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Feedback Linearization Control for Path Tracking of Articulated Dump Truck

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Abstract

The articulated dump truck is a widespread transport vehicle for narrow rough terrain environment. To achieve the autonomous driving in the underground tunnel, this article proposes a path following strategy -for articulated vehicle based on feedback linearization algorithm. First of all, the kinematic model of articulated vehicle, which reflects the relationship between the structure parameters and state variables, has been established. Referring to the model, the nonlinear errors equation between real path and reference path, which are as the feedback from the path tracking process, has been solved and linearized. After estimating the system controllability, the path following controller with feedback linearization algorithm has been designed through calculating the parameters with the pole assignment according to the error equation. Finally, the Hardware-In-the-Loop simulation on NI cRIO and PXI controller has been -conducted for verifying the control quality and real-time path tracking performance. The result shows that the path tracking controller with feedback linearization can track the reference path accurately.

Keywords: articulated vehicle, path tracking, feedback linearization, hardware-In-the-loop

1. Introduction

The articulated dump truck is a transport vehicle which is widely used in mining, water conservancy and construction. It especially adapts to the narrow space, rough terrain and severe weather, for example, the tunnel in the underground mining. The autonomous driving system, an advanced automatic technology, can improve the vehicle productivity, operator safety and exhaust cleanliness [1]. The purpose of this article is to find a path following strategy for autonomous driving system on articulated vehicle.

Before developing the path following strategy, the articulated vehicle model, path following algorithm and controller test method need to be clear. The articulated vehicle modeling and path following control application have been investigated in many studies. In [2-4] kinematic model of articulated vehicle and error model between real and reference path are presented, and the path tracking simulation with model predictive control is applied, while in [5] the feedback controller based on Lyapunov approach is designed and the stability of the closed-loop system is proved in theory. The simulations in these literatures are non-real-time and the real-time performance has not been verified. Moreover, in [6] a trajectory tracking strateagy fora new structure automated guided vehicle is presented, while in [7] a feedback linearization control for almost global output-feedback tracking is provided for the underactuated autonomous quadrotor. Both the literatures have designed the controller of feedback linearization. However, the models of the plants are distinct from the articulated vehicle. This article need to design a path following algorithm controller with a kinematic model of articulated vehicle and test in a real-time environment.

An articulated dump truck consists of a tractor, a trailer and an articulated body. This structure have two degrees of freedom, yaw and roll, for a shorter steering radius and keeping all tires contacting the ground on the rough terrainrespectively. However, compared with the traditional Ackermann steering mechanism, this articulated steering structure has complex steering process. To solve this problem, the mathematical model of articulated vehicle is derived

in which the velocity of the front frame is considered as a measurable variable. Furthermore, the articulated vehicle model, as a highly nonlinear model, is hard to control. However, the feedback linearization can simplify the controller design. Meanwhile, controller real-time performance is important, while traditional offline simulation method cannot test. Hardware-In-the-Loop simulation is thus conducted toverify the real-time performance [8].

In this article, a 35 tons underground articulated mining truck with electical transmission designed by University of Science & Technology Beijing is selected as the research prototype. In Section 2, a kinematic model of the articulated vehicle has been builtthrough the geometrical relationship. In addition, the definition of the errors between real path and reference path has beendiscribed and modelled. In Section 3,according to the feedback linearization method, the state equation and output equation of the nonlinear articulated vehicle have been transformed to a linear controllable and observable system. Then the controller has been configured by linear control method to control the model for the precise path tracking. In Section 4, the Hardware-In-the-Loop simulation devices have been shownand the simulation procedure with different platforms have been introduced. In Section 5, the simulation has been launched to verify the quality of the controller. The results have been shown with the graphs. And the interpretation of the results has been discuss. In Section 6, the conclusion presents the findings of this article.

2. Articulated Vehicle Kinemics Modeling
2.1. Articulated Vehicle Mathematical Model

The articulated vehicle turning state is presented in Figure 1. In this figure, O is the instantaneous center of movement. $P_f(x_f, y_f)$ and $P_r(x_r, y_r)$, denote the corresponding center points of tractor and trailer. $l_f$and l_r are the length of the front and rear units. $\theta_f$andθ_r denote the units orientation. $\gamma$is the articulated angle which is defined as the difference between the front and rear orientation. Usually, considering simplifying calculation, P_f is the whole vehicle reference point, because the velocity v_f orientation of this point is coincident with the whole vehicle orientation [9]. The velocity of articulated vehicle is defined as
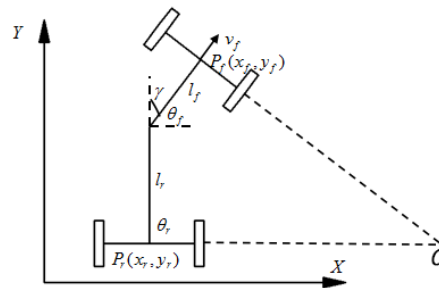
$$v = v_f \tag{1}$$

Figure 1. Articulated vehicle scheme

The coordinates of P_f is:

$$\begin{aligned} \dot{x}_f &= v_f \cos\theta_f \\ \dot{y}_f &= v_f \sin\theta_f \end{aligned} \tag{2}$$

The rate ofθ_f is composed of the angular velocity in constant radius turning and yaw velocity in fixed P_f pivot steering.

$$\dot{\theta}_f = \frac{v_f \sin\gamma + l_r\dot{\gamma}}{l_f \cos\gamma + l_r} \tag{3}$$

The articulated vehicle state equation $P_f=[x_f, y_f, \theta_f, \gamma]$ is:

$$
\begin{bmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{\theta}_f \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \cos\theta_f \\ \sin\theta_f \\ \dfrac{\sin\gamma}{l_f\cos\gamma + l_r} \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ \dfrac{l_r}{l_f\cos\gamma + l_r} \\ 1 \end{bmatrix} \dot{\gamma} \tag{4}
$$

2.2. Path Description

Figure 2 shows the errors between the real and reference path. The small circle of which the center is *c* is the real vehiclepath, while the big circle of which the center is *C* is the reference path. Ideally, the vehicle should pass P_1, P_2, P_3. The variables are defined as:

a) Lateral displacement error ε_d:the lateral displacement errorbetween the vehicle reference point *p*and the corresponding point *P* (the nearest point on the reference path);

b) Orientation error ε_θ:the orientation error between the velocity orientation of *p* and the tangential orientation of *P*;

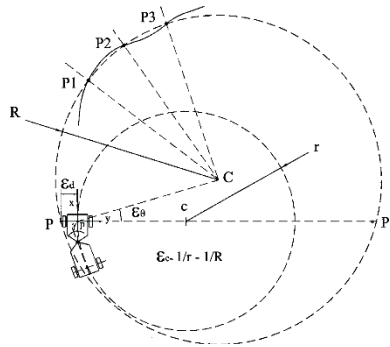c) Curvature error ε_c: the curvature error between the curvature of *p* and *P*.

Figure 2. Articulated vehicle plan-view

2.3. Error Modeling

Figure 3 is a part of Figure 2. When the vehicle moves from *p* to *p'*, it revolves dθ around instantaneous center *c* at radius *r*. Radial lines *cp* and *Cp'* intersect the reference path circle at *P* and *P'* respectively. The angle between *CP* and *CP'* is denoted dϕ.
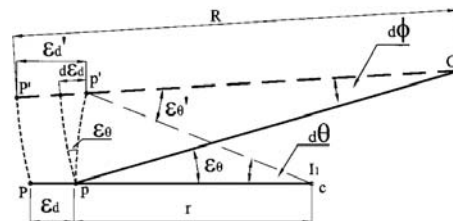

Figure 3. Geometric relationship of errors

a) Lateral displacement error ε_d
Assuming both d$\theta$and dϕ are small angles, the change of lateral displacement error is

$$
d\varepsilon_d = rd\theta\varepsilon_\theta \tag{5}
$$

Substituted with $v=rd\theta/dt$, then the rate of lateral displacement error is:

$$\dot{\varepsilon}_d = v\varepsilon_\theta \tag{6}$$

b) Orientation error ε_θ
According tothe geometric relationship,

$$d\varepsilon_\theta = d\theta - d\phi \tag{7}$$

$$(R-\varepsilon_d)d\phi = rd\theta \tag{8}$$

From equation(7)(8), the steady-state steeringdifferential of orientation error is

$$d\varepsilon_\theta = d\theta\left(1 - \frac{r}{R-\varepsilon_d}\right) \tag{9}$$

Generally, the width of the tunnel is about 5 m and width of the vehicle is 3.4 m. So $\varepsilon_d \leq 0.6$ m, while the vehicle steering radius$r \leq 6.6$ mthat means $R \geq 6.6$ m. Thus, assume that$R \gg \varepsilon_d$. Andconsidering the additional yaw angle velocity when $\dot{\gamma} \neq 0$, as well as substituting for $v=rd\theta/dt$ and $\varepsilon_c = r^{-1} - R^{-1}$, the rate of orientation error is:

$$\dot{\varepsilon}_\theta = v\varepsilon_c + \dot{\gamma}\left(\frac{l_r}{l_r + l_f\cos\gamma}\right) \tag{10}$$

c) Curvature error ε_c
Known vehicle velocity v and reference path radius R, the real path radius is:

$$r = \frac{v}{\dot{\theta}_f} = v\left(\frac{l_r + l_f\cos\gamma}{v\sin\gamma + l_r\dot{\gamma}}\right) \tag{11}$$

Differentiatingreciprocal of Equation (11) with respect to time t gives,

$$\dot{\varepsilon}_c = \frac{d\left(r^{-1}\right)}{dt} = \frac{v(l_f + l_r\cos\gamma)\dot{\gamma} + l_r(l_r + l_f\cos\gamma)\ddot{\gamma}}{v(l_r + l_f\cos\gamma)^2} + \frac{(l_f l_r\sin\gamma)\dot{\gamma}^2}{v(l_r + l_f\cos\gamma)^2} \tag{12}$$

From Equation (6), (10), (12), the linearized state equation is:

$$\begin{bmatrix} \dot{\varepsilon}_d \\ \dot{\varepsilon}_\theta \\ \dot{\varepsilon}_c \end{bmatrix} = \begin{bmatrix} 0 & v & 0 \\ 0 & 0 & v \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \varepsilon_d \\ \varepsilon_\theta \\ \varepsilon_c \end{bmatrix} + \begin{bmatrix} 0 \\ l_r\left(l_r + l_f\cos\gamma\right)^{-1} \\ \left(l_r + l_f\cos\gamma\right)^{-1} \end{bmatrix}\dot{\gamma} + \begin{bmatrix} 0 \\ 0 \\ l_r\left[v\left(l_r + l_f\cos\gamma\right)\right]^{-1} \end{bmatrix}\ddot{\gamma} \tag{13}$$

Known$-0.25\pi < \gamma < 0.25\pi$from vehicle structure, an assumption is made that$\gamma$is a small angle measured in radians and $L = l_f + l_r$. Redefine Equation (13) asthe articulated vehicle state equation for path tracking SMC design.

$$\begin{bmatrix} \dot{\varepsilon}_d \\ \dot{\varepsilon}_\theta \\ \dot{\varepsilon}_c \end{bmatrix} = \begin{bmatrix} 0 & v & 0 \\ 0 & 0 & v \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \varepsilon_d \\ \varepsilon_\theta \\ \varepsilon_c \end{bmatrix} + \begin{bmatrix} 0 \\ l_r L^{-1} \\ L^{-1} \end{bmatrix}\dot{\gamma} \tag{14}$$

3. Feedback Linearization Control Algorithm Design
3.1. Feedback Linearization Method
Feedback linearization is a common method used in controlling nonlinear systems. The approach involves coming up with a transformation of the nonlinear system into an equivalent linear system through a change of variables and a suitable control input. The state feedback is implimented on the basis of vehicle kinematic model.The nonlinear kinematic model can be transformed into a closed-loop linear system via introducing appropriate state feedbacks. The well-established linear control theories are then applicable for the nonlinear systems with state feedback.

Moreover, the feedback linearized systems are observable as well as controllable. A linear control method is then proposed in order to control the vehicle kinematic model.

3.2. Controller Design
Before designing the controller, the system controllability must be estimated. If the r th derivative of system can express the relationship between output $y(t)$ and input $u(t)$, r is defined as the relative degree. The system is controllable when $r \leq n$ where n is system degree [11].

Analytically, from equation(14),ε_c can be controlled by $\dot{\gamma}$ and $\varepsilon_\theta$can be controlled by $\dot{\gamma}$ and ε_c as well as ε_d can be controlled by ε_θ. Thus, the articulated vehicle system is controllable when the articulated angle rate $\dot{\gamma}$is as the input. Assuming that the state variable vector x=$[\varepsilon_d, \varepsilon_\theta, \varepsilon_c]^T$ is measurable, the path tracking controller can be designed with the state variable feedback [12].

Theoretically, the closed-loop system can be formatted as wish. The state feedback is denoted as u=-Kx, where $K$is feedback gain and K=$[k_1,k_2,k_3]$. The optimization objective is to find an appropriate K that can lead any arbitrary x to the desired value in time [13].

Supposed that the object controlled is a linear time-invariant time system expressed with state space equations as follow[14]

$$\dot{x} = Ax + Bu \tag{15}$$

The linear feedback is:

$$u = -Kx \tag{16}$$
$$\dot{x} = (A - BK)x \tag{17}$$

Where $A = \begin{bmatrix} 0 & v & 0 \\ 0 & 0 & v \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ l_r L^{-1} \\ L^{-1} \end{bmatrix}, x = \begin{bmatrix} \varepsilon_d \\ \varepsilon_\theta \\ \varepsilon_c \end{bmatrix}, u = \dot{\gamma}, \ K = [k_1, k_2, k_3].$

The system transition performance should be considered, such as response time, setting time and overshoot. The pole assignmentis the solution. In this article, differential transformation method is used for assigning the pole to the arbitrary location on plane S for system stability and transition performance. So the closed-loop pole can be set on the desired location in order to keep the system having 2nd order dynamic response that natural frequency ω_n and damping ratio $\xi$is dominant. Meanwhile, the 3rd pole should be far away for these two poles on the left side half open complex plane S [15].

Considering the closed-loop stability of this system, the eigenvalue $s$of the eigenmatrix A-$BK$is on the left side of plane S [16].

$$s^3 + (\frac{l_r k_2 + k_3}{L})s^2 + v(\frac{l_r k_1 + k_2}{L})s + \frac{k_1}{L}v^2 = 0 \tag{18}$$

The truck sizes are l_f=1.68m and$L$=5.12m and it drives with the constant velocity$v$=3 m/s. Solving the Equation (18), the State feedback gain can be obtain as:

$$K = [k_1, k_2, k_3] = [0.7, 3.9, 15.6] \tag{19}$$

4. Hardware-In-the-Loop Simulation

Hardware-In-the-Loop is a form of real-time simulation. Hardware-In-the-Loop differs from pure real-time simulation by the addition of a "real" component in the loop.This component may be an electronic control unit (ECU for automotive, FADEC for Aerospace) or a real engine. The purpose of a Hardware-In-the-Loop simulation is to provide all of the electrical stimuli needed to fully exercise the ECU. In effect, "fooling" the ECU into thinking that it is indeed connected to a real plant.In this article the component is NI cRIO as a path following controller. In this article,the Hardware-In-the-Loop devices are shown in Figure 4(a) and flowchart is in Figure 4(b). It shows that the plant, articulated vehicle modelled by MapleSim, is simulated in PXI and the cRIO controller, in which program is compiled by LabView, is real. To observe directly, all the data is uploaded to a PC to display with a graphical user interface programmed by LabView.
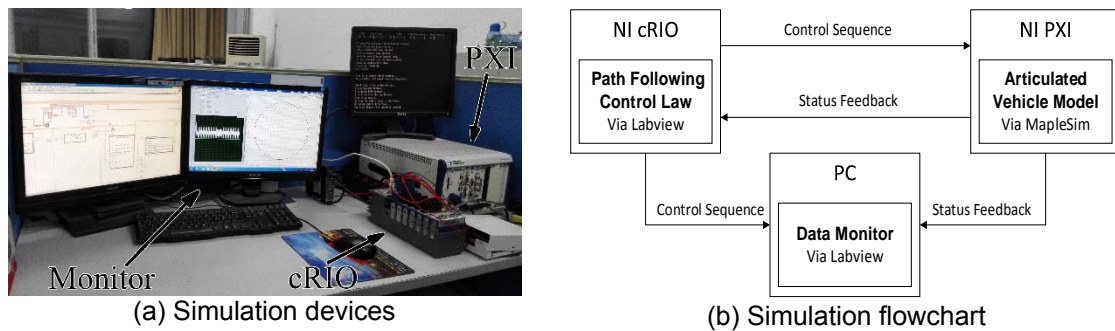

(a) Simulation devices   (b) Simulation flowchart

Figure 4. Hardware-In-the-Loopsimulation

5. Results and Discussion

During the simulation, the truck follows a circle path with radius r=25 m in the constant speed v=3 m/s. The original point of the globle coordinate system is on the centre of the circle path. The starting point is (-3,-25), and the initial direction is -∞ of X axle.The simulation duration is 100 secends.

As Figure 5 and Figure 6 shown, the real path is almost coincident with reference path. Considering thesteering wheel angle is the input varaible on the manned vehicle, and in order to show the vehicle steering procedure clearly, the desired articulated angleγ,integration of $\dot{\gamma}$, is as the input of the articulated vehicle model instead of articulated angle rate $\dot{\gamma}$. All the graphs has overshoot in Figure 6 before 10 seconds, because the vehicle needs to accelerate from 0 to 3 m/s and the initial errors are relative large. Desired articulated angle from controller approaches to 0.21 rad (12°) in 10 secends. The desired articulated angle is calculated by ε_d, ε_θ and ε_c whose changing trends are not identical. When γ leads three variables changes to different directions, the chattering appears while it is slight.Lateral displacement error reduces respectively from 200 mm to 100mm after the short overshoot. Referring to the front wheel track 2280 mm, the error is only 4%. Orientation error stays near 0.01 rad(0.5°), and curvature error is almost 0. All the variables finally tend to be stable in 10 seconds with a little chattering for adjusing. The result shows that the feedback linearization controller can track the reference path effectively in real-time environment.
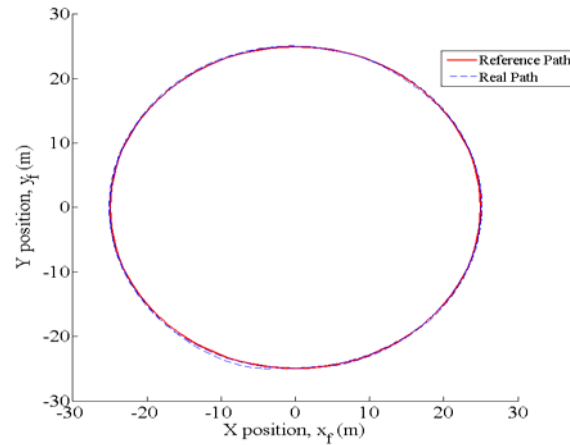
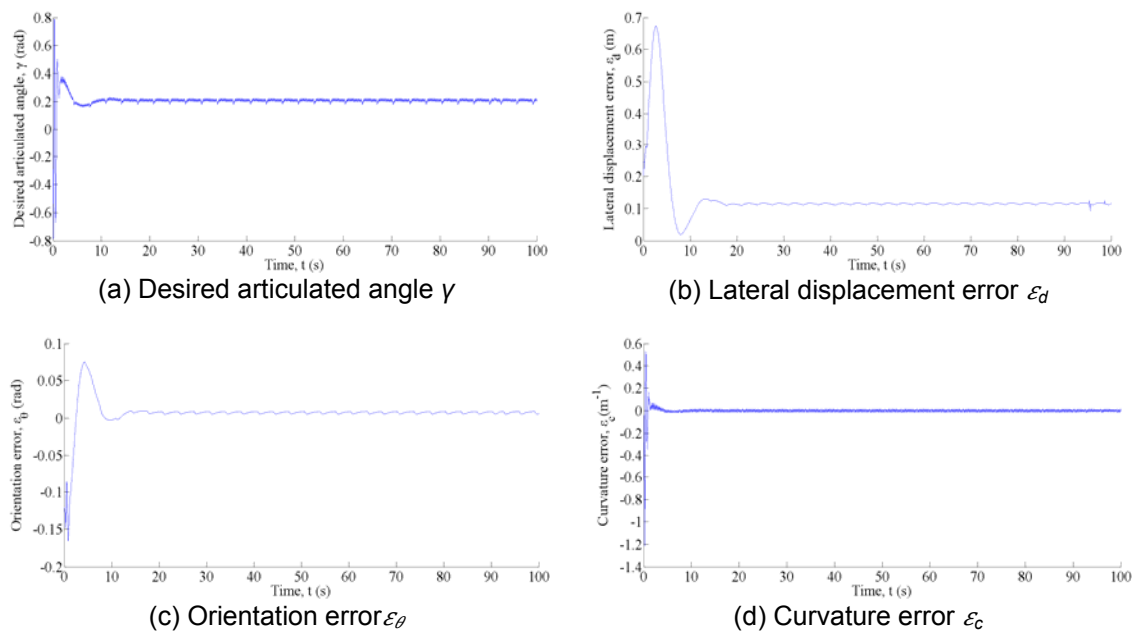Figure 5. Contrast between reference and real path

(a) Desired articulated angle γ

(b) Lateral displacement error ε_d

(c) Orientation error ε_θ

(d) Curvature error ε_c

Figure 6. Articulated angle and errors change

5. Conclusion

In this article, the research prototype is a 35-tonne electrical transmission underground mining articulated dump truck. And the hardware in the loop simulation is based on NI cRIO and PXI controller. The feedback linearization is used for designing the path tracking controller.

The conclusions of this article are:

a) For articulated vehicle model, a highly nonlinear model,the feedback linearization controllercantrack the reference path accurately. Both the dynamic and steady characteristicscan fulfill the demand.

b) The feedback linearization controller developed by the kinematics model can control the vehicle to follow the reference path without accurate dynamic model. The real path is smooth and little chattering.

c) Compared with the real vehicle test, Hardware-In-the-Loop simulation is economical and efficient. And the real-time performance is better than the off-line simulation. The characteristic of the controller is tested comprehensively.

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