Characteristic's analysis of associative switching system

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ABSTRACT

This paper introduced new method and model of telecommunication switching system design which can be applied to wavelength-division multiplexing (WDM) optical networks, circuit-switching networks or virtual channel/path connections in an asynchronous transfer mode (ATM) networks. Modern data switching systems such as electronic private branch exchange (PBX), routers and switches include switching matrix which are constructed in the form of bipartite graphs. In such systems, the issues of requests' processing are considered from the queuing theory point of view. Associative switching systems are fundamentally new structures, therefore it is necessary to develop adequate methods for their throughput determination. Article covered matters of throughput determination basics of an associative switching system and the obtained formulas used for state probability calculation of switching modules and system throughput.

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1. INTRODUCTION

Plurality of scientific researches states that packet switching principle provides the possibility for optimal usage of network channels throughput and traffic unification. This leads to the implementation of single switching principle for different traffic types. Data network convergence means the realization of unified multi-protocol Medias, which are capable to process and transfer the heterogeneous traffic regardless of network and transport protocols [1]. Fiber-optic communication provides higher information exchange rate and greater system reliability comparing to wired electronic communications and radio communications. Switching and routing process rate in optical networks is limited by electronic components of optical network devices.

One of the network devices performance increase methods, in particular packet data processing rate, is grouping based on specific characteristics of internet protocol (IP) packages into data bursts and organization optical burst-switched wavelength-division multiplexing (WDM) network [2]. Optical burst switching allows to avoid the potential time limitation of electronic processing in optical WDM networks. Optical routers should possess low losses coefficient, low optical crosstalk, wide bandwidth in optical range. Also, these devices should operate regardless of polarization, have homogeneous performance and possess the possibility for future network expansion.

Additional approach for packet data processing rate increase is the associative switching theory. This theory provides the possibility to describe the structure of perspective switching matrix based on associative calculus. Associative calculus is based on Boolean algebra. Associative switching systems (ASS) are the brand-new structures, therefore is it necessary to develop new methods of throughput calculation [3], [4].

Optimal structure of switching systems and overall network should allow the operation based on as-low-as-possible time and technical resources expenditures between any types of subscribers. Also, such structure should guarantee the adequate system life cycle. Close attention should be kept on development of switching system (switching matrix) used in electronic exchanges and switches/routers.

Modern data switching systems such as electronic private branch exchange (PBX), routers and switches include switching matrix which are constructed in the form of bipartite graphs [5]-[9]. In such systems, the issues of servicing requests are considered from the queuing theory point of view. ASS are fundamentally new structures, so it is necessary to develop adequate methods for determining throughput for them. This article covers matters of throughput determination basics of an associative switching system and the obtained formulas for calculating the probability of the state of switching modules and the throughput of the system.

2. RESEARCH METHOD

An ASS is a certain graph (V, T) with a vertex set V and a set of connecting paths edges T, constructed as a set of switching modules and a defined link system of connecting paths between vertices in accordance with the associative calculus from group theory (it should be noted the term "associative calculus" seems to be restricted to the Russian literature; in the Western literature the term "Thue system" is often used) [10]-[12]. Each switching module has the property of variability or programmability, the switching is defined as a connection matrix $R(t)$, which establishes the connections of the external poles S_i.

In this case call handling is providing some network path the between the switching module source and switching module receiver for the call duration. If this path cannot be set up, the call is lost. So, a random configuration of separate connecting paths had been created on the ASS, these connecting paths are engaged and immediately released after the connection end.

Due to the complexity of determining the ASS structure and its throughput, it is difficult to derive the formula for throughput calculation in a closed form, so the following way of problem solving is proposed. The connecting path between the source and receiver of the switching module is represented as a deductive chain passing through the transit switching modules, which formation probability of can be described by a Markov chain [13], [14]. Markov chain operates probabilistic States of switching modules. Each module has m connections with neighboring modules. Its probabilistic States can be determined by the Hasse diagram [15]-[17]. The Hasse diagram for an ASS with parameter $i = 4$ is shown in Figure 1.

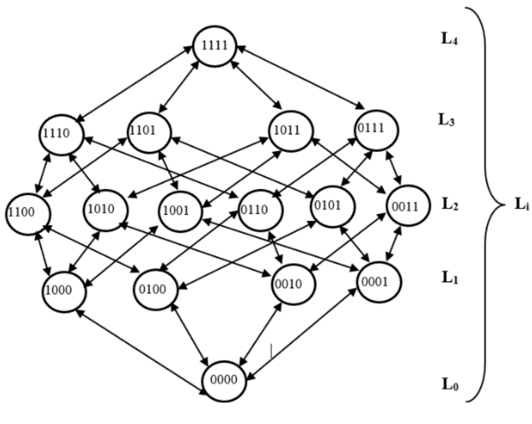

Figure 1. The Hasse diagram for an ASS with $i = 4$ parameter

In [18], the author formulated a theorem on the probable states of the ASS switching module. In this article, we will give proof of theorem.
Theorem

If all the i poles of ASS switching module are independent from each other, then the probabilistic States of ASS switching module can be defined by (1)

$$P_{x_1,x_2,\dots,x_m} = \frac{\prod_{i=1}^{m} \lambda_i^{x_i} \mu_i^{\bar{x}_i}}{\prod_{i=1}^{m}(\lambda_i+\mu_i)}, \quad (1)$$

where x_i is the state of the i pole of the switching module; 0 – idle, 1 – busy; λ_i is the intensity of incoming calls at the ith pole; μ_i is the intensity of processing the incoming load at the i pole of the module, the $i = \overline{1, m}$. We will prove the theorem by the method of mathematical induction [19].
Proof of theorem

– For $i = 2$, the system of differential equations describing the system state has the form

$$\begin{aligned} p_{00}^1(t) &= -(\lambda_1 + \lambda_2)p_{00}(t) + \mu_1 p_{10}(t) + \mu_2 p_{01}(t), \\ p_{10}^1(t) &= -(\mu_1 + \lambda_2)p_{10}(t) + \lambda_1 p_{00}(t) + \mu_2 p_{01}(t), \\ p_{01}^1(t) &= -(\lambda_1 + \mu_2)p_{01}(t) + \lambda_2 p_{00}(t) + \mu_2 p_{10}(t), \\ p_{11}^1(t) &= -(\mu_1 + \mu_2)p_{11}(t) + \lambda_1 p_{10}(t) + \lambda_2 p_{01}(t). \end{aligned} \tag{2}$$

– When moving to the limit $t \to \infty$, the solution of this system comes down to the solution of algebraic equations system. Add to this system the normalizing conditions

$$p_{00} + p_{10} + p_{01} + p_{11} = 1. \tag{3}$$

– After solving the probabilistic state, we get

$$P_{x_1,x_2} = \frac{\prod_{i=1}^{2} \lambda_i^{x_i} \mu_i^{\bar{x}_i}}{\prod_{i=1}^{2}(\lambda_i + \mu_i)} \tag{4}$$

– After performing a similar calculation for $i = 3,\ i = 4$, we can write

$$P_{x_1,x_2,x_3} = \frac{\prod_{i=1}^{3} \lambda_i^{x_i} \mu_i^{\bar{x}_i}}{\prod_{i=1}^{3}(\lambda_i + \mu_i)} \tag{5}$$

$$P_{x_1,x_2,x_3,x_4} = \frac{\prod_{i=1}^{4} \lambda_i^{x_i} \mu_i^{\bar{x}_i}}{\prod_{i=1}^{4}(\lambda_i + \mu_i)}$$

The proof is complete.

On each transit switching module, the transmission direction is determined according to the search algorithms, which are engaged with probability $P_{x_1 x_2 \dots x_m}$. The connecting path from the switching module source to the switching module receiver through the transit nodes can be represented as a deductive chain (see Figure 2). A call originating from the transmitter's switching module moves in the direction of the receiver, stopping at the transit switching modules to determine the next step according to the specified algorithm. The initial position of the call and the source of the combination module can be set by a vector

$$\xi = (1, 0, 0 \dots 0) \tag{6}$$

The first component is the probability that the call is initially located in the module source. As time passes, the probability of call location is described by the Markov chain. To do this, imagine the entire deductive chain as a device with a finite set of States

$$S = \{S_1, S_2, \dots, S_n\}, \tag{7}$$

where n is the number of switching modules in the chain. We introduce a transition matrix of order $n \times n$ as (8)

$$P = \begin{pmatrix} p_{11} & p_{12} \cdots & p_{1n} \\ p_{12} & p_{22} \cdots & p_{2n} \\ p_{n1} & p_{n2} \cdots & p_{nn} \end{pmatrix} \tag{8}$$

For matrix elements the following States are met

$$\left.\begin{aligned} 0 \le P_{ij} \le 1, \\ \sum_{j=1}^{n} P_{ij} = 1 \end{aligned}\right] i, j = \overline{1, n}. \tag{9}$$

The p_{ij} value is the probability that the call pass from SM_i to SM_j during t_i time. If ξ is the initial vector-string, then the call location after t_i time is determined by the vector ξp, and after k "steps" call location is determined by the vector ξp^k. It means that the i component of the ξp^k vector determines the probability that the call will reach the SM_k module after the kt_i time expires.

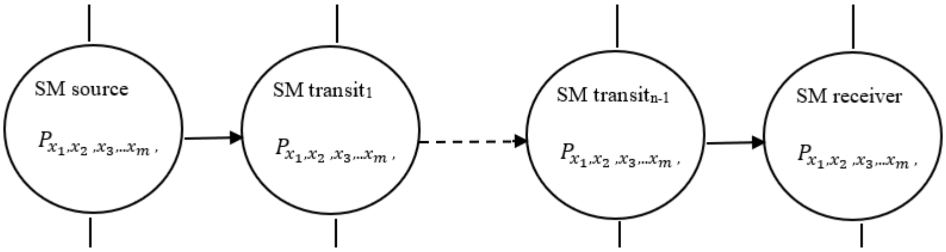

Figure 2. Connecting path from the switching module source to the switching module receiver through the transit nodes

Analyzing the Hasse diagram (see Figure 1) you can note that the call can move in the module receiver direction if it finds the transit modules in the state L_0, L_1, L_2. The probability of this event is equal to

$$p_{п} = p_{0000} + p_{1000} + p_{01000} + p_{1100} + p_{1010} + p_{1001} + p_{0110} + p_{0101} + p_{0011}.$$

The probability that the call will "go" in the opposite direction is equal to $p_0 = 1 - p_n$. Given this expression, the (6) and (8) transform into

$$\left.\begin{array}{l} \xi = (1,0,0,\dots,0,0,0), \\ p = \begin{vmatrix} p_0 p_n 0 & \dots & 000 \\ p_0 0 p_n & \cdots & 000 \\ 000 & \cdots & p_2 0 p_n \\ 000 & \cdots & 0 p_0 p_n \end{vmatrix} \end{array}\right\} \tag{10}$$

If the size of p matrix is large, the calculations required to determine the value p_{ij}^k can be cumbersome.

According to the Markov chains theory, proposed method allows find p^k without calculating the p matrix degrees. By using the Peron formula [20], [21] and introducing the concept of the characteristic equation of the matrix, we find

$$p(\lambda) = \begin{bmatrix} p_{11,}p_{12} \dots p_{1n} \\ p_{21,}p_{22} \dots p_{2n} \\ \vdots \qquad\quad \vdots \\ p_{n1,}p_{n2} \dots p_{nn} \end{bmatrix}. \tag{11}$$

By solving the equation $|p(\lambda)| = 0$ we find roots of the equation – the proper values of $p(\lambda)$. Then the elements of the P_{ij}^k matrix can be found from the relation

$$P_{ij}^k = \sum_{i=1}^{q} \frac{1}{(m_i-1)!D_\lambda^{m_i-1}[(\lambda^k M_{ji}(\lambda)/\psi_i(\lambda))]_{\lambda=\lambda_i}}, \tag{12}$$

where $\lambda_1, \lambda_2 \dots \lambda_m$ – roots proper value of the p matrix characteristic equation; $m_1 m_2 \dots m_q$ – multiplicity eigenvalues λ_i; $M_{ji}(\lambda)$ – algebraic complement of the element standing in the i row and j column of the matrix $p(\lambda)$; $D_\lambda^{m_1-1}$ – λ derivative of the $(m_i - 1)$ order.

Write down all realization cases of the connection setup probability between the source and receiver switching modules in the form of:

$$p_y = \xi p^n \tag{13}$$

We can determine the overall probability of the connection setup in the ASS by performing following method. Let’s define the all connecting paths number from the source switching module to the receiver switching module in a two-dimensional grid as described in [22], [23].

$$\upsilon = C_l^d + d, \tag{14}$$

Where d and l are absolute values of the differences between digits tens and ones of the receiver switching module number and the source switching module number.

The probability graphs showing all possible connection paths between the source and receiver switching modules are shown in Figure 3. Assuming that the i edge blocking does not depend on the blocking state of other edges, we determine the probability of call loss in the ASS by the (15).

$$\mathrm{p_{ASS}} = (1 - \mathrm{P_y})^{\upsilon} = (1 - \xi \mathrm{p^n}). \tag{15}$$

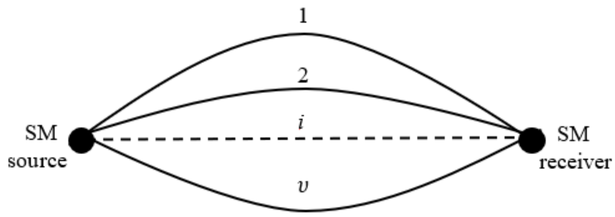

Figure 3. Probability graphs of all possible connection paths between the source and receiver SM's

3. RESULTS AND ANALYSIS

We explaine the results of research in small Example. Let the ASS is given as an abstract two-dimensional network with parameters m = 4, as shown in Figure 4 [18]. Coordinate of the source switching module is 14, coordinate of the receiver switching module is 32. Parameters of module call flow is follows $\lambda = 1, \mu = 2,\ p = \frac{\lambda}{\mu} = 0.5$. It is necessary to determine the probability of loss in ASS.

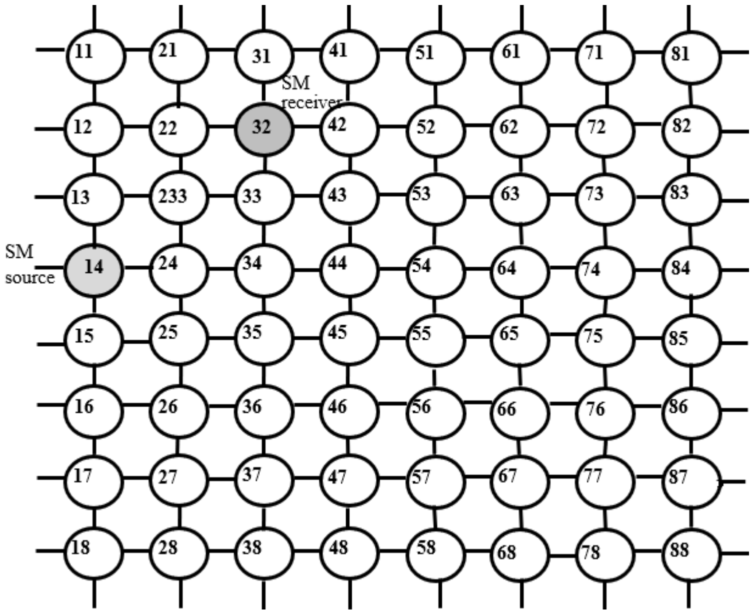

Figure 4. Effects of selecting different switching under dynamic condition

Decision

- Define the parameters l – absolute value of the difference between digits ones of the receiver switching module number and the source switching module number, d – absolute value of the difference between digits tens of the receiver switching module number and the source switching module number and ϑ – max number of all possible connection paths between the source and receiver SM's; $d = 3 - 1 = 2$, $l = 3 - 1 = 3,\ \upsilon = 6$.
- Using the (1), find $P_{x_1}P_{x_2}P_{x_3}P_{x_4}$ and calculate that $p_0 = 0.1$.
- Make a Markov chain by (10).

$$\left.\begin{array}{l}\xi = (1,0,0,0,0,0),\\ p = \begin{vmatrix} 0,1 & 0,9 & 0 & 0 & 0 & 0\\ 0,1 & 0 & 0,9 & 0 & 0 & 0\\ 0 & 0,1 & 0 & 0,9 & 0 & 0\\ 0 & 0 & 0,1 & 0 & 0,9 & 0\\ 0 & 0 & 0 & 0,1 & 0 & 0,9\\ 0 & 0 & 0 & 0 & 0,1 & 0,9 \end{vmatrix}\end{array}\right\} \quad (16)$$

The solution from (16) we gives the result

$p = p^6 = 0.54$

– According to the (15) determine the loss in ASS.

$p_{ASS} = (1 - p) = 0.46^6 = 0.0156$

Based on above listed calculation we considered the throughput of associative switching system. As a conclusion, we obtained the formulas for determination of switching module state and system throughput. These calculations might be used for future fixed network desing and planning of new architecture based on associative switching fabric [24], [25].

4. CONCLUSION

The obtained results show the implementation possibility of broadband digital system model based on fiber-optic communication lines and integrated optical switching elements. In this paper, we aim to propose new network design method for iterative associative multi-dimensional switching systems with high regular structures and many alternative shortest paths between every pair of nodes. The offered method is applicable in different types of networks and aimed for high rates of processing and distribution of information. Architectures considered within our research is the fist stage toward development of multi-layer associative switching matrix, named associative switching fabric.

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