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Projective and hybrid projective synchronization of 4-D hyperchaotic system via nonlinear controller strategy

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ABSTRACT

Nonlinear control strategy was established to realize the Projective Synchronization (PS) and Hybrid Projective Synchronization (HPS) for 4-D hyperchaotic system at different scaling matrices. This strategy, which is able to achieve projective and hybrid projective synchronization by more precise and adaptable method to provide a novel control scheme. On First stage, three scaling matrices were given in order to achieving various projective synchronization phenomena. While the HPS was implemented at specific scaling matrix in the second stage. Ultimately, the precision of controllers were compared and analyzed theoretically and numerically. The long-range precision of the proposed controllers are confirmed by third stage.

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1. INTRODUCTION

In nonlinear dynamic systems, chaos synchronization is the first phenomenon which discovered by Fujisaka and Yamada in 1983, but did not receive great interest until 1990 when Pecora and Carrol developed this phenomenon between two identical chaotic systems with different initial condition [1-4]. Chaos synchronization has attracted considerable attention due to its important applications in physical systems [1], biological systems [5], Encryption [6] and secure communications [7], etc. After then, several attempts were made to create many types of synchronization phenomena such as Complete Synchronization (CS) [2, 4, 8], Anti-Synchronization (AS) [9, 10], Hybrid Synchronization (HS) [11], Generalized Synchronization (GS) [12], Projective Synchronization (PS) [13], Hybrid Projective Synchronization (HPS) [14] and Generalized Projective Synchronization (GPS) [15]. Amongst all types of synchronization schemes, PS and HPS attracted lots of attention because it can obtain faster communication in application to secure communication [13, 14]. Both of them are characterized that the two systems could be synchronized up to a constant diagonal matrix, but in the first feature, all diagonal elements of scaling matrix should be equal whereas these diagonal elements are different in the second feature. Obviously, choosing the constant matrix as unity will lead to CS. So, CS and AS are the special cases of PS and HS belong to the special case of hybrid projective synchronization. HPS gives more complexity to the controller and the message cannot be easily decoded by the intruder.

In projective and HPS processes, various strategies have been introduced to stabilize dynamic error systems, including adaptive control [16], active control, nonlinear control [17-20] and linear feedback control [21-23]. Among many control strategies, the nonlinear control strategy has been continuously for which great interest to many scientists, due to its effectiveness, reliability, and widely has been used as a single powerful strategy for synchronizing different class of the nonlinear dynamic systems [24, 25]. But, the control input design should be based on the functions of the controlled system according to the traditional nonlinear control. In order to simplify the control input, adaptive nonlinear control has been designed to facilitate the control input process. To ensure that the designed controller has a good control effect, the controller is designed for a non-linear control system based on the theory of stability Lyapunov with known and unknown parameters. Then, controllers designed to synchronize a hyperchaotic system were used. These findings may be important in understanding and controlling problems in modern society. Also the effectiveness and strength of controllers are verified by numerical simulation results.

2. PROJECTIVE AND HYBRID PROJECTIVE SYNCHRONIZATION

The PS and HPS are illustrated in this section. There are two nonlinear dynamical systems, the first is called drive system, whereas the second, is called response system, and the response system controls the drive system. The drive system and response system yields the (1) and (2), respectively [18] and [21].

$$\dot{x} = Ax + f(x) \tag{1}$$

$$\dot{y} = By + g(y) + U \tag{2}$$

where A, B are a $n \times n$ parameters matrices, $x = [x_1, x_2, \dots, x_n]^T \in R^{n\times 1}$, $y = [y_1, y_2, \dots, y_n]^T \in R^{n\times 1}$ are state vector, $f(x)$ and $g(y)$ are the nonlinear functions for system 1 and system 2, respectively. Also, $U = [u_1, u_2, \dots, u_n]^T \in R^n$ is a control input vector. Whereas the error dynamical system is defined as

$$e = y - Sx \tag{3}$$

where $e = [e_1, e_2, \dots, e_n]^T \in R^{n\times 1}$ in general, S is n-order diagonal matrix i.e. $S = diag(s_1, s_2, \dots., s_n)$, S is called scaling matrix and s_1, $s_2, \dots, s_n$ are called scaling factor. Our goal is to propose a suitable controller U to make the response system asymptotically approaches the drive system, and finally the synchronization phenomena will be achieved in the sense that the limit of the error dynamical system approaches zero i.e.

$$lim_{t\to\infty}\|e\| = lim_{t\to\infty}\|y - Sx\| = 0 \tag{4}$$

The scaling matrix S play an important role to determine the phenomenon of synchronization, such as if S is constant matrix and

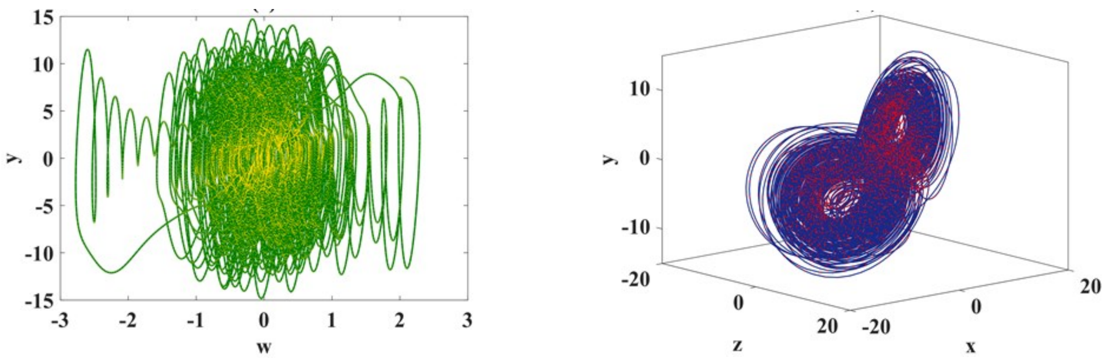
- $s_1 = s_2 = \cdots = s_n$, then this phenomenon is called PS
- $s_1 \neq s_2 \neq \cdots \neq s_n$, then this phenomenon is called HPS
- $\forall s_i = 1$, then this phenomenon is called CS
- $\forall s_i = -1$, then this phenomenon is called AS
- $\forall s_i = \pm 1$, then this phenomenon is called HS

3. APPLICATIONS

In this section, we take 4-D non-linear dynamical system which discover by Zhang et al in 2017 [26], for example to show how to use the results obtained in this paper to analyse the synchronization class of hyperchaotic systems. The mathematical model is representing by the following:

$$\begin{cases} \dot{x} = a(y - x) - fw \\ \dot{y} = xz - qy \\ \dot{z} = b - xy - cz \\ \dot{w} = ry - dw \end{cases} \tag{5}$$

where x, y, z and w are state variable, xz and xy are two nonlinear terms and a, b, c, d, f, r and q are positive parameters. And the system (5) is hyperchaotic attractors due to has possess two positive Lyapunov exponents as $LE_1 = 0.24, LE_2 = 0.23$ at $a = 5,\ b = 20,\ c = 1,\ d = 0.1,\ f = 20.6,\ r = 0.1$ and $q = 1$. The projections of hyperchaotic attractor of the above system are shown in Figure 1.



(a) (b)

Figure 1. The attracters of the system (1) in: (a) y-w plane, (b) x-z-y space

According to (1) and (2), system (5) can be represent as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\underbrace{\begin{bmatrix}-a & a & 0 & -f\\ 0 & -q & 0 & 0\\ 0 & 0 & -c & 0\\ 0 & r & 0 & -d\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\underbrace{\begin{bmatrix}0\\ 0\\ b\\ 0\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\ x_1x_3\\ x_1x_2\\ 0\end{bmatrix}}_{f(x)} \tag{6}$$

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\\ \dot{y}_4\end{bmatrix}=\underbrace{\begin{bmatrix}-a & a & 0 & -f\\ 0 & -q & 0 & 0\\ 0 & 0 & -c & 0\\ 0 & r & 0 & -d\end{bmatrix}}_{B}\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\end{bmatrix}+\underbrace{\begin{bmatrix}0\\ 0\\ b\\ 0\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\ y_1y_3\\ y_1y_2\\ 0\end{bmatrix}}_{g(y)}+\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{bmatrix} \tag{7}$$

where $U=[u_1,u_2,u_3,u_4]^T$ is the controller to be designed

4. PROJECTIVE SYNCHRONIZATION (PS)

This phenomenon take place under the condition “that all diagonal elements s_i of the constant scaling matrix $S(t)$, possess the same value”. Herein, three cases are considered as

- $\forall\, s_i = 3$
- $\forall\, s_i = 1$
- $\forall\, s_i = -1$

4.1. The Controllers at scaling factor $\forall\, s_i = 3$

According to (3), the error of PS $\dot{e}_i \in R^4$ between the system (6) and the system (7) is depict by:

$$\dot{e}_i = y_i - 3I_i\, x_i, \quad I = daig[1,1,\dots,1],\ i = 1,2,3,4$$

and lead to

$$\begin{cases}\dot{e}_1 = a(e_2-e_1) - fe_4 + u_1\\ \dot{e}_2 = -qe_2 + e_3y_1 + 3(y_1-x_1)x_3 + u_2\\ \dot{e}_3 = -ce_3 - e_2y_1 + 3(x_1-y_1)x_2 - 2b + u_3\\ \dot{e}_4 = -de_4 + re_2 + u_4\end{cases} \tag{8}$$

Theorem 1. If design control $U=[u_1,u_2,u_3,u_4]^T$ as:

$$\begin{cases}u_1 = 0\\ u_2 = -ae_1 - re_4 + 3(x_1-y_1)x_3\\ u_3 = 2b + 3(y_1-x_1)x_2\\ u_4 = fe_1\end{cases} \tag{9}$$

Then the system (8) can be controlled i.e., PS between system (6) and system (7) is achieved.
Proof: By inserting the controller (9) in the error system (8) we get:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 \\ \dot{e}_2 = -qe_2 - re_4 - ae_1 + e_3y_1 \\ \dot{e}_3 = -ce_3 - e_2y_1 \\ \dot{e}_4 = -de_4 + re_2 + fe_1 \end{cases} \quad (10)$$

Construct the Lyapunov function as the following:

$$V(e) = e^T P\, e, \quad P = diag(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}) \quad (11)$$

and derivative $V\,(e)$ along time of (10) is:

$$\dot{V}(e) = e_1(a(e_2 - e_1) - fe_4) + e_2(-qe_2 - re_4 - ae_1 + e_3y_1) \\ +e_3(-ce_3 - e_2y_1) + e_4(-de_4 + re_2 + fe_1)$$

Above equation can reduce as:

$$\dot{V}(e) = -ae_1^2 - qe_2^2 - ce_3^2 - de_4^2 = -e^T Q\, e, \quad Q = diag[a\ q\ c\ d]$$

the matrix Q is positive definite. So, $\dot{V}(e)$ is negative definite. The Lyapunov's direct method is satisfied. Therefore, the response system (7) is PS with the drive system (6) asymptotically, the proof is complete.

4.2. The Controllers at scaling factor $\forall\, s_i = 1$

For all scaling factor are equal 1, the error of PS between the drive system (6) and the response system (7) is given by:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 + u_1 \\ \dot{e}_2 = -qe_2 + e_1x_3 + e_3x_1 + e_1e_3 + u_2 \\ \dot{e}_3 = -ce_3 - e_1x_2 - e_2x_1 - e_1e_2 + u_3 \\ \dot{e}_4 = -de_4 + re_2 + u_4 \end{cases} \quad (12)$$

Theorem 2. The systems (6) & (7) will be asymptotically stable, if the controller is designed as follows:

$$\begin{cases} u_1 = -e_2x_3 + e_3x_2 \\ u_2 = -ae_1 - re_4 \\ u_3 = 0 \\ u_4 = fe_1 \end{cases} \quad (13)$$

Proof: By substituting the controllers (13) in the system (12), we can obtain:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 - e_2x_3 + e_3x_2 \\ \dot{e}_2 = -qe_2 + e_1x_3 + e_3x_1 + e_1e_3 - ae_1 - re_4 \\ \dot{e}_3 = -ce_3 - e_1x_2 - e_2x_1 - e_1e_2 \\ \dot{e}_4 = -de_4 + re_2 + fe_1 \end{cases} \quad (14)$$

the Lyapunov function and its derivative are yields Eqs. (15) and (16), respectively

$$V(e) = \tfrac{1}{2}\textstyle\sum_{i=4}^{4} e_i^2 = \tfrac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) \quad (15)$$

$$\dot{V}(e) = -ae_1^2 - qe_2^2 - ce_3^2 - de_4^2 < 0 \quad (16)$$

since $V(e)$ is a positive function and $\dot{V}(e)$ is negative. So, the response of system (7) is PS with the drive system (6) asymptotically. The proof is complete.

4.3. The Controllers at scaling factor $\forall\, s_i = -1$

For all scaling factor are taken the values -1, the error of PS between the system (6) and the system (7) is given by:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 + u_1 \\ \dot{e}_2 = -qe_2 + e_1x_3 + e_3x_1 + (y_1 - x_1)(y_3 - x_3) + u_2 \\ \dot{e}_3 = -ce_3 - e_1x_2 - e_2x_1 - (y_1 - x_1)(y_2 - x_2) + 2b + u_3 \\ \dot{e}_4 = -de_4 + re_2 + u_4 \end{cases} \tag{17}$$

Theorem 3. Choose the controller U_i as:

$$\begin{cases} u_1 = fe_4 - e_2x_3 + e_3x_2 \\ u_2 = -ae_1 - re_4 - (x_1 - y_1)(x_3 - y_3) \\ u_3 = (y_1 - x_1)(y_2 - x_2) - 2b \\ u_4 = 0 \end{cases} \tag{18}$$

The error dynamic system (17) can be controlled.
Proof: With this control, (17) can be rewritten as:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - e_2x_3 + e_3x_2 \\ \dot{e}_2 = -qe_2 - re_4 - ae_1 + e_1x_3 + e_3x_1 \\ \dot{e}_3 = -ce_3 - e_1x_2 - e_2x_1 \\ \dot{e}_4 = -de_4 + re_2 \end{cases} \tag{19}$$

The time derivative of the Lyapunov function is:

$$\dot{V}(e) = -ae_1^2 - qe_2^2 - ce_3^2 - de_4^2 < 0$$

which is negative definite So, $\dot{V}(e) < 0$. Therefore, PS of the two systems can be achieved simultaneously.

5. HYBRID PROJECTIVE SYNCHRONIZATION (HPS)

If at least one of scaling factor is different, this phenomenon is called HPS. Herein, two cases are considered as

- $s_1 = 1,\ s_2 = 2,\ s_3 = 3,\ s_4 = 4$
- $s_{1,3} = -1\ ,\ s_{2,4} = 1$

5.1. The Controllers at scaling factor $s_1 = 1, s_2 = 2, s_3 = 3, s_4 = 4$

If the matrix S is chosen as $S = diag(1, 2, 3, 4)$, i.e.

$$e = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}}_{S} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{20}$$

According to the (20), the error of HPS system between the system (6) and the system (7), is given as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 + ax_2 - 3fx_4 + u_1 \\ \dot{e}_2 = -qe_2 + e_1y_3 + (y_3 - 2x_3)x_1 + u_2 \\ \dot{e}_3 = -ce_3 - e_2y_1 - (2y_1 - 3x_1)x_2 - 2b + u_3 \\ \dot{e}_4 = -de_4 + re_2 - 2rx_2 + u_4 \end{cases} \tag{21}$$

Theorem 4. If the following controller is designed as:

$$\begin{cases} u_1 = -ax_2 + 3fx_4 - e_2y_3 \\ u_2 = -5qe_1 + (2x_3 - y_3)x_1 + e_3y_1 - \frac{1}{200}be_4 \\ u_3 = 2b + (2y_1 - 3x_1)x_2 \\ u_4 = 206re_1 + 2rx_2 \end{cases} \tag{22}$$

Then the system (21) will be controlled.
Proof: Insert above control in (21), we get:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 - e_2y_3 \\ \dot{e}_2 = -qe_2 - 5qe_1 - \frac{1}{200}be_4 + e_1y_3 + e_3y_1 \\ \dot{e}_3 = -ce_3 - e_2y_1 \\ \dot{e}_4 = -de_4 + re_2 + 206re_1 \end{cases} \tag{23}$$

The time derivative of the Lyapunov function is:

$$\dot{V}(e) = -ae_1^2 - qe_2^2 - ce_3^2 - de_4^2 + (a - 5q)e_1e_2 + \left(r - \frac{1}{200}b\right)e_2e_4 + (206r - f)e_1e_4 \tag{24}$$

So, $\dot{V}(e)$ is negative definite, the system (21) was controlled based on control system (22).

5.2. The Controllers at scaling factor $s_1 = 1, s_2 = -1, s_3 = 1, s_4 = -1$

If the matrix S is chosen as $S = diag(1, -1, 1, -1)$, i.e.

$$e = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}_{S} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

According to the above equation, the error of hybrid projective synchronization system between the system (6) and the system (7), is given as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 - ax_2 + fx_4 + u_1 \\ \dot{e}_2 = -qe_2 + e_3y_1 + (y_1 + x_1)x_3 + u_2 \\ \dot{e}_3 = -ce_3 - e_2y_1 + (y_1 + x_1)x_2 + u_3 \\ \dot{e}_4 = -de_4 + re_2 + u_4 \end{cases} \tag{25}$$

Theorem 5. If design the following controller (26):

$$\begin{cases} u_1 = ax_2 - fx_4 \\ u_2 = -(x_1 + y_1)x_3 - ae_1 - re_4 \\ u_3 = (-x_1 - y_1)x_2 \\ u_4 = fe_1 \end{cases} \tag{26}$$

Then the system (25) will be controlled.
Proof: The time derivative of the Lyapunov function is:

$$\dot{V}(e) = -ae_1^2 - qe_2^2 - ce_3^2 - de_4^2 \tag{27}$$

So, $\dot{V}(e)$ is negative definite, the system (25) was controlled based on control (26).

6. NUMERICAL SIMULATION

For simulation, the MATLAB version R2017a is adopted to solve the differential equation of controlled error dynamical system (8), system (12) and system (17) for PS and controlled error dynamical system (21), system (25) for HPS based on fourth-order Runge-Kutta scheme with time step $h = 0.01$ and the and the initial values of the drive system and the response system are following $(3.2, 8.5, 3.5, 2.0)$ and $(-3.2, -8.5, -3.5, -2.0)$ respectively. We choose the parameters $a = 5, b = 20, c = 1, d = 0.1, f = 20.6$, $r = 0.1$ and $q = 1$.

- For scaling factor $\forall\, s_i = 3$. Figure 2 show the PS of the systems (6) and (7) with control (9).
- For scaling factor $\forall\, s_i = 1$. Figure 3 show the PS of the systems (6) and (7) with control (13).
- For scaling factor $\forall\, s_i = -1$. Figure 4 show the PS of the systems (6) and (4) with control (18).
- For scaling factor $s_1 = 1,\ s_2 = 2,\ s_3 = 3,\ s_4 = 4$. Figure 5 show the HPS of the systems (1) and (4) with control (22).
- For scaling factor $s_{1,3} = -1$, $s_{2,4} = 1$. Figure 6 show the HPS of the systems (1) and (4) with control (26).

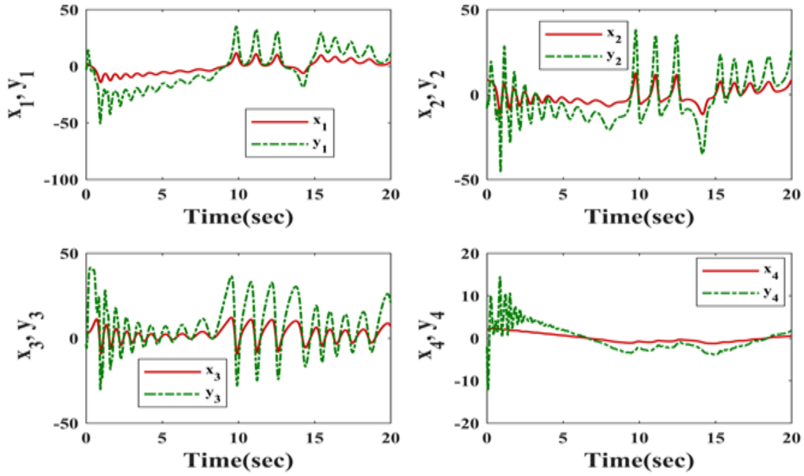


Figure 2. The PS for the state variables with control (9) at scaling factors ∀ $\boldsymbol{s_i} = \mathbf{3}$

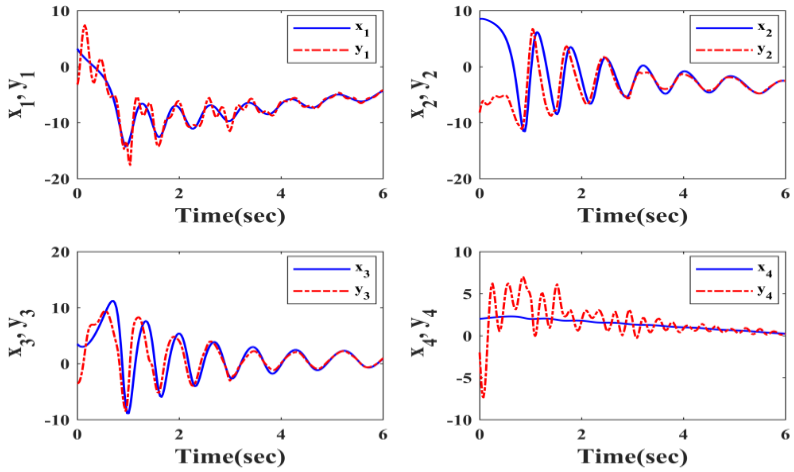


Figure 3. The PS for the state variables with control (13) at scaling factors ∀ $\boldsymbol{s_i} = \mathbf{1}$

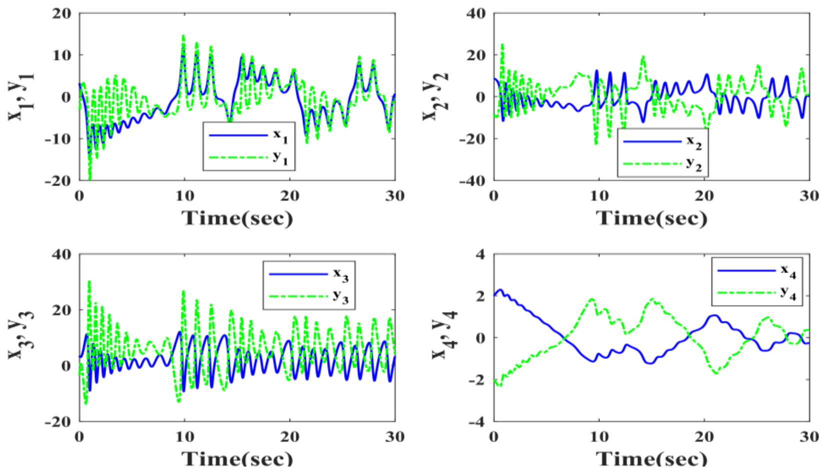


Figure 4. The PS for the state variables with control (18) at scaling factors ∀ $\boldsymbol{s_i} = \mathbf{-1}$

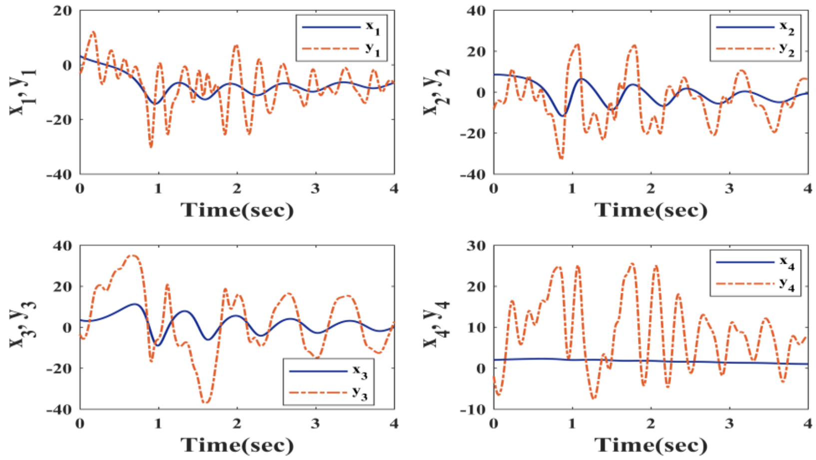


Figure 5. The HPS for the state variables with control (22) at scaling factors $\boldsymbol{s_1 = 1, s_2 = 2, s_3 = 3, s_4 = 4}$

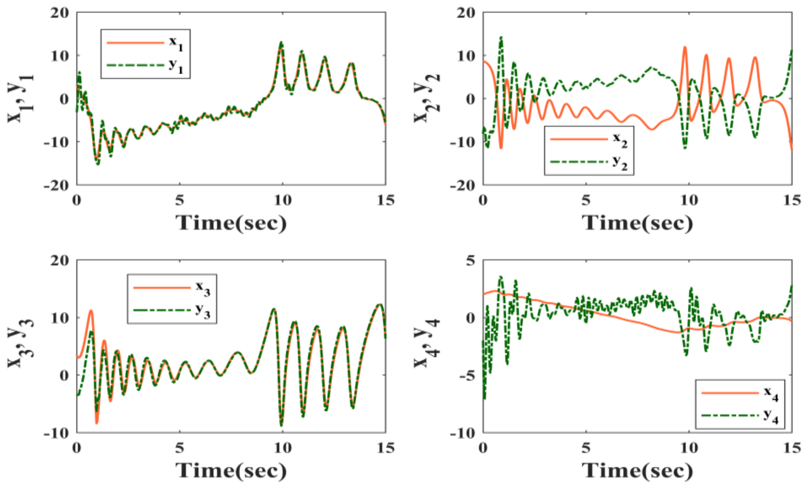


Figure 6. The HPS for the state variables with control (26) at scaling factors $\boldsymbol{s_i = \pm 1}$

7. CONCLUSION

Based on the scaling matrix S, two types of synchronization phenomena are achieved, namely PS and HPS. Three error systems of PS and two error systems of HPS with controller have been proposed for obtaining PS and HPS between two identical 4-D hyperchaotic systems with unknown parameters based on Lyapunov's method and the nonlinear control strategy. Certainly, the projective synchronization, were achieved CS, AS as well as PS via this phenomenon. Whereas, the HPS, was achieved HS. The effectiveness of these proposed control strategies was validated by numerical simulation results.

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