenomena have been applied in various dynamical systems such as full/complete synchronization (CS) [7-10], anti-synchronization (AS) [11], hybrid synchronization (HS) [12], lag synchronization, phase synchronization, projective synchronization (PS) [13], modified projective synchronization (MPS) [14] and generalized projective synchronization (GPS) [15]. Full synchronization and anti-synchronization are the most commonly used [9, 11] and play an important role in engineering applications [16, 17].

These phenomena are achieved via different various types of synchronizations schemes including active control [18], adaptive control [16], nonlinear control [19-22] and linear feedback control [23-25]. Among the aforementioned schemes, the active control and the nonlinear control have been widely used as two powerful strategies for synchronization of different class of nonlinear dynamical systems. However, the active control suffers from many terms corresponding to relevant variables of drive and response systems. To overcome this problem, the nonlinear control strategy is used with the minimum of terms and speed synchronization whereas nonlinear control strategy has demonstrated excellent performance in synchronization schemes.

In this paper, we implement complete synchronization between two 4D identical Rabinovich hyperchaotic systems based on active and nonlinear control strategies via Lyapunov second method and observed that the speed of convergence of control in the second a strategy faster and number of terms less than the first strategy. The proposed control with low terms is more interesting and easily applied and implemented. The contributions of this paper are mainly the following:

– A necessary and sufficient condition is proposed to show how many relevant variables of drive and response systems can achieve synchronization under active and nonlinear controller.
– New results are derived from a number of terms of relevant variables of drive and response systems that can lead to synchronization.
– New results are derived from speed synchronization via relevant variables of drive and response systems.

The rest of this paper is organized as follows. Section 2 is the description of hyperchaotic Rabinovich System. Section 3 presents the problem of complete synchronization for the hyperchaotic Rabinovich. Section 4 is the conclusions this paper.

2. DESCRIPTION OF HYPERCHAOTIC RABINOVICH SYSTEM

Rabinovich system is a four-dimensional hyperchaotic which include ten terms, three of them are nonlinearity with three parameters and descript by the following form [26, 27]:

$$\begin{cases} \dot{x}_1 = -ax_1 + hx_2 + x_2x_3 \\ \dot{x}_2 = hx_1 - x_2 - x_1x_3 + x_4 \\ \dot{x}_3 = -x_3 + x_1x_2 \\ \dot{x}_4 = -kx_2 \end{cases} \tag{1}$$

where x_1, x_2, x_3, x_4 are the state variables and a, h, k are positive constants this system possesses hyperchaotic attractors when the parameters taken $a = 4, h = 6.75, k = 2$ as show in Figures 1-4.

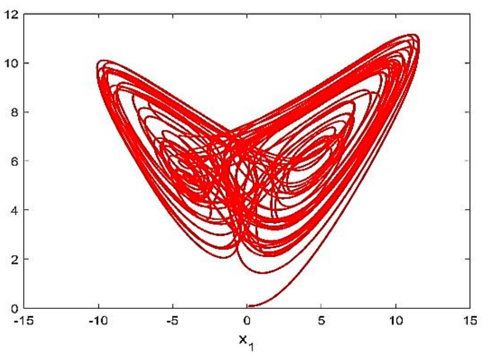

Figure 1. The attractor of system (1) in $x_1 - x_3$ plane

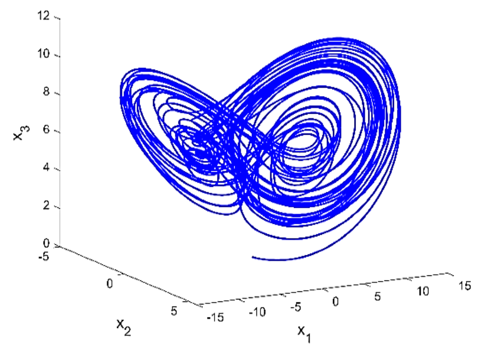

Figure 2. The attractor of system (1) in x_1, x_2, x_3 space

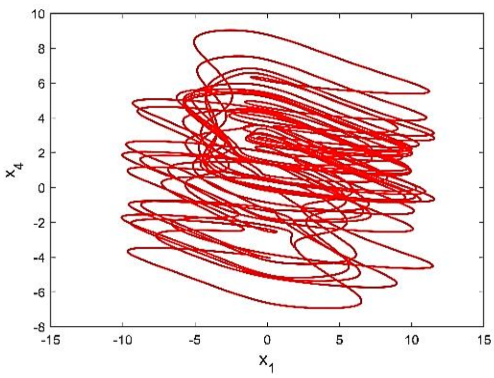

Figure 3. The attractor of system (1) in $x_1 - x_4$ plane

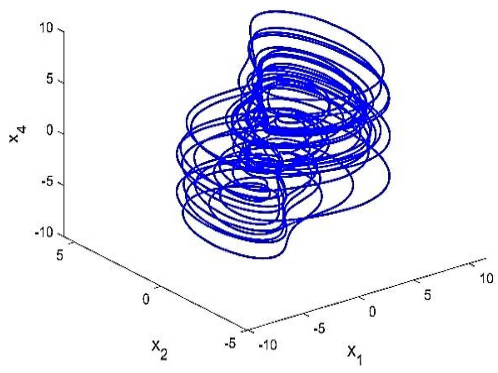

Figure 4. The attractor of system (1) in x_1, x_2, x_4 space

3. SYNCHRONIZATION BETWEEN TWO IDENTICAL HYPERCHAOTIC RABINOVICH SYSTEM

In order to achieve complete synchronization for Rabinovich system, two systems are needed, the first system is called drive system, and the second system is called response system. The drive and response systems for Rabinovich system depict in (2) and (3) respectively.

$$\begin{cases} \dot{x}_1 = -ax_1 + hy_1 + y_1 z_1 \\ \dot{y}_1 = hx_1 - y_1 - x_1 z_1 + w_1 \\ \dot{z}_1 = -z_1 + x_1 y_1 \\ \dot{w}_1 = -ky_1 \end{cases} \tag{2}$$

$$\begin{cases} \dot{x}_2 = -ax_2 + hy_2 + y_2 z_2 + u_1 \\ \dot{y}_2 = hx_2 - y_2 - x_2 z_2 + w_2 + u_2 \\ \dot{z}_2 = -z_2 + x_2 y_2 + u_3 \\ \dot{w}_2 = -ky_2 + u_4 \end{cases} \tag{3}$$

Where $u = [u_1, u_2, u_3, u_4]^T$ is the controller to be designed, The synchronization error $e \in R^4$ is defined as:

$$e_1 = x_2 - \alpha_i x_1 \ , \ e_2 = y_2 - \alpha_i y_1 \ , \ e_3 = z_2 - \alpha_i z_1 \ , \ e_4 = w_2 - \alpha_i w_1 \ \ , \forall \alpha_i = 1 \ , \ i = 1,2,3,4$$

so, the error dynamical system which is given by:

$$\begin{cases} \dot{e}_1 = -ae_1 + he_2 + e_2 e_3 + z_1 e_2 + y_1 e_3 + u_1 \\ \dot{e}_2 = he_1 - e_2 + e_4 - e_1 e_3 - z_1 e_1 - x_1 e_3 + u_2 \\ \dot{e}_3 = -e_3 + e_1 e_2 + y_1 e_1 + x_1 e_2 + u_3 \\ \dot{e}_4 = -ke_2 + u_4 \end{cases} \tag{4}$$

3.1. Synchronization based on active control

To realize the complete synchronization, we need to design suitable nonlinear control. Therefore, the control functions are chosen as the following:

$$\begin{cases} u_1 = -e_2 e_3 - z_1 e_2 - y_1 e_3 + v_1 \\ u_2 = e_1 e_3 + z_1 e_1 + x_1 e_3 + v_2 \\ u_3 = -e_1 e_2 - y_1 e_1 - x_1 e_2 + v_3 \\ u_4 = v_4 \end{cases} \tag{5}$$

inserting the control (5) in (4) we get:

$$\begin{cases} \dot{e}_1 = -ae_1 + he_2 + v_1 \\ \dot{e}_2 = he_1 - e_2 + e_4 + v_2 \\ \dot{e}_3 = -e_3 + v_3 \\ \dot{e}_4 = -ke_2 + v_4 \end{cases} , \qquad v = [v_1 \quad v_2 \quad v_3 \quad v_4]^T = A[e_1 \quad e_2 \quad e_3 \quad e_4]^T \tag{6}$$

where v is linear control, A is a constant matrix, to make the system (6) stable, the matrix A should be selected by the following:

$$A = \begin{bmatrix} 0 & -h & 0 & 0 \\ -h & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & k & 0 & -1 \end{bmatrix} \tag{7}$$

hence, the error dynamical system (4) with above matrix becomes:

$$\begin{cases} \dot{e}_1 = -ae_1 \\ \dot{e}_2 = -e_2 \\ \dot{e}_3 = -e_3 \\ \dot{e}_4 = -e_4 \end{cases} \tag{8}$$

Therefore, the above system has all eigenvalues with negative real parts, these eigenvalues guarantees the stability of the system (8). So, the response system (3) synchronizes the drive system. Hence, we reach at the following results:
Theorem 1: If the matrix (7) is combine with system (6), then, the response system (3) follows the drive system via the following nonlinear active control which consists of 16 terms.

$$\begin{cases} u_1 = -he_2 - e_2e_3 - z_1e_2 - y_1e_3 \\ u_2 = -he_1 - e_4 + e_1e_3 + z_1e_1 + x_1e_3 \\ u_3 = -e_1e_2 - y_1e_1 - x_1e_2 \\ u_4 = ke_2 - e_4 \end{cases} \tag{9}$$

Proof: Based on the Lyapunov second method, we construct a positive definite Lyapunov candidate function as:

$$\mathrm{V(e)} = e^T pe = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}e_4^2 \tag{10}$$

where $P = diag\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]$, the derivative of the Lyapunov function $\mathrm{V(e)}$ with respect to time is:

$$\dot{V}(e) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4$$

$$\dot{V}(e) = e_1(-ae_1) + e_2(-e_2) + e_3(-e_3) + e_4(-e_4)$$

$$\dot{V}(e) = -ae_1^2 - e_2^2 - e_3^2 - e_4^2 = -e^TQe\ ,\ Q = diag[a\ , 1, 1, 1] \tag{11}$$

According to [14], every diagonal matrix with positive diagonal elements is positive definite. So $Q > 0$. Therefore, $\dot{V}(e)$ is negative definite. And according to the Lyapunov asymptotical stability theory, the nonlinear active controller is implemented and the synchronization of the hyperchaotic system is achieved. The proof is now complete. The theorem 1 showed that proposed control which consists of (14) terms achieved synchronization. The time converges synchronization at (6) as shown in Figure 5.

3.2. Synchronization based on nonlinear control strategy

In this section, complete synchronization between system (2) and system (3) is considered by using other strategy which is called nonlinear control. Theorem 2: If design a controller consists of six terms as follows:

$$\begin{cases} u_1 = -2he_2 \\ u_2 = ke_4 \\ u_3 = -2y_1e_1 - e_1e_2 \\ u_4 = -e_2 - e_4 \end{cases} \tag{12}$$

then the error dynamical system (4) is convergent to zero as time (4). Proof**:** With this choice, the error dynamical system (4) becomes:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} -a & -h + z_1 & e_2 + y_1 & 0 \\ h - z_1 & -1 & -e_1 - x_1 & k + 1 \\ e_2 - y_1 & x_1 & -1 & 0 \\ 0 & -(k + 1) & 0 & -1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$$

i.e.,

$$\begin{cases} \dot{e}_1 = -ae_1 - he_2 + e_2e_3 + z_1e_2 + y_1e_3 \\ \dot{e}_2 = he_1 - e_2 + (k + 1)e_4 - e_1e_3 - z_1e_1 - x_1e_3 \\ \dot{e}_3 = -e_3 + e_1e_2 - y_1e_1 + x_1e_2 \\ \dot{e}_4 = -(k + 1)e_2 - e_4 \end{cases} \tag{13}$$

this derivative is:

$$\dot{V}(e) = -ae_1^2 - e_2^2 - e_3^2 - e_4^2 = -e^TQe \tag{14}$$

So $Q > 0$. Therefore, $\dot{V}(e)$ is negative definite. The theorem 2 showed that proposed control which consists of six terms achieved synchronization and the time synchronization at (4) time(sec) as shown in Figure 6.

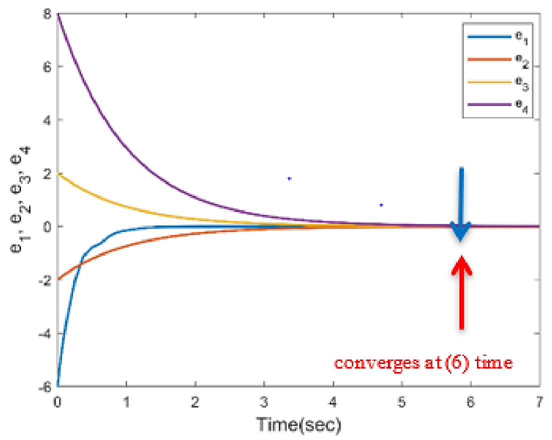

Figure 5. The converges of system(4) with controller (9)

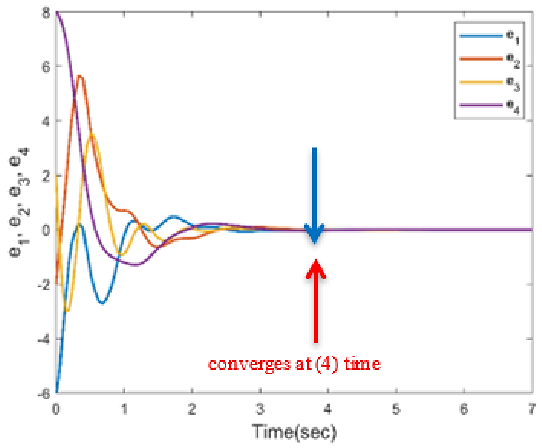

Figure 6. The converges of system (13) with controller (12)

Theorem 3: If the controller is designed with six terms as follows:

$$\begin{cases} u_1 = -2he_2 - 2y_1e_3 - e_2e_3 \\ u_2 = 0 \\ u_3 = 0 \\ u_4 = ke_2 - e_2 - e_4 \end{cases} \tag{15}$$

then the system (4) will be convergent to zero as the time (4). Proof: when substituting the controllers (15) in the system (4), we get:

$$\begin{cases} \dot{e}_1 = -ae_1 - he_2 + z_1e_2 - y_1e_3 \\ \dot{e}_2 = he_1 - e_2 + e_4 - e_1e_3 - z_1e_1 - x_1e_3 \\ \dot{e}_3 = -e_3 + e_1e_2 - y_1e_1 + x_1e_2 \\ \dot{e}_4 = -e_2 - e_4 \end{cases} \tag{16}$$

construct Lyapunov function as:

$$\mathrm{V(e)} = e^Tpe = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}e_4^2$$

then,

$$\dot{V}(e) = -ae_1^2 - e_2^2 - e_3^2 - e_4^2 = -e^TQe$$

so, $\mathrm{V(e)} > 0$ and $\dot{V}(e) < 0$, the nonlinear controller is implemented. The theorem 3 showed that proposed control which consists of six terms achieved synchronization. The time synchronization at (4) time(sec) as shown in Figure 7.

Theorem 4: the system (4) is achieved and converges to zero at the time (4.20), if the controller is designed as:

$$\begin{cases} u_1 = -2he_2 - 2y_1e_3 \\ u_2 = -e_1e_3 + ke_4 \\ u_3 = 0 \\ u_4 = -e_2 - e_4 \end{cases} \tag{17}$$

Proof: Using system (4) with the controller (17), is given by.

$$\begin{cases} \dot{e}_1 = -ae_1 - he_2 + e_2e_3 + z_1e_2 - y_1e_3 \\ \dot{e}_2 = -he_1 - e_2 + (k+1)e_4 - 2e_1e_3 - z_1e_1 - x_1e_3 \\ \dot{e}_3 = -e_3 + e_1e_2 + y_1e_1 + x_1e_2 \\ \dot{e}_4 = -(k+1)e_2 - e_4 \end{cases} \quad (18)$$

the same results were found in theorem (3). The theorem 4 showed that proposed control which consists of six terms achieved synchronization. The time synchronization at (4.20) time (sec) illustrated in Figure 8.

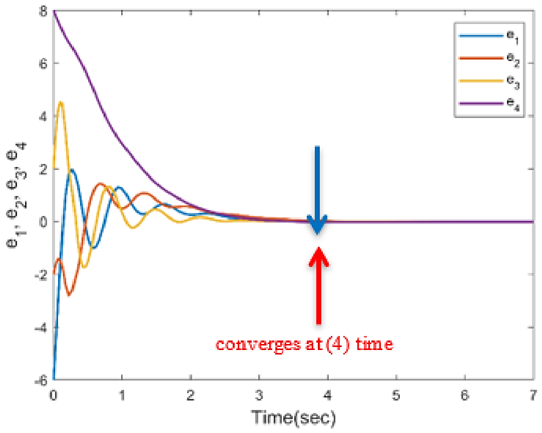

Figure 7. The converges of system (16) with controller (15)

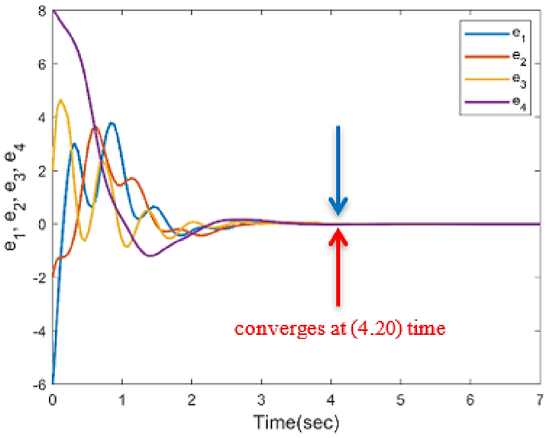

Figure 8. The converges of system (18) with controllers (17)

4. CONCLUSION

In the paper, synchronaiztion problem for 4-D Rabinovich hyperchaotic system is considered, based on two strategies: active and non-linear controller. The stability of error dynimcal systems are established based on the Lyapunov theory and compared between these strategies and found that both of them lead to synchronization but the performance of the nonlinear controller is faster and better than the active control. Also, the number of terms of controller is less than first strategy.

ACKNOWLEDGEMENTS

The authors are very grateful to University of Mosul/College of Computer Sciences and Mathematics for their provided facilities, which helped to improve the quality of this work.

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