

# Physical security with power beacon assisted in half-duplex relaying networks over Rayleigh fading channel: performance analysis

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## ABSTRACT

In this research, we proposed and investigated physical security with power beacon assisted in half-duplex relaying networks over a Rayleigh fading channel. In this model, the source (S) node communicates with the destination (D) node via the helping of the intermediate relay (R) node. The D and R nodes harvest energy from the power beacon (PB) node in the presence of a passive eavesdropper (E) node. Then we derived the integral form of the system outage probability (OP) and closed form of the intercept probability (IP). The correctness of the analytical of the OP and IP is verified by the Monte Carlo simulation. The influence of the main system parameters on the OP and IP also is investigated. The research results indicated that the analytical results are the same as the simulation ones.

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## 1. INTRODUCTION

Radiofrequency (RF) energy harvesting (EH) in the wireless communication network is become a novel solution for the communication network with battery-limited devices and has attracted massive attention in research and industrial directions. The communication network with the battery-limited devices or devices and wireless sensors, which are working in the dangerous conditions is the main inside human bodies object of RF EH wireless communication network application. This solution can be considered as the main one because of carrying both energy and information of the EF, to help the battery-limited devices to harvest energy for information transmission to the destination. This technic is called simultaneous wireless information and power transfer (SWIPT) [1-11]. Nowadays, the physical layer security (PLS) in EH communication cooperative relaying network is popularly studied with considerable interest. The first concept of PLS was proposed by authors in [12, 13]. In this paper, the author proposed the secret communication between the source and destination nodes with the presence of the eavesdropper channel. Furthermore, the cooperative jammer is used for secure the cooperative relaying network by degrading the eavesdropper's channel is proposed and studied in [14, 15]. In this cooperative network, the jammer not

only is used to degrade the eavesdropper’s channel but also is helpful for increasing the EH process of the energy receiver in the cooperative relaying network. Moreover, a harvest-and-jam (HJ) protocol with multi-relay and multi-node in the cooperative relaying network was proposed in [16] to improve the secrecy rate of the energy harvesting and information transmission. Also, different secure relay beam-forming algorithms for SWIPT were discussed in [17-20].

In this research, we proposed and investigated physical security with power beacon assisted in half-duplex relaying networks over a Rayleigh fading channel. In this model, the source (S) node communicates with the destination (D) node via the helping of the intermediate relay (R) node. The D and R nodes harvest energy from the power beacon (PB) node in the presence of a passive Eavesdropper (E) node. Then we derived the integral form of the system outage probability (OP) and closed form of the intercept probability (IP). The correctness of the analytical of the OP and IP is verified by the Monte Carlo simulation. The influence of the main system parameters on the OP and IP also is investigated. The research results indicated that the analytical results are the same as the simulation ones.

**2. SYSTEM MODEL**

In Figure 1, the source (S) node communicates with the destination (D) node via the helping of the intermediate relay (R) node. The D and R nodes harvest energy from the power beacon (PB) node in the presence of a passive Eavesdropper (E) node. Figure 2 draws the energy harvesting (EH) and information processing (IT) of the model system. In this protocol, the transmission is divided into blocks of length T, which consists of three-time slots. In the first time slot  $\alpha T$  ( $\alpha$  is the time switching factor,  $0 < \alpha < 1$ ), the S and R harvest energy from the PB node. In the remaining intervals time  $(1-\alpha)T/2$ , the source S transfers the information to R, and R transfers information to D node [21-25].

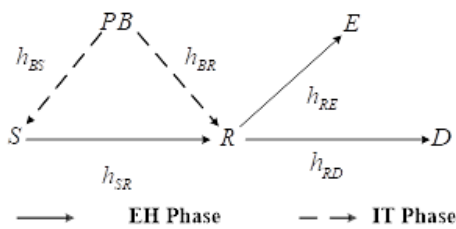


Figure 1. System model

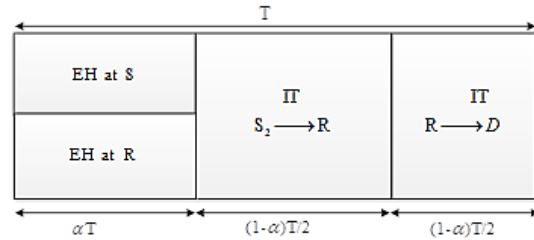


Figure 2. Time switching protocol

**2.1. Energy harvesting phase**

In the first phase, the power beacon will supply the energy for both S and R nodes. Hence, the harvested energy at the source and relay can be given as, respectively

$$E_s = \eta P_B \alpha T |h_{BS}|^2 \tag{1}$$

$$E_R = \eta P_B \alpha T |h_{BR}|^2 \tag{2}$$

where  $0 < \eta \leq 1$  is energy conversion efficiency,  $P_B$  is the average transmitted power at the power beacon, and  $h_{BS}$ ,  $h_{BR}$  are the channel gain of B-S link, B-R link, respectively. The average transmitted power at the source and relay nodes can be obtained from (1) and (2), respectively

$$P_s = \frac{E_s}{(1-\alpha)T/2} = \frac{\eta P_B \alpha T |h_{BS}|^2}{(1-\alpha)T/2} = \kappa P_B |h_{BS}|^2 \tag{3}$$

$$P_R = \kappa P_B |h_{BR}|^2 \tag{4}$$

where  $= \frac{2\eta\alpha}{1-\alpha}$ .

**2.2. Information transmission phase**

In the second phase, the received signal at the relay can be rewritten as

$$y_r = h_{SR} x_s + n_r \tag{5}$$

where  $h_{SR}$  is the channel gain of S-R link,  $x_s$  is the transmitted signal from source and  $n_r$  is additive white Gaussian noise (AWGN) with variance  $N_0$ . In the third phase, the received signal at the destination can be given by

$$y_d = h_{RD}x_r + n_d \quad (6)$$

where  $h_{RD}$  is the channel gain of R-D link,  $x_r$  is the transmitted signal from relay and  $n_d$  is (AWGN) with variance  $N_0$ . Here, we consider amplify and forward (AF) mode at R. Hence, the amplifying factor can be given as

$$\chi = \frac{x_r}{y_r} = \sqrt{\frac{P_R}{P_S|h_{SR}|^2 + N_0}} \quad (7)$$

substituting (7) into (6), we have

$$y_d = h_{RD}\chi y_r = h_{RD}\chi[h_{SR}x_s + n_r] + n_d = \underbrace{h_{SR}h_{RD}\chi x_s}_{\text{signal}} + \underbrace{h_{RD}\chi n_r + n_d}_{\text{noise}} \quad (8)$$

### 3. SYSTEM PERFORMANCE ANALYSIS

From (8), the end to end signal to noise ratio (SNR) of S-R-D link can be calculated as (9).

$$\gamma_{SRD} = \frac{E\{\text{signal}^2\}}{E\{\text{noise}^2\}} = \frac{|h_{SR}|^2|h_{RD}|^2\chi^2 P_S}{|h_{RD}|^2\chi^2 N_0 + N_0} = \frac{|h_{SR}|^2|h_{RD}|^2 P_S}{|h_{RD}|^2 N_0 + \frac{N_0}{\chi^2}} \quad (9)$$

After doing some algebra and using the fact that  $N_0 \ll P_R$ , the (9) can be rewritten as (10).

$$\gamma_{SRD} = \frac{|h_{SR}|^2|h_{RD}|^2 P_S P_R}{P_R|h_{RD}|^2 N_0 + N_0 P_S|h_{SR}|^2} \quad (10)$$

Substituting (3) and (4) into (10), the SNR can be reformulated as (11).

$$\gamma_{SRD} = \frac{|h_{SR}|^2|h_{RD}|^2 \kappa \Psi |h_{BS}|^2 |h_{BR}|^2}{|h_{BR}|^2 |h_{RD}|^2 + |h_{BS}|^2 |h_{SR}|^2} = \frac{\kappa \Psi X Y}{X + Y} \quad (11)$$

Where  $= \frac{P_R}{N_0}$ ,  $X = |h_{BR}|^2 |h_{RD}|^2$ ,  $Y = |h_{BS}|^2 |h_{SR}|^2$ . The received signal at the eavesdropper can be given by

$$y_E = h_{RE}x_r + n_E \quad (12)$$

where  $h_{RE}$  is the channel gain of R-E link and  $n_E$  is AWGN with variance  $N_0$ . The SNR at the eavesdropper can be expressed as

$$\gamma_E = \frac{|h_{RE}|^2 P_R}{N_0} \quad (13)$$

substituting (4) into (13), we have:

$$\gamma_E = \kappa \Psi |h_{BR}|^2 |h_{RE}|^2 = \kappa \Psi Z \quad (14)$$

Where  $Z = |h_{BR}|^2 |h_{RE}|^2$

**Lemma1.** Please note that all channel are the Rayleigh fading channels, so the probability density function (PDF) of  $|h_i|^2$  can be given by:

$$f_{|h_i|^2}(x) = \lambda_i e^{-\lambda_i x} \quad (15)$$

where  $i \in (SR, RD, BS, BR, RE)$ . Moreover, the cumulative distribution function (CDF) of  $|h_i|^2$  also can be obtained by

$$F_{|h_i|^2}(x) = 1 - e^{-\lambda_i x} \quad (16)$$

where  $\lambda_i$  is the mean value of the exponential random variable  $|h_i|^2$ .

**Lemma 2.** The CDF of  $X$  and  $Y$  can be computed as respectively

$$F_X(a) = \int_0^\infty F_{|h_{RD}|^2} \left( \frac{a}{|h_{BR}|^2} \mid |h_{BR}|^2 = x \right) f_{|h_{BR}|^2}(x) dx \quad (17)$$

$$F_Y(b) = \int_0^\infty F_{|h_{SR}|^2} \left( \frac{b}{|h_{BS}|^2} \mid |h_{BS}|^2 = y \right) f_{|h_{BS}|^2}(y) dy \quad (18)$$

utilizing the result in [26], the CDF of  $X$  and  $Y$  can be shown as the below equation, respectively

$$F_X(a) = 1 - 2\sqrt{\lambda_{RD}\lambda_{BR}a} K_1(2\sqrt{\lambda_{RD}\lambda_{BR}a}) \quad (19)$$

$$F_Y(b) = 1 - 2\sqrt{\lambda_{SR}\lambda_{BS}b} K_1(2\sqrt{\lambda_{SR}\lambda_{BS}b}) \quad (20)$$

where  $K_\nu(\cdot)$  is the modified Bessel function of the second kind and  $\nu^{\text{th}}$  order. From (21) and (22), the PDF of  $X$  and  $Y$  can be calculated as, respectively after applying the formula

$$\frac{\partial K_n(z)}{\partial z} = -K_{n-1}(z) - \frac{n}{z} K_n(z)$$

$$f_X(a) = \frac{\partial F_X(a)}{\partial a} = 2\sqrt{\lambda_{RD}\lambda_{BR}} K_0(2\sqrt{\lambda_{RD}\lambda_{BR}a}) \quad (21)$$

$$f_Y(b) = \frac{\partial F_Y(b)}{\partial b} = 2\sqrt{\lambda_{SR}\lambda_{BS}} K_0(2\sqrt{\lambda_{SR}\lambda_{BS}b}) \quad (22)$$

### 3.1. Outage probability (OP)

The OP of system can be given as

$$OP = Pr(\gamma_{SRD} < \gamma_{th}) \quad (23)$$

where  $\gamma_{th} = 2^{2R} - 1$  is the threshold of system and  $R$  is source rate. Substituting (11) into (23), we have

$$OP = Pr\left(\frac{\kappa^{\Psi}XY}{X+Y} < \gamma_{th}\right) = Pr(X[\kappa^{\Psi}Y - \gamma_{th}] < \gamma_{th}Y)$$

$$= \begin{cases} Pr\left(X < \frac{\gamma_{th}Y}{\kappa^{\Psi}Y - \gamma_{th}}\right), Y > \frac{\gamma_{th}}{\kappa^{\Psi}} \\ 1, Y \leq \frac{\gamma_{th}}{\kappa^{\Psi}} \end{cases}$$

$$= \int_0^{\frac{\gamma_{th}}{\kappa^{\Psi}}} f_Y(y) dy + \int_{\frac{\gamma_{th}}{\kappa^{\Psi}}}^\infty F_X\left(\frac{\gamma_{th}Y}{\kappa^{\Psi}Y - \gamma_{th}} \mid Y = y\right) \times f_Y(y) dy \quad (24)$$

Applying (19-22), (24) can be rewritten as (25).

$$OP = \int_0^{\frac{\gamma_{th}}{\kappa^{\Psi}}} 2\sqrt{\lambda_{SR}\lambda_{BS}} K_0(2\sqrt{\lambda_{SR}\lambda_{BS}y}) dy - 4 \int_{\frac{\gamma_{th}}{\kappa^{\Psi}}}^\infty \sqrt{\lambda_{SR}\lambda_{BS}\lambda_{RD}\lambda_{BR} \left[\frac{\gamma_{th}y}{\kappa^{\Psi}y - \gamma_{th}}\right]} \times K_0(2\sqrt{\lambda_{SR}\lambda_{BS}y}) K_1\left(2\sqrt{\lambda_{RD}\lambda_{BR} \left[\frac{\gamma_{th}y}{\kappa^{\Psi}y - \gamma_{th}}\right]}\right) dy \quad (25)$$

By changing the variable for the first term of (25):  $t = 2\sqrt{\lambda_{SR}\lambda_{BS}y}$ .

$$OP = \frac{1}{\sqrt{\lambda_{SR}\lambda_{BS}}} \int_0^{\frac{\gamma_{th}}{\kappa^{\Psi}}} t \times K_0(t) dt - 4 \int_{\frac{\gamma_{th}}{\kappa^{\Psi}}}^\infty \sqrt{\lambda_{SR}\lambda_{BS}\lambda_{RD}\lambda_{BR} \left[\frac{\gamma_{th}y}{\kappa^{\Psi}y - \gamma_{th}}\right]} \times K_0(2\sqrt{\lambda_{SR}\lambda_{BS}y}) K_1\left(2\sqrt{\lambda_{RD}\lambda_{BR} \left[\frac{\gamma_{th}y}{\kappa^{\Psi}y - \gamma_{th}}\right]}\right) dy \quad (26)$$

Apply (6.561, 16) of the table of integral, (26) can be reformulated as

$$OP = \frac{1}{\sqrt{\lambda_{SR}\lambda_{BS}}} - 4 \int_{\frac{\gamma_{th}}{\kappa^{\Psi}}}^\infty \sqrt{\lambda_{SR}\lambda_{BS}\lambda_{RD}\lambda_{BR} \left[\frac{\gamma_{th}y}{\kappa^{\Psi}y - \gamma_{th}}\right]} \times K_0(2\sqrt{\lambda_{SR}\lambda_{BS}y}) K_1\left(2\sqrt{\lambda_{RD}\lambda_{BR} \left[\frac{\gamma_{th}y}{\kappa^{\Psi}y - \gamma_{th}}\right]}\right) dy \quad (27)$$

### 3.2. Intercept probability (IP)

The IP can be defined by

$$IP = Pr(\gamma_E \geq \gamma_{th}) \tag{28}$$

substituting (14) into (28) and then applying formulas in (19) or (20), finally we have:

$$IP = Pr(\kappa\Psi Z \geq \gamma_{th}) = 1 - Pr\left(Z < \frac{\gamma_{th}}{\kappa\Psi}\right) = 2\sqrt{\frac{\lambda_{BR}\lambda_{RE}\gamma_{th}}{\kappa\Psi}} \times K_1\left(2\sqrt{\frac{\lambda_{BR}\lambda_{RE}\gamma_{th}}{\kappa\Psi}}\right) \tag{29}$$

## 4. NUMERICAL RESULTS AND DISCUSSION

The influence of  $\eta$  on the OP and IP is plotted in Figure 3 and Figure 4 for various values of  $\psi$ . Here we set  $\alpha = 0.5$ ,  $R = 0.5$  bps/Hz and  $\psi = 1, 5, 10$  dB. The first observation one can see from these results is that the analytical match very well with the simulation results. Figure 5 and Figure 6 illustrated the optimal switching time system OP and IP versus  $\psi$  for  $\eta = 0.8$ ,  $R=0.5$  bps/Hz, and  $\alpha = 0.25, 0.5, 0.85$  respectively. It should be pointed out that the simulation and analytical results are the same.

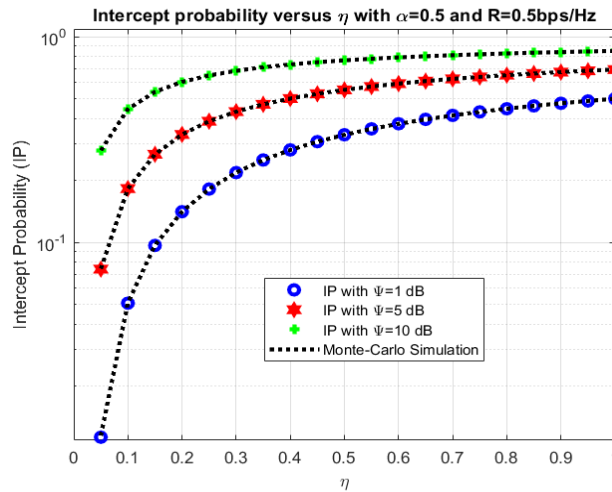


Figure 3. IP versus  $\eta$

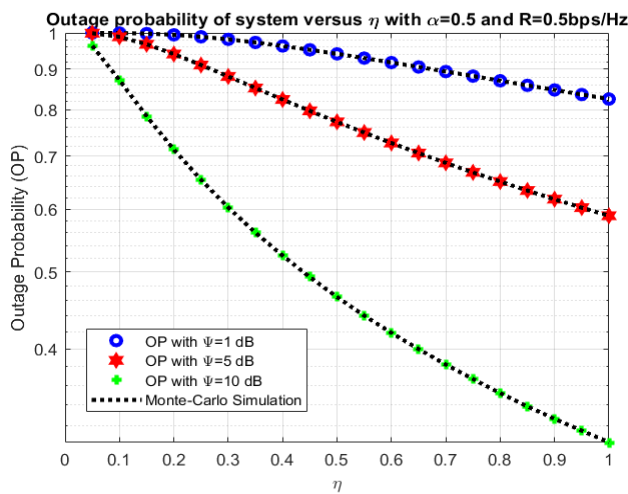
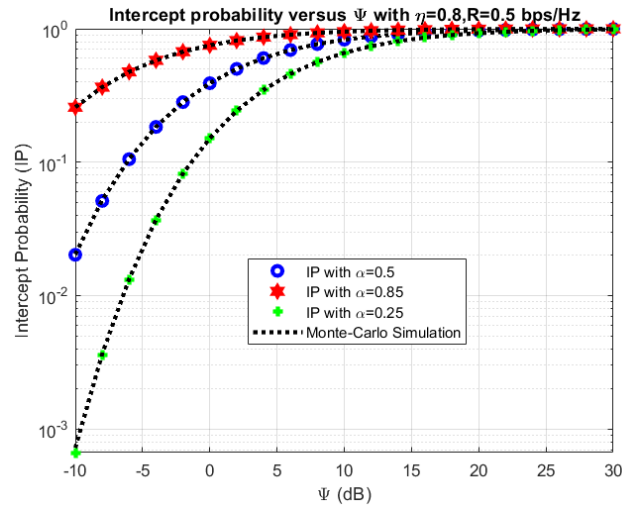
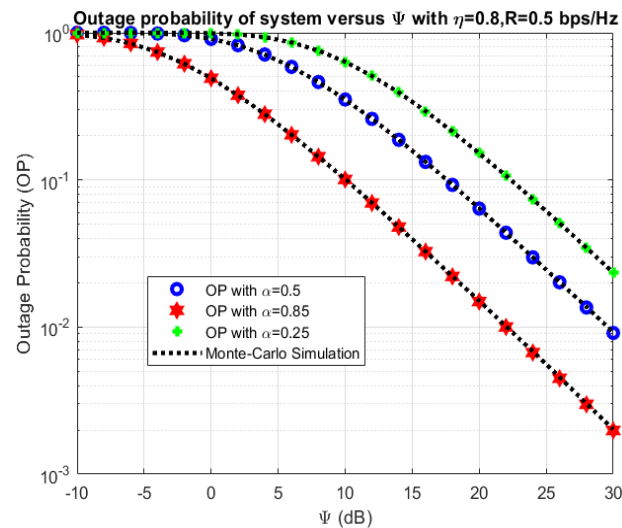


Figure 4. OP versus  $\eta$

Figure 5. IP versus  $\psi$ Figure 6. OP versus  $\psi$ 

## 5. CONCLUSION

In this research, we proposed and investigated physical security with power beacon assisted in half-duplex relaying networks over a Rayleigh fading channel. In this model, the source (S) node communicates with the destination (D) node via the helping of the intermediate relay (R) node. The D and R nodes harvest energy from the power beacon (PB) node in the presence of a passive Eavesdropper (E) node. Then we derived the integral form of the system outage probability (OP) and closed form of the intercept probability (IP). The correctness of the analytical of the OP and IP is verified by the Monte Carlo simulation. The influence of the main system parameters on the OP and IP also is investigated. The research results indicated that the analytical results are the same as the simulation ones.

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