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Application of model reduction for robust control of self-balancing two-wheeled bicycle

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Article Info	ABSTRACT
	In recent years, balance control of two-wheeled bicycle has received more attention of scientists. One difficulty of this problem is the control object is unstable and constantly impacted by noise. To solve this problem, the authors often use robust control algorithms. However, robust controller of self-balancing two-wheeled bicycle are often complex and higher order so affect to quality during real controlling. The article introduces the stochastic balanced truncation algorithm based on Schur analysis and applies this algorithm to reduce order higher order robust controller in control balancing two-wheeled bicycle problem. The simulation results show that the reduced 4th and 5th order controller arcoording to the stochastic balanced truncation algorithm based on Schur analysis can control the two-wheeled bicycle model. The reduced 3rd order controller cannot control the balance of the two-wheeled bicycle model. The reduced 4th and 5th order controller can replace the original controller while the performance of the control system is ensured. Using reduced 5th, 4th order controller will make the program code simpler, reducing the calculation time of the self-balancing two-wheel control system. The simulation results show the correctness of the model reduction algorithm and the robust control algorithm of two-wheeled self-balancing two-wheeled bicycle.

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1. INTRODUCTION

In recent years, research on self-balancing two-wheeled bicycle has been interested by many scientists. In particular, a difficult problem is the study of self-balancing problem of the robot. To solve the problem of balancing two-wheeled bicycle, there are three basic methods as follows; (a) controlling balance by the flywheel, as in the studies of Beznos [1], Xu [2], and Kim [3]. Lee [4] Gallaspy [5], and Suprapto [6]; Thanh [7], (b) controlling balance by centrifugal force as in the study of Tanaka and Murakami [8], and (c) controlling balance by changing the center of gravity as Lee and Ham's research [9]. Among these three methods, control of balance using the flywheel has the advantage of being responsive and can be balanced even when the vehicle is not moving.

In two-wheeled robot models that control the balance by using the flywheel, two-wheeled bicycle uses the flywheel according to the principle of gyroscope [1, 5-7] to create a balanced torque for the wheels. The momentum usually revolves at high speed, so the flywheel dissipates a large amount of energy. This problem leads to a difficulty in the operation power of the robot as the robot is only powered by a battery with a limited

capacity. In contrast, the two-wheeled bicycle model uses the flywheel according to the principle of inverted pendulum [2-4], to create a balanced torque for the car, the flywheel rotates only at very small speed, so the energy dissipated by the flywheel is low. Due to the reason, this model is suitable in terms of energy saving for the car. Therefore, the authors proposed the self-balancing two-wheeled robot using the flywheel based on the principle of inverted pendulum.

Because two-wheeled bicycles often have to work in different conditions, the carrying capacity may vary, the external forces acting on the vehicles may change. It is difficult to find the model of self-balancing two-wheeled bicycle, and Two-wheel bicycle can be considered as indeterminate objects [5]. Several control algorithms of two-wheeled bicycle have been proposed such as: nonlinear control by Beznol [1], Lee và Ham [9], the compensated design using the orbital approach by Gallaspy [5], PD controller by Surpato [6]. Due to the uncertainty of two-wheel model, the robust control method in [7] is the most suitable. However, in the robust control design method RH_∞ first introduced by McFarlane and Glover in 1992 [10], the controllers usually have a high order (controller level is defined as the denominator). The high order controller introduces the disadvantage when we use it to control the bicycle. The program is complex. The calculation time is long, so the response of the system is slow. Therefore, reducing the order of the controller while ensuring the quality of the controller has a significant meaning in practical applications. In order to reduce the controller order, there are 2 methods can be followed:

The first method: this method selects a fixed structure of the order reduction controller and then applies optimal algorithms to find the parameters of the order reduction controller so that the standards of the robust control are met. The second method: designing a robust controller for an uncertain object will obtain a high-order controller, then perform a high-order controller reduction according to the order reduction algorithms to obtain a reduced order controller.

According to the authors, in the first method, the controller can be a low order controller [7], but two optimization problems need to be sovle simultaneously (problems in fiding parameters of the controller and robust control). This issue leads to difficuty of this method. The parameter of the low order controller may not be found if the chosen controller is not suitable. In the second method, the order reduction problem is an independent problem, so it always gives the order reduction result as in [11]. Due to that reason, the second method has the advantage over the first method because the low order controller can be found in any senario.

In this paper, the authors proposed the control method of two-wheeled bicycle using model reduction algorithm in two steps as follow: (a) design the RH_∞ controller to control the balance of two-wheeled bicycle, the found controller is called a full-level controller, and (b) applying order reduction algorithm to reduce order of RH_∞ controller to lower order controller while ensuring quality. This step reduction is meant to reduce the system response time.

2. DYNAMIC MODEL AND MATHEMATICAL MODEL OF THE SELF-BALANCING TWO-WHEELED BICYCLE

2.1. Dynamic model of the self-balancing two-wheeled bicycle

The two-wheeled bicycle model is developed based on the principle of balance using flywheel according to the principle of inverted pendulum [2-4]. It is briefly described the principle of balancing of the vehicle as follows: if no external torque (torque) is applied to an object or system (or the total torque applied to an object is zero), then the total torque of the object will be preserved.

The vehicle moving by 2 wheels, when the vehicle deviates from the balance position (corresponding to a q angle according to vertical axis). The gravity of the vehicle creates a torque that makes the car tend to fall down. To maintain a state of equilibrium, we put on the vehicle a flywheel that operates on the principle of "the inverted pendulum". This flywheel will rotate around the axis (with an angular acceleration of α) and create a torque to compensate the torque generated by the vehicle's gravity. To control the acceleration of the flywheel, we uses a DC dc motor with the voltage applied to the motor being U. Then, the problem of balancing control becomes the problem of controlling the θ angle (output) by controlling the voltage U (input) applying to the motor. The problem requires that the θ angle (output) always go to zero. The self-balancing two-wheeled bicycle that the authors built is shown in Figure 1.

The model machanical parameters: long: 1.19 m; height: 0.5 m; width: 0.4 m; the flywheel weight: 3.976 kg, diameter: 0.26m; Driving the flywheel using DC motor: 100W-15V-3400 rpm with H-bridge driver; Measuring the flywheel velocity by Encoder Sharo 100 pulse; Measuring the q amgle by sensor GY-521 MPU-6050; Forward and reverse system consists of a DC motor, H-bridge driver and a remote controller. The hardware system is connected to Ardruino microprocessor according to the following block diagram as shown in Figure 2.
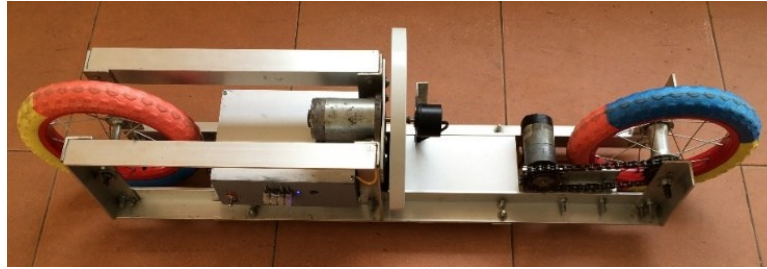
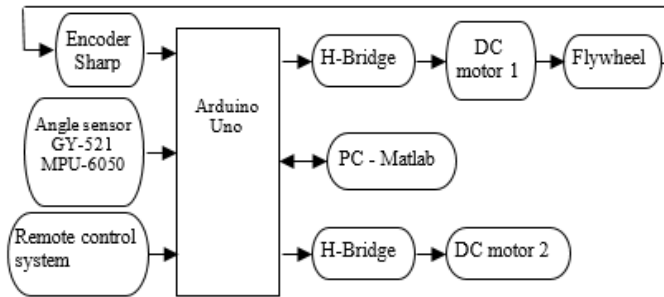Figure 1. The self-balancing two-wheeled bicycle model

Figure 2. Schematic structure of bicycle controller

2.1. Mathematical model of the self-balancing two-wheeled bicycle

Dynamic model of the self-balancing two-wheeled bicycle is shown in Figure 3. Where: m_1 is the bicycle weight (including DC motor), m_2 is the flywheel weight, h_1 is the height of the center gravity of the bicycle (excluding the flywheel), h_2 is the height of the center gravity of the flywheel, I_1 is the inertia torque of the bicycle, I_2 is the inertia torque of the flywheel, q is the tilt angle of the bicycle corresponding to the vertical axis, j is the rotation angle of the flywheel. We have: the absolute velocity of point A is $|v_A| = h_1\dot{\theta}$. The absolute velocity of point B is $|v_B| = h_2\dot{\theta}$. In [5], the author used Lagrange equation to develop the dynamic model of the vehicle.

$$\frac{d}{dt}\left\{\frac{\partial T}{\partial \dot{q}_i}\right\} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \qquad (1)$$

where: T is the total kinetic energy of the system, V is the total potential energy of the system, Q_i is the external force, q_i is the generalized coordinate.
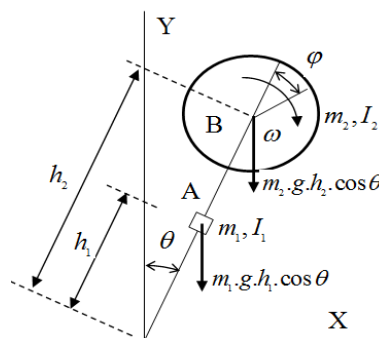

Figure 3. Self-balancing two-wheel bicycle model

The total kinetic energy of the system defined by: $T = T_1 + T_2$. T_1, which is the kinetic energy of the two-wheeled vehicles, is determined by the following formula:

$$T_1 = \frac{1}{2}m_2|v_B|^2 + \frac{1}{2}I_1\dot{\theta}^2$$

T_2, which is the flywheel kinetic energy, is determined by the following formula:

$$T_2 = \frac{1}{2} m_2 |v_B|^2 + \frac{1}{2} I_2 (\dot{\theta} + \dot{\varphi})$$

We have:

$$T = \frac{1}{2} m_1 |v_A|^2 + \frac{1}{2} m_2 |v_B|^2 + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} I_2 \dot{\theta}^2 + \frac{1}{2} I_2 \dot{\varphi}^2 + I_2 \dot{\varphi} \dot{\theta} \tag{2}$$

$$\Rightarrow T = \frac{1}{2} (m_1 h_1^2 + m_2 h_2^2 + I_1 + I_2) \dot{\theta}^2 + \frac{1}{2} I_2 \dot{\varphi}^2 + I_2 \dot{\varphi} \dot{\theta} \tag{3}$$

The total potential energy of the system:

$$V = g. \cos\theta . (m_1 h_1 + m_2 h_2) \tag{4}$$

With $q_i = q$, taking (1-4), we get:

$$(m_1 h_1^2 + m_2 h_2^2 + I_1 + I_2) \ddot{\theta} + I_2 \ddot{\varphi} - g. \sin\theta . (m_1 h_1 + m_2 h_2) = 0 \tag{5}$$

With $q_i = j$, taking (1–4), we get:

$$I_2 \ddot{\varphi} + I_2 \ddot{\theta} = T_m. \tag{6}$$

With T_m is the motor shaft torqe.

Considering a DC dc motor with a gear ratio of a:1, the torque of the DC motor driving the flywheel is as follows:

$$T_m = a K_m i = a K_m \left[\frac{U - K_e \dot{\varphi}}{R} \right], \tag{7}$$

with K_m is the motor torque constant, K_e is the back-emf constant, R is the resistance of the motor. Substitute (7) into (6), we get:

$$I_2 \ddot{\varphi} + I_2 \ddot{\theta} = T_m = a K_m \left[\frac{U - K_e \dot{\varphi}}{R} \right]. \tag{8}$$

In (5) and (8) are the dynamic system equation. It is clear that the system is nonlinear. Linearizing the model and turn it into a state space model. Assume that when the vehicle is operating, the vehicle's inclination angle is very small ($\theta < 10^0$). Linearizing in (5) around the equilibrium point ($\theta = \varphi = 0, \sin\theta = \theta$), we have:

$$(m_1 h_1^2 + m_2 h_2^2 + I_1 + I_2) \ddot{\theta} + I_2 \ddot{\varphi} - g. \theta . (m_1 h_1 + m_2 h_2) = 0 \tag{9}$$

$$I_2 \ddot{\varphi} + I_2 \ddot{\theta} = T_m = a K_m \left[\frac{U - K_e \dot{\varphi}}{R} \right] \tag{10}$$

Taking $A_1 = (m_1 h_1^2 + m_2 h_2^2 + I_1 + I_2); B_1 = (m_1 h_1 + m_2 h_2)$

Taking $x = \begin{bmatrix} \theta = x_1 \\ \dot{\theta} = x_2 \\ \dot{\varphi} = x_3 \end{bmatrix}$, is state variable, $y = \theta, u = U$

We have the state space model describing the system as follow:

$$\dot{x} = Ax + Bu \tag{11}$$

$$y = Cx + Du$$

with:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{B_1 g}{(A_1 - I_2)} & 0 & \frac{a K_m K_e}{R(A_1 - I_2)} \\ -\frac{B_1 g}{(A_1 - I_2)} & 0 & -a K_m K_e \frac{A_1}{I_2 R(A_1 - I_2)} \end{bmatrix}; B = \begin{bmatrix} 0 \\ -\frac{a K_m}{R(A_1 - I_2)} \\ a K_m \frac{A_1}{I_2 R(A_1 - I_2)} \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}; D = [0]$$

The nominal parameters of the two-wheeled bicycle model are shown in Table 1 as follows: explaining research chronological, including research design, research procedure (in the form of algorithms, Pseudocode or other), how to test and data acquisition [1-3]. The description of the course of research should be supported references, so the explanation can be accepted scientifically [2, 4]. Tables and figures are presented center, as shown below and cited in the manuscript. Substituting for the system of (11), we obtain the following parameters:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 47.2048 & 0 & 0.0100 \\ -47.2048 & 0 & -0.1248 \end{bmatrix}; B = \begin{bmatrix} 0 \\ -0.2230 \\ 2.8541 \end{bmatrix}; C = [1 \quad 0 \quad 0].$$

Convert the vehicle model into transfer function:

$$S(s) = \frac{\theta(s)}{U(s)} = \frac{-0.223s}{s^3 + 0.1284s^2 - 47.2s - 5.589} \tag{12}$$

Table 1. The parameters of the two-wheeled bicycle mode

Parameter	Value	Unit
I_1	0.1105	Kg.m^2
h_1	0.105	m
I_2	0.03289	Kg.m^2
h_2	0.205	m
m_1	10.024	Kg
m_2	3.976	Kg
K_e	0.045	V.s/Rad
K_m	0.045	Nm/A
R	0.52	Ω
a	1:1	
g	9.81	m/s^2

Remark on two-wheel drive models. The self-balancing two-wheeled bicycle model shows that some parameters of self-balancing two-wheeled bicycle are uncertain such as: the changing load volume (leading to a change in the center of gravity of the car), the inertia torque of the bicycle changed. Additionally, operating two-wheeled bicycle may be influenced by external uncertainties such as: the external force and uncertain noise due to the changing of topography. Therefore, The two-wheeled bicycle is the uncertain object. In particular, the authors pay the most attention to the uncertainty due to the change of load weight. Specifically, the authors consider 4 cases of two-wheeled bicycle carrying different loads as shown in the Table 2.

Uncertain factors may reduce the accuracy of two-wheeled mathematical models. Therefore, the control quality is reduced and the system can even become unstable. Due to the uncertain properties, the various control algorithm for the two wheeled bicycle has been proposed: nonlinear control by Beznol [1], Lee và Ham [4], the compensated design using the orbital approach by Gallaspy [5], PD controller by Surpato [8]. The most suitable algorithm to control the uncertain object was the algorithm in [10].

Table 2. Parameters of the two-wheeled bicycle model as the load is different

Case	Load volume m_t (kg)	Height of the center of gravity h_1 (m)	Moment of inertia I_1 (Kg.m^2)
1	5	0.205	0.6314
2	5	0.155	0.3609
3	7	0.055	0.0515
4	7	0.155	0.409

3. OPTIMAL DESIGN RH∞ FOR BALANCE WHEEL PROBLEM

The structure of the balancing control system for self-balancing two-wheeled bicycle is shown in Figure 4. The balancing control system consists of 3 loop controls , namely, loop control the rotation angle of the flywheel, loop control the velocity tilt angle of bicycle and loop control the tilt angle of bicylce. The robust controller R(s) is used in loop control the tilt angle of bicylce. To design a robust control system for self-balancing two-wheeled bicycle, the control structure diagram shown in Figure 4 is used by the authors.
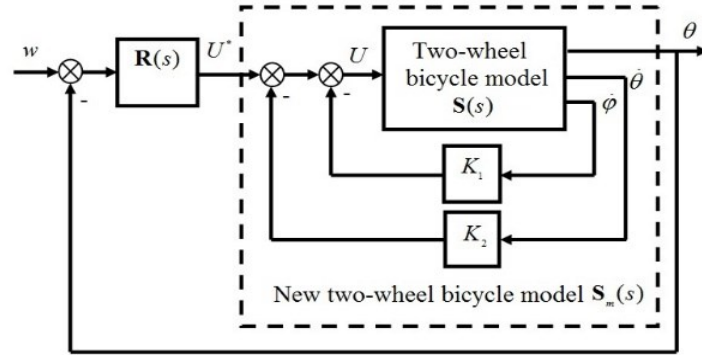
Figure 4. The robust control structure for self-balancing two-wheeled bicycle

3.1. Developing the self-balancing two-wheeled bicycle model $S_m(s)$

Assuming that when the vehicle is in operation, the inclination of the bicycle is very small, we linearize in (5) around the equilibrium point ($\theta = \varphi = 0, sin\,\theta = \theta$). We have:

$$(m_1 h_1^2 + m_2 h_2^2 + I_1 + I_2)\ddot{\theta} + I_2\ddot{\varphi} - g.\theta.(m_1 h_1 + m_2 h_2) = 0 \tag{13}$$

$$I_2\ddot{\varphi} + I_2\ddot{\theta} = T_m = aK_m\left[\frac{U^* - (K_e + K_1)\dot{\varphi} + K_2\dot{\theta}}{R}\right] \tag{14}$$

Taking: $A_1 = (m_1 h_1^2 + m_2 h_2^2 + I_1 + I_2)$; $B_1 = (m_1 h_1 + m_2 h_2)$

Taking $x = \begin{bmatrix} \theta = x_1 \\ \dot{\theta} = x_2 \\ \dot{\varphi} = x_3 \end{bmatrix}$, is state variable, $y = \theta, u = U^*$

We have the state space model describing the system as follow:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \tag{15}$$

The system parameters:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{B_1 g}{(A_1 - I_2)} & \frac{aK_m K_2}{R(A_1 - I_2)} & \frac{aK_m(K_e + K_1)}{R(A_1 - I_2)} \\ -\frac{B_1 g}{(A_1 - I_2)} & -aK_m K_2\frac{A_1}{I_2 R(A_1 - I_2)} & -aK_m(K_e + K_1)\frac{A_1}{I_2 R(A_1 - I_2)} \end{bmatrix}; B = \begin{bmatrix} 0 \\ -\frac{aK_m}{R(A_1 - I_2)} \\ aK_m\frac{A_1}{I_2 R(A_1 - I_2)} \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}; D = [0].$$

Chosing $K_1 = 2, K_2 = 5$. Substituting the parameters in Table 1 into (15), the model is converted to the transfer function form:

$$S_m(s) = \frac{\theta(s)}{U(s)} = \frac{-0.223s}{s^3 + 4.722s^2 - 47.2s - 254} \tag{16}$$

To design a robust controller for self-balancing two-wheeled bicycle, the authors followed the steps of designing a robust controller RH$_\infty$ according to [10, 12]. We get the robust controller:

$$R(s) = \frac{H(s)}{D(s)} \tag{17}$$

with

$$H(s) = -2.23.10^{-7}s^{30} - 4.67.10^{-4}s^{29} - 0.266s^{28} - 22.96s^{27} - 1006s^{26} - 2.853.10^4 s^{25}$$

$$-5.837.10^5 s^{24} - 4.199.10^{11}s^{18} - 9.144.10^6 s^{23} - 1.139.10^8 s^{22} - 1.158.10^9 s^{21} - 9.776.10^9 s^{20}$$
$$-6.949.10^{10}s^{19} - 2.172.10^{12}s^{17} - 9.663.10^{12}s^{16} - 3.71.10^{13}s^{15} - 1.231.10^{14}s^{14}$$

$$-3.53.10^{14}s^{13} - 8.74.10^{14}s^{12} - 1.862.10^{15}s^{11} - 3.398.10^{15}s^{10} - 5.276.10^{15}s^9 - 6.903.10^{15}s^8$$

$$-7.511.10^{15}s^7 - 6.676.10^{15}s^6 - 4.721.10^{15}s^5 - 2.556.10^{15}s^4 - 9.953.10^{14}s^3 - 2.482.10^{14}s^2$$

$$-2.977.10^{13}s - 0.00439$$

$$D(s) = 4.971.10^{-14}s^{30} + 2.032.10^{-10}s^{29} + 2.663.10^{-7}s^{28} + 1.221.10^{-4}s^{27} + 9.72.10^{-3}s^{26}$$

$$+0.3918s^{25} + 10.14s^{24} + 187.1s^{23} + 2612s^{22} + 2.862.10^4 s^{21} + 1.088.10^7 s^{18} + 2.523.10^5 s^{20}$$

$$+1.82.10^6 s^{19} + 5.428.10^7 s^{17} + 2.273.10^8 s^{16} + 8.005.10^8 s^{15} + 2.372.10^9 s^{14} + 5.9.10^9 s^{13}$$

$$+1.225.10^{10}s^{12} + 2.107.10^{10}s^{11} + 2.962.10^{10}s^{10} + 3.341.10^{10}s^9 + 2.941.10^{10}s^8$$

$$+1.931.10^{10}s^7 + 8.743.10^9 s^6 + 2.286.10^9 s^5 + 1.519.10^8 s^4 - 5.226.10^7 s^3 + 3.6.10^{-6}s^2$$

$$+5.32.10^{-22}s$$

3.2. Compare the robust controller with another controller

The bicycle with the varying parameter is controlled by PID control method. The result is then used to compare to the case which the robust controller is applied. Simulation diagram of self-balancing two-wheeled bicycle control system using robust controller and PID controller are shown in Figure 5. Simulation results of self-balancing two-wheeled bicycle control system when the parameters of model are rated and when the model parameters change, Initially, the bicycle deviates $\theta = \frac{\pi}{180}(rad)$ from the vertical axis. Parameters of PID controller: K_p = -450, K_I = -30, K_D = -15. The results shown in Figure 6.
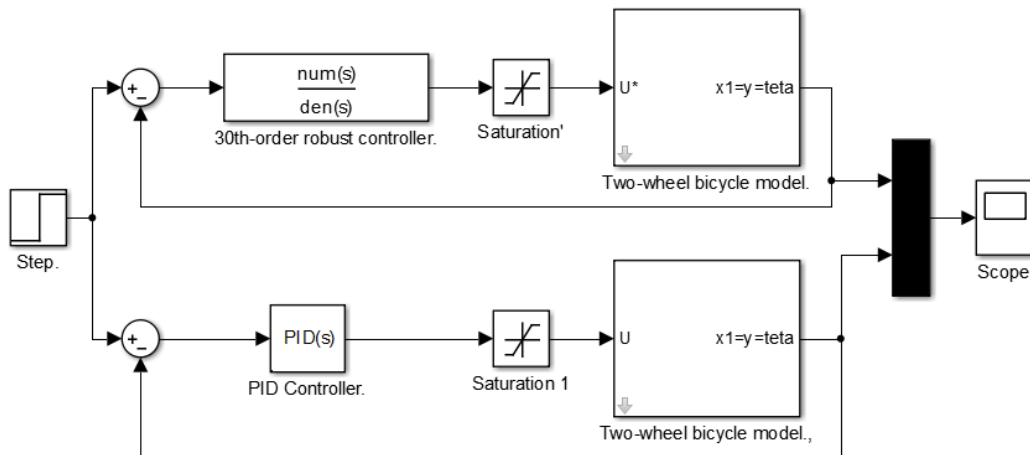

Figure 5. Simulation diagram of self-balancing two-wheeled bicycle control system using robust controller and PID controller

Remark by the simulation result in both cases the nominal parameters of the bicycle and the variable parameters of the bicycle due to the load varied. PID controller can only balance the bicycle as the bicycle parameters is niminal and in case 3. PID controller has not worked in the case 1, 2, and 4. The robust controller did work in all 4 cases. It can be seen that the robust controller was able to balance the system even the system parameters are varied (load and the height of the center of gravity of the bicycle). The robust controller has the advantage over the PID controller

Howerver, the 30th order controller could lead to the difficulty of operating the balancing process. Due to the complex program, the long processing time, the low system response, the system will not be able to adapt the requirements of real-time applications and can become unstable. For that reason, reducing the order of the controller is needed to simplify the program. The system response is thereby increased, while the robustness is ensured.
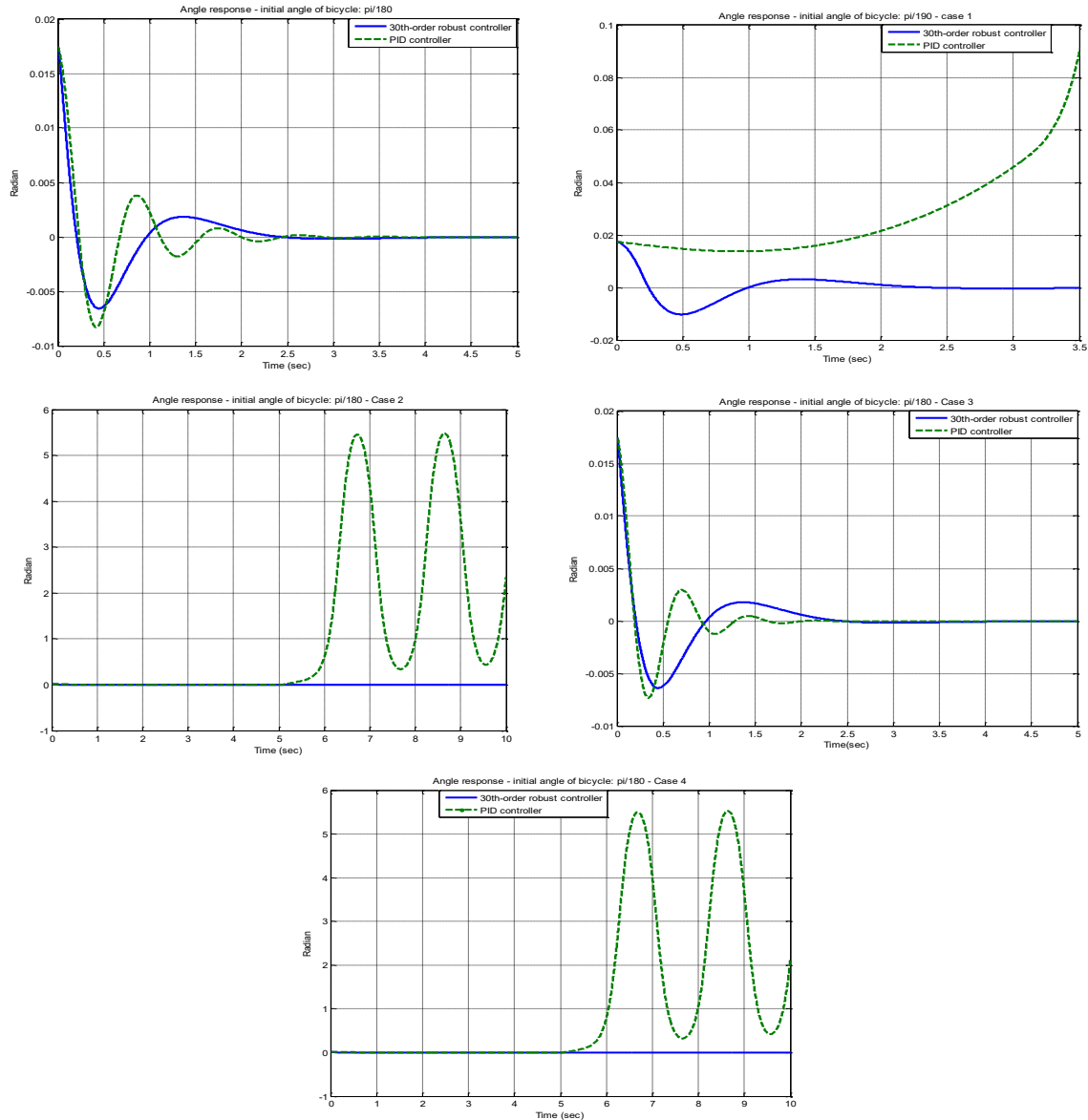
Figure 6. The system output response of self-balancing two-wheeled bicycle control system using robust controller and PID controller

4.   STOCHASTIC BALANCE TRUNCATION ALGORITHM BASED ON SCHUR ANALYSIS
4.1. Model reduction problem

Given a linear, continuous, time-invariant, MIMO system described by the following state space model:

$$\dot{x} = Ax + Bu \quad y = Cx \tag{18}$$

where, $x \in R^n, u \in R^p, y \in R^q, A \in R^{nxn}, B \in R^{nxp}, C \in R^{qxn}$. The goal of the order reduction problem for the model described by state space model given in (17) is to find the model described by state space model:

$$\dot{x}_r = A_r x_r + B_r u$$

$$y_r = C_r x_r \tag{19}$$

where, $x_r \in R^r, u_r \in R^p, y_r \in R^q, A_r \in R^{rxr}, B_r \in R^{rxp}, C_r \in R^{qxr}$ với $r \ll n$. So that the model described by in (19) can replace the model described by the (18) applications in analysis, design, and control of the system.

4.2. Stochastic balanced truncation algorithm based on Schur analysis

Most of the model reduction algorithms have published in the world only apply to stable high order linear models (the roots of the charateristic equation are negative) [13-15]. However, many high order mathematical models are unstable in reality such as the model in section 3. Therefore, the order reduction algorithm should be applicable to reduce the order for the unstable linear system. There are two basic methods for model reduction of unstable system. The first method (indirect order reduction algorithm). This algorithm divides the unstable original system into stable and unstable components, then applies the order reduction algorithm to the stable components [16-24]. At the end, to get the order of reduction of the root system, we add the reduced stable components with the unstable components.

The second method (direct order reduction algorithm). This algorithm modifies and adjusts the order reduction algorithms so that these algorithms can perform order reduction regardless of whether the original system is stable or unstable [25-29]. In the content of this paper, the author introduces the stochastic balanced truncation algorithm based on Schur analysis [23, 24]. This is a order reduction algorithm applied to the unstable system by indirect order reduction method. The specific contents of the algorithm are as follows:

Input: The system (A, B, C) (stable or unstable) described in (18) has a representation of the form of the transfer function: $G(s) := C(sI - A)^{-1}B$.

Step 1: Find the controllability grammian P and observability grammian Q by solving the following Lyapunov and Riccati equations:

$$AP + PA^T + BB^T = 0; B_W = PC^T + BD^T; QA + A^TQ + (QB_W - C^T)(-DD^T)(QB_W - C^T)^T = 0$$

Step 2: Find the Schur decomposition for PQ in both ascending and descending order, respectively,

$$V_A^T PQ V_A = \begin{bmatrix} \lambda_1 & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & 0 & \lambda_n \end{bmatrix}; \qquad V_D^T PQ V_D = \begin{bmatrix} \lambda_n & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & 0 & \lambda_1 \end{bmatrix}$$

Step 3: Find the left/right orthonormal eigen-bases of PQ associated with the kth big Hankel singular values of the all-pass phase matrix $(W * (s))^{-1}G(s)$.

$$V_A = \left[V_{R,SMALL}, \overbrace{V_{L,BIG}}^{k} \right] \qquad V_D = \left[\overbrace{V_{R,BIG}}^{k}, V_{R,SMALL} \right]$$

;

Step 4: Find the SVD of $\left(V_{L,BIG}^T V_{R,BIG} \right) = U\Sigma V$

Step 5: Form the left/right transformation for the final k^{th} order reduced model.

$$S_{L,BIG} = V_{L,BIG} U\Sigma(1:k, 1:k)^{-1/2}; S_{R,BIG} = V_{R,BIG} V\Sigma(1:k, 1:k)^{-1/2}$$

Step 6: Calculate $(A_r, B_r, C_r) = \left(S_{L,BIG}^T A S_{R,BIG}, S_{L,BIG}^T B, C S_{R,BIG} \right)$.

Output: The reduced system (A_r, B_r, C_r).

5. APPLIED LQG ALGORITHM FOR ROBUST CONTROL PROBLEM OF TWO-WHEELED BICYCLE

5.1. The reduced controller of sef-balancing two-wheeled bicycle

The full order RH_∞ controller is designed as (17), which is a 30th order controller. To obtain low controller, we perform order reduction of RH_∞ controller in accordance with the stochastic balanced truncation algorithm based on Schur analysis in section 4. The results of the order reduction controller are shown in Table 3.

5.2. Controlling the two-wheeled bicycle using the reduced 4th and 5th order controller

Using the reduced 5th order controller in Table 3 controls the balancing system for two-wheeled bicycle having the model as (16). The performance is compared to the performance of the original (30th order) controller. The simulation diagram of two-wheeled bicycle system using the original controller and reduced controllers is shown in Figure 7.

Table 3. Results of model reduction of the 30th order robust controller

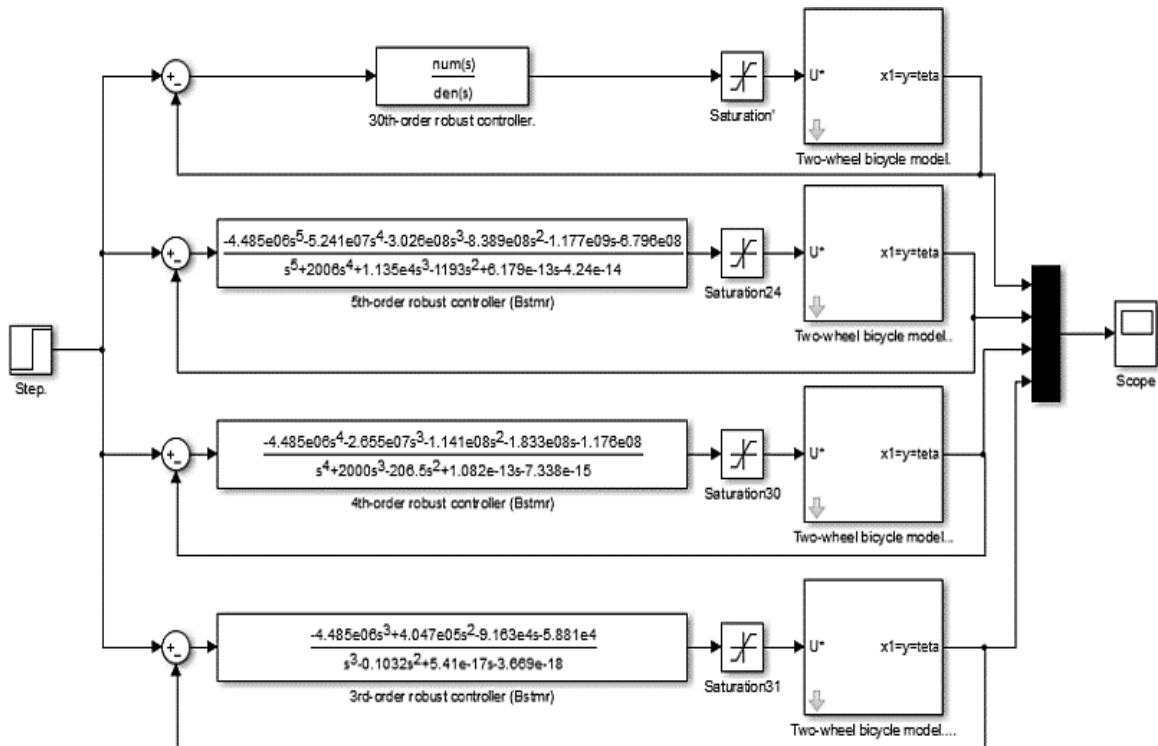| Order	Transfer function – $R_r(s)$
5	$\dfrac{-4.485.10^6 s^5 - 5.241.10^7 s^4 - 3.026.10^8 s^3 - 8.389.10^8 s^2 - 1.177.10^9 s - 6.796.10^8}{s^5 + 2006 s^4 + 1.135.10^4 s^3 - 1193 s^2 + 6.179.10^{-13} s - 4.24.10^{-14}}$
4	$\dfrac{-4.485.10^6 s^4 - 2.655.10^7 s^3 - 1.141.10^8 s^2 - 1.833.10^8 s - 1.176.10^8}{s^4 + 2000 s^3 - 206.5 s^2 + 2.369.10^{-14} s - 3.026.10^{-15}}$
3	$\dfrac{-4.485.10^6 s^3 + 4.047.10^5 s^2 - 9.163.10^4 s - 5.881.10^4}{s^3 - 0.1032 s^2 + 5.41.10^{-17} s - 3.669.10^{-18}}$

Figure 7. The simulation diagram of two-wheeled bicycle system using the original controller and reduced controllers

The simulation results of self-balancing two-wheeled bicycle control system when the parameters model are nominal and the bicycle is initially deviated $\theta = \pi/180$ (rad) from the vertical axis. The results shown in Figure 8. The simulation results of self-balancing two-wheeled bicycle control system when the vehicle is loaded and the bicycle is initially deviated $\theta = \pi/180$ (rad) from the vertical axis. The results shown in Figure 9.
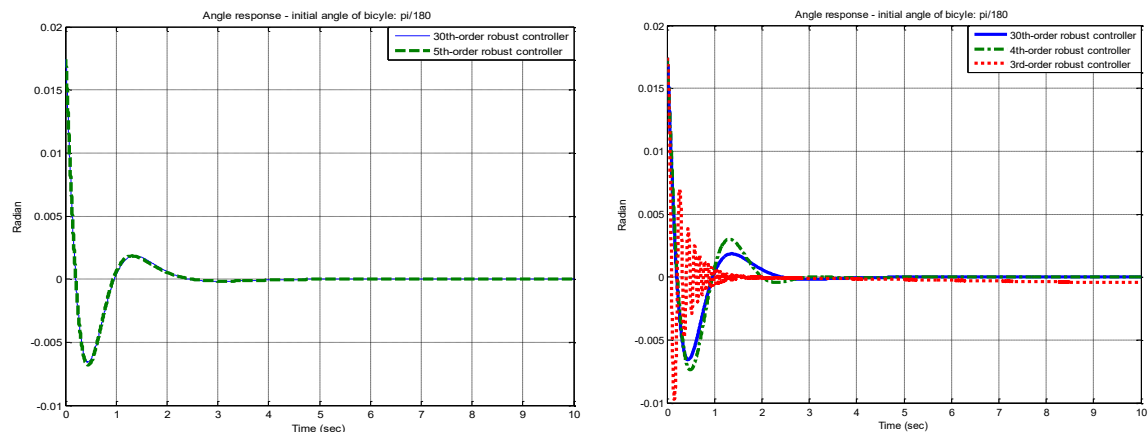

Figure 8. Output response of the self-balancing two-wheeled bicycle control system in no load condition
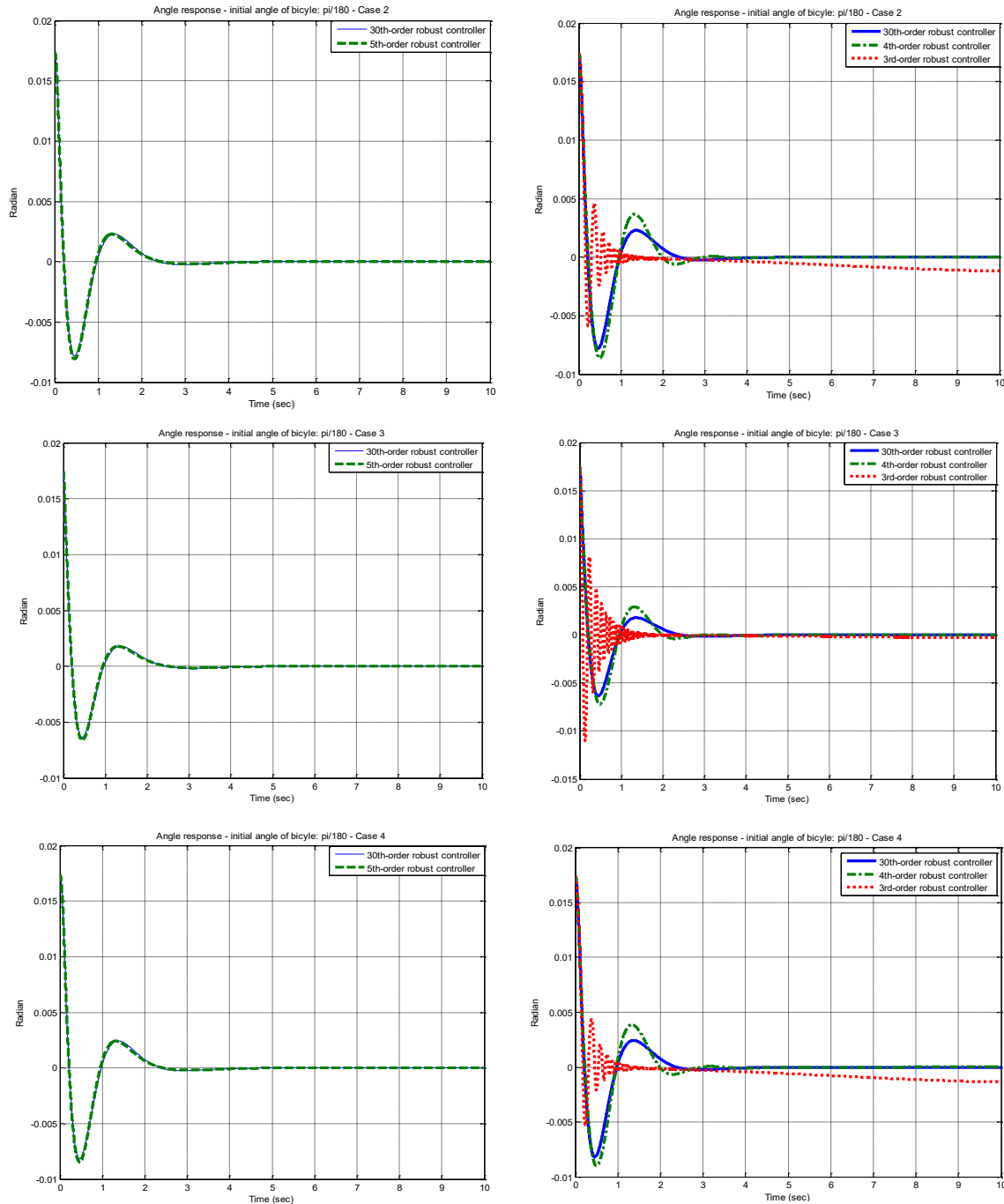
Figure 9. Output response of the self-balancing two-wheeled bicycle control system when the system is loaded condition (continue)

Compared the result between the system using the original controller and the system using the reduced controller according to the balanced truncation and other algorithms. The author compared the stochastic balanced truncation algorithm based on Schur analysis with the balanced truncation algorithm proposed by Moore [30]. This is the most commonly used order reduction algorithm. In Matlab, the command balancmr is used to perform the balanced truncation. We get the reduced 4th order controller.

$$R_r(s) = \frac{-4.485.10^6 s^4 - 5.351.10^8 s^3 + 7.513.10^7 s^2 + 2.822.10^7 s + 1.307.10^7}{s^4 + 2000 s^3 - 206.5 s^2 + 1.258.10^{-10} s - 4.767.10^{-12}}$$

The simulation is performed with the nominal parameters of the two-wheeled bicycle model and initially deviated from the vertical by an angle $\theta = \frac{\pi}{180}\ (rad)$. The result shown in Figure 10.
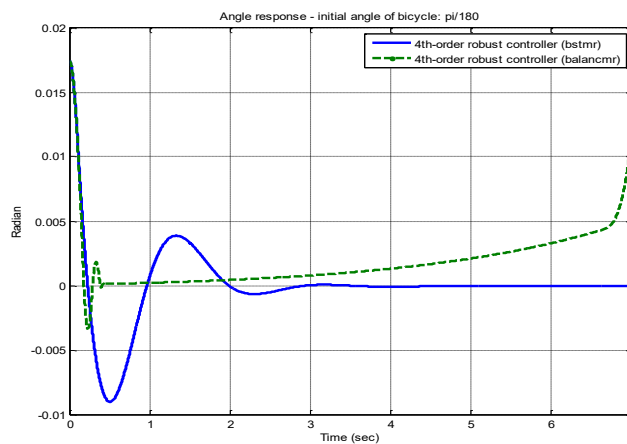

Figure 10. Output response of the self-balancing two-wheeled bicycle control system using the reduced 4th order controller

5.3. Evaluated results

The reduced 4th and 5th order controller arcooring to the stochastic balanced truncation algorithm based on Schur analysis can be used to control the two-wheeled bicycle model. The output response of the reduced 5th order controller is almost identical to the output response of original controller. The output response of the reduced 4th order controller is different from that of the original controller. The reduced 3rd order controller cannot control the balance of the two-wheeled bicycle model.

Compared the result of the two-wheel balancing control system between the system using the reduced controller according to the balanced truncation algorithm based on Schur analysis and the system using the balanced truncation algorithm (balancmr): We see that the control system using the reduced 4th order controller according to the balanced truncation algorithm based on Schur analysis ensure the stable balance of the two-wheeled bicycle when the bicycle deviates from vertical and when the parameters of the model change, while the control system using the reduced 4th order controller according to the balanced truncation algorithm (balancmr) does not.

6. CONCLUSION

The paper has developed, modeled a two-wheeled self-balancing bicycle model and designed a robust controller to control the balance of two-wheeled bicyle. The paper also introduces the stochastic balanced truncation algorithm based on Schur analysis and applies this algorithm to reduce the high order robust controller using to control the balance of two-wheeled bicyle. In particularly, the reduced 4th and 5th order controller can replace the original controller (30th-order) while the performance of the control system is ensured. Using the reduced controller simplify the program, so the computational time is reduced. Therefore, the system respose is improved, and the requirements in real-time application are met. The simulation results show the correctness of the model reduction algorithm and the robust control algorithm of two-wheeled self-balancing two-wheeled bicycle.

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