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An LMI approach to Mixed H_∞/H_- fault detection observer design for linear fractional-order systems

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ABSTRACT

This study deals with the problem of robust fault detection for linear time-invariant fractional-order systems (FOSs) assumed to be affected by sensor, actuator and process faults as well as disturbances. The observer-based method was employed to solve the problem**,** where the detector is an observer. The problem was transformed into the mixed H_∞/H_- robust optimization problem to make the system disturbance-resistant on one hand and fault-sensitive on the other hand. Then, sufficient conditions were obtained to solve the problem in the linear matrix inequality (LMI) mode. Finally, the effectiveness and superiority of the method were demonstrated by simulating the solutions on a single-input multi-output thermal testing bench.

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1. INTRODUCTION

Fraction calculations made their way through engineering and application after 300 years and merely theoretical studies in mathematics [1]-[18]. Given different and new mathematics provided in fraction calculations, debates in various fields such as control theory require new proofs and theorems. As a result, the fundamental aspects of fractional-order systems (FOSs) were investigated, and stability theorems were proposed [19]-[22]. However, many aspects remain open, with some of them being currently studied. One of such aspects is fault detection (FD) in FOSs, which is of great importance. According to searches, there have been few studies in this area. Aribi *et al.* [23] present three methods to evaluate fractional residual. Aribi *et al.* [24], diagnosis methods in FO thermal systems have been proposed. The FD control of FOSs is investigated in [25]. Zhong *et al.* [26] tried to find a way to solve the fault detection observer design problem for fractional-order systems. Their subject is precisely the same as the subject of this article. However, they were utterly unsuccessful because they could not prove the stability of the closed-loop system, and the published article has very undeniable flaws.

Various methods have been proposed to detect faults [27]-[33]. One of these methods is the model-based FD technique, which has been practically employed in many industrial applications [34]-[38]. Figure 1 illustrates the algorithm of the model-based FD method. Disturbances disable typical fault detection systems in real-life systems since they are treated as faults. So, alarms are activated while there has been no fault in the system. For this reason, robust fault detection systems have been designed. In a fault detection system, robustness is defined as the system's sensitivity to faults and resistance against unknown inputs [39]-[42].

The main challenge is now to implement the model-based fault detection algorithm on FOSs and make the system disturbance-resistant on the one hand and fault-sensitive on the other hand. Additionally, it is well-known that the use of linear matrix inequality (LMI) can eliminate restrictions on conventional approaches, and can be used to solve problems involving multiple matrix variables. Besides those, different structures can be imposed on these matrices [43]-[46]. To this end, the problem was transformed into the mixed H_∞/H_- robust optimization problem, also present the results in linear matrix inequalities (LMIs) robust control theoretical framework.

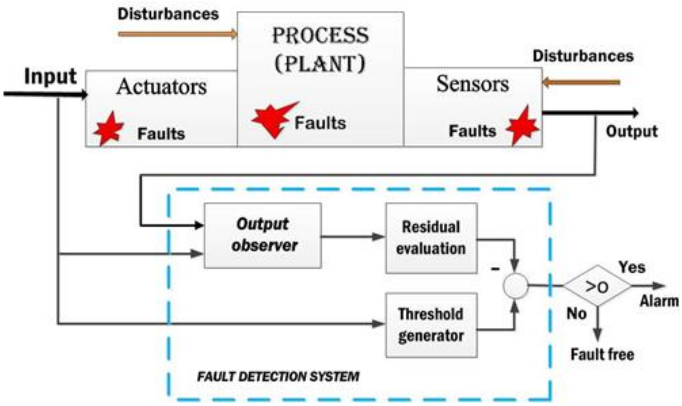

Figure 1. Model-based fault diagnosis algorithm

The rest of the paper is organized as follows. In section 2, implementation of the model-based technique on the FOS, as well as, the preliminaries and the problem statement are given. The solutions to the FD problem for FOSs are presented in section 3. Also, some simulation examples are given in section 5 to illustrate the results. Finally, some concluding remarks are provided in section 6. Notations: A^T denoted the transpose of a matrix A, its conjugate $\bar{A}$ and its conjugate transpose A^*. $Her(A)$ is short for $A + A^*$, and $\sigma\max(A)$ represents the maximum singular value of A.

2. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

Consider the following FOS:

$$G: \begin{cases} D^{\alpha}x(t) = Ax(t) + Bu(t) + B_d d(t) + B_f f(t) \\ y(t) = Cx(t) + Du(t) + D_d d(t) + D_f f(t) \\ \quad x(t) = x(0) \qquad t \in [-h_1, 0] \end{cases} \tag{1}$$

System G is the state space form of a time-invariant linear FOS where D is the differ-integral operator and $0 < \alpha < 1$. $x(t) \in \mathbb{R}^n$ denotes the pseudo-state vector. $y(t) \in \mathbb{R}^r$ denotes the measured output. $A \in \mathbb{R}^{n\times n}$, $B_d \in \mathbb{R}^{n\times p}$, $B_f \in \mathbb{R}^{n\times q}$, $C \in \mathbb{R}^{r\times n}$, $D_d \in \mathbb{R}^{r\times p}$ and $D_f \in \mathbb{R}^{r\times q}$ are constant matrices. The $x(0)$, stand for initial condition defined on $[-h_1, 0]$ where $h_1 \in \mathbb{R}$ and $0 < h_1$. This FOS affected by disturbance $d(t) \in \mathbb{R}^p$ as an unwanted input and fault $f(t) \in \mathbb{R}^q$ input as a bug in the system. If there is a problem in reading and sending data, or in measurement, it is referred to as a sensor fault $f_s(t)$, which is represented by considering $B_f = I$ in the output equation of the system as (2) [47].

$$y(t) = Cx(t) + D_d d(t) + f_s(t) \tag{2}$$

If there is a problem with actuators' performance, it affects the input of the system and calls it an actuator fault f_A:

$$\begin{aligned} D^{\alpha} x(t) &= A\, x(t) + B\big(u(t) + f_A(t)\big) + B_d d(t) \\ y(t) &= C\, x(t) + D(u(t) + f_A(t)) + D_d\, d(t) \end{aligned} \tag{3}$$

by adding the process fault $f_p(\mathrm{t})$ according to its location and type and considering $\mathrm{B_f} = B_p$ and $\mathrm{D_f} = D_p$, generally, in this method, additive faults for describing the fault is considered for the system with sensor, actuator and process faults. As a result, the faults can be rewritten as:

$$f(t) = \begin{bmatrix} f_A(\mathrm{t}) \\ f_p(\mathrm{t}) \\ f_s(t) \end{bmatrix}, \qquad \mathrm{B_f} = [B \quad B_p \quad 0], \qquad \mathrm{D_f} = [D \quad D_p \quad I],$$

one of the best definitions of fractional derivatives so far in control applications is the Caputo's definition [48]:

$${}_{\mathrm{a}}\mathrm{D}_{\mathrm{t}}^{\alpha} \triangleq \frac{1}{\Gamma(\mathrm{k}-\alpha)} \int_{\mathrm{a}}^{\mathrm{t}} \frac{\mathrm{f}^{(\mathrm{k})}(\tau)}{(\mathrm{t}-\tau)^{\alpha+1-\mathrm{k}}} \, \mathrm{d}\tau \tag{4}$$

if the FOS (1) is relaxed at $t = 0$, the transfer functions of the system, in which the fault and disturbance are as inputs and the output of the system considered as output are as (5) and (6), respectively [49]:

$$G_{yf}(s) = C(S^{\alpha} I - A)^{-1} B_f + D_f \tag{5}$$

$$G_{yd}(s) = C(S^{\alpha} I - A)^{-1} B_d + D_d \tag{6}$$

according to Figure 1, after determining the dynamical equations of the system with the fault, the next important step is to define a stable observer for the system (1). For this purpose, the observer F has been designed as (7).

$$F: \begin{cases} D^{\alpha} \hat{x}(t) = A \hat{x}(t) + L \; r(t) \\ \hat{y}(t) = C \hat{x}(t) \\ r(t) = y(t) - \hat{y}(t) \\ \hat{x}(t) = \varphi(t) \qquad t \in [-h_2, 0] \end{cases} \tag{7}$$

Where $\hat{x}(t) \in \mathbb{R}^n$ denotes the detection observer state vector, $\hat{y}(t) \in \mathbb{R}^r$ represents the output estimation vectors, $r(t) \in \mathbb{R}^r$ is residual, and $L \in \mathbb{R}^{n \times r}$ is the observer gain.

By the combination of the filter (7), the system (1) and considering $e(t) = x(t) - \hat{x}(t)$ the following augmented FOS is obtained:

$$\begin{cases} D^{\alpha} e(t) = (A - LC)e(t) + (B_d - LD_d)d(t) + \left(B_f - LD_f\right)f(t) \\ r(t) = Ce(t) + D_d d(t) + D_f f(t) \end{cases} \tag{8}$$

To have a robust FD system, the design should be carried out in such a way that the following conditions are established:

- The observer (7) must be designed such that asymptotically stability of the augmented system (8) is guaranteed. To achieve this condition, $\left|Arg\left(spec\big((A - LC)\big)\right)\right| > \alpha \frac{\pi}{2}$, where spec $((A - LC))$ is the set of eigenvalues of $(A - LC)$ or there exist $P > 0$ and $Q > 0$ such that $sym(rAP + \bar{r}AQ) < 0$ where $r = e^{j(1-\alpha)\frac{\pi}{2}}$ is asymptotically stable [50]. Figure 2 shows the stability region for this system.
- Robustness to disturbance input is one of the main design points of the FD. By using H_{∞} optimization criteria, this performance index expressed as (9) [51].

$$sup \frac{\|r(t)\|_2}{\|d(t)\|_2} < \gamma, \gamma > 0 \tag{9}$$

- Robust control by H_{-} optimizations criteria is the best idea for solving system sensitivity to faults. Performance index (10) guarantees the residual's sensitivity to faults, which is expressed as (10) [52].

$$sup \frac{\|r(t)\|_2}{\|f(t)\|_2} > \beta, \beta > 0 \tag{10}$$

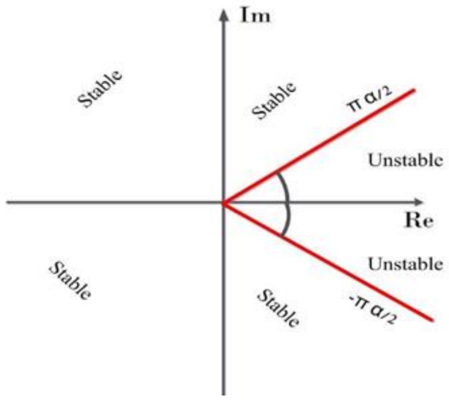

Figure 2. Stable region illustration

Based on the three above assumptions, mixing H_∞/H_- is the proposed method in this work for FDI design. The following definitions and lemmas are used for implementing the proposed method. Definition 1. [53] The H_∞ norm of $G_{yd}(s)$ for FOS (1) is defined as (11).

$$\|G_{yd}\|\_(H\_\infty) \triangleq \sup_{Re(s)\geq 0} \sigma_{max}(G_{yd}(s)) \tag{11}$$

Lemma 1: *(H-BR):* [54] Consider the FOS (1) and $G_{yu}(s) = C(S^{\alpha}I - A)^{-1}B + D$ then $\|G_{yu}(s)\|_{H\infty} < \gamma$ if only if there exist $P > 0$ and $Q > 0$ such that:

$$\begin{bmatrix} sym(AX) & * & * \\ CX & -\gamma I & * \\ B^T & D^T & -\gamma I \end{bmatrix} < 0 \tag{12}$$

where $X = \begin{cases} e^{j\theta}P + e^{-j\theta}Q, & if\ 0 < \alpha < 1 \\ e^{j\theta} \quad if\ 1 \leq \alpha < 1 \end{cases}$ $\theta = \frac{\pi}{2}(1-\alpha)$

Lemma 2*:* [55] Let matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $\Phi \in H_2$ $\Theta \in H_{(n+m)}$ and $\psi \in H_2$. Set Λ is defined as (13).

$$\Lambda(\Phi,\Psi) \triangleq \left\{\lambda \in C \left| \begin{bmatrix}\lambda \\ I\end{bmatrix}^* \Phi \begin{bmatrix}\lambda \\ I\end{bmatrix} = 0, \begin{bmatrix}\lambda \\ I\end{bmatrix}^* \Psi \begin{bmatrix}\lambda \\ I\end{bmatrix} \geq 0 \right.\right\} \tag{13}$$

For $H(\lambda) \triangleq (\lambda I_n - A)^{-1}$, there holds:

$$\begin{bmatrix}H(\lambda) \\ I_m\end{bmatrix}^* \Theta \begin{bmatrix}H(\lambda) \\ I_m\end{bmatrix} < 0, \quad \forall\lambda \in \Lambda \tag{14}$$

there exist $P, Q \in H_n$ and $Q > 0$ such that:

$$\begin{bmatrix}A & B \\ I_n & 0\end{bmatrix}^* (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix}A & B \\ I_n & 0\end{bmatrix} + \Theta < 0 \tag{15}$$

then "$(15) \Longrightarrow (14)$". Furthermore, if Λ represents a curve in the complex plane, then holds"$(15) \Longleftrightarrow (14)$".
Lemma 3*:* [55] The set $\Lambda(\Phi,\Psi)$ is defined as:

$$\Lambda(\Phi,\Psi) \triangleq \left\{\lambda \in C \left| \begin{bmatrix}\lambda \\ I\end{bmatrix}^* \Phi \begin{bmatrix}\lambda \\ I\end{bmatrix} \geq 0, \begin{bmatrix}\lambda \\ I\end{bmatrix}^* \Psi \begin{bmatrix}\lambda \\ I\end{bmatrix} \geq 0 \right.\right\} \tag{16}$$

if matrices $P > 0$ and $Q > 0$ exist such that LMI condition (15) holds, then condition (14) holds $\forall\lambda \in \Lambda$.
Lemma 4: (Projection lemma) [56]. Unstructured matrix X satisfies the following equations if a symmetric matrix $Z \in S_m$ and column dimension m, U and V matrices exist:

$$U^T X V + V^T X^T U + Z < 0 \quad (17)$$

if and only if:

$$N_U^T Z N_U < 0 \quad (18)$$

and

$$N_V^T Z N_V < 0 \quad (19)$$

concerning to X are satisfied. Where N_U and N_V are arbitrary matrices, whose columns form a basis of the null spaces of U and V, respectively.
Lemma 5: [57]. The FOS G(s)is stable if and only if $\|G(s)\|_{H\infty}$ is bounded.

3. MAIN RESULTS

In this section, conditions ii and iii are transformed into LMIs. Corollary 1 unifies the theorems. Theorem 1. The system (8) is stable, and the performance indices (9) is guaranteed, if there exist positive scalar γ positive definite symmetric matrices $P_{1,}\ Q_1$ and matrices $X,\ N$ such that the following LMIs hold:

$$\begin{bmatrix} Her(\Pi) + C^T C & \Xi_2 & \Omega + C^T D_d \\ * & -\lambda(X + X^T) & \lambda\Omega \\ * & * & D_d^T D_d - \gamma^2\ I \end{bmatrix} < 0 \quad (20)$$

where

$$\Pi = A^T X - C^T N^T, \Omega = X^T B_d - N D_d\ , \lambda > 0, \Xi_2 = \lambda\Pi - X^T + \overline{r} P_1 + r Q_1, r = e^{j\theta}, \theta = (1-\alpha)\frac{\pi}{2}.$$

holds and the filter gain L is obtained:

$$L = X^{-T} N \quad (21)$$

Proof: Based on definition 1:

$$\|G(s)_{rd}\|_{H\infty} \triangleq \sup_{Re(s)\geq 0} \overline{\sigma}\,(G(s)_{rd}) \ = \sup_{Re(s)\geq 0} \overline{\sigma}\,(C\big(S^\alpha I - \tilde{A}\big)^{-1}\tilde{B}_d + D_d) \quad (22)$$

where

$$\begin{aligned} \tilde{A} &= A - LC \\ \tilde{B}_d &= B_d - L D_d \end{aligned} \quad (23)$$

by some basic matrix calculations:

$$\left\|G(s)_{rd}\right\|_{H_\infty} < \gamma \Leftrightarrow G(s)_{rd} G(s)_{rd}^* - \gamma^2 I < 0 \ \ \forall\, Re(s) \geq 0 \Leftrightarrow \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix}^* \Theta \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix} < 0,\ \forall \lambda \in \Lambda \quad (24)$$

where $H(\lambda) \triangleq \big(\lambda I_n - \tilde{A}\big)^{-1}, \lambda = S^\alpha$, and $\Lambda(\Phi, \Psi)$ is defined in (13), also:

$$\Theta = \begin{bmatrix} C^T C & C^T D_d \\ D_d^T C & D_d^T D_d - \gamma^2 I \end{bmatrix} \quad (25)$$

then according to Lemma 2, the last part of (24) is also equivalent to the statement that $\exists\, P_1,\ Q_1 \in H_n$, $P_1 > 0$ and $Q_1 > 0$ such that the LMI (24) holds.

$$\begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_d \end{bmatrix}^T (\Phi \otimes P_1 + \Psi \otimes Q_1) \begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_d \end{bmatrix} + \Theta < 0 \quad (26)$$

similar to [54],

$$\Phi = \begin{bmatrix} 0 & \bar{r} \\ r & 0 \end{bmatrix} \qquad \Psi = \begin{bmatrix} 0 & r \\ \bar{r} & 0 \end{bmatrix} \tag{27}$$

now the inequality (26) can be reformulated as $N_U^T Z N_U < 0$ where N_U and Z are given by:

$$Z = \begin{bmatrix} C^T C & \bar{r}P_1 + rQ_1 & C^T D_d \\ rP_1 + \bar{r}Q_1 & 0 & 0 \\ D_d{}^T C & 0 & D_d^T D_d - \gamma^2 I \end{bmatrix} \qquad N_U = \begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_d \\ 0 & I \end{bmatrix} \tag{28}$$

by defining the matrices N_V and V as (29).

$$N_V = \begin{bmatrix} \lambda I & 0 \\ -I & 0 \\ 0 & I \end{bmatrix} \rightarrow V = [I \quad \lambda I \quad 0] \tag{29}$$

It can be obtained by Lemma 4 that inequality $N_U^T Z N_U < 0$ is equivalent to:

$$Z + \begin{bmatrix} \tilde{A}^T \\ -I \\ \tilde{B}_d \end{bmatrix} [X \quad \lambda X \quad 0] + \begin{bmatrix} X^T \\ \lambda X^T \\ 0 \end{bmatrix} [\tilde{A}^T \quad -I \quad \tilde{B}_d] < 0 \tag{30}$$

Now by substituting $N = X^T L$ inequality (20) is obtained, and the proof is completed.
Theorem 2. The augmented fractional-order system (8) is stable and it guarantees the performance index (10), if there exist positive scalar $\beta > 0$ and symmetric matrices P_2, Q_2 and matrices X, N such that the following LMI:

$$\begin{bmatrix} Her(\Pi) - C^T C & \Xi_2 & \Omega - C^T D_f \\ * & \lambda(X - X^T) & \lambda\Omega \\ * & * & -D_f^T D_f + \beta^2 I \end{bmatrix} < 0 \tag{31}$$

where

$$\Pi = A^T X - C^T N^T, \Omega = X^T B_f - ND_f \quad , \lambda > 0 \ , \Xi_2 = \lambda\Pi - X^T + \bar{r}P_2 + rQ_2, \ r = e^{j\theta}, \theta = (1-\alpha)\frac{\pi}{2}.$$

The filter gain L is given by (21).
Proof: Although the principles of proving this theorem are very similar to that Theorem 1, since it contains small and essential points, the proof of this theorem is fully addressed.
Based on definition 1:

$$\|G(s)_{rf}\|_{H_\infty} \triangleq \sup_{Re(s)\geq 0} \bar{\sigma}\left(G(s)_{rf}\right) = \sup_{Re(s)\geq 0} \bar{\sigma}\left(C\left(S^\alpha I - \tilde{A}\right)^{-1}\tilde{B}_f + D_f\right) \tag{32}$$

where $\tilde{B}_f = B_f - LD_f$,
By analyzing $\left\|G(s)_{rf}\right\|_{H_\infty}$:

$$\left\|G(s)_{rf}\right\|_{H_\infty} < \gamma \Leftrightarrow G(s)_{rf}G(s)_{rf}^* - \gamma^2 I < 0 \ \ \forall Re(s) \geq 0 \Leftrightarrow \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix}^* \Theta \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix} < 0 \ . \forall\lambda \in \Lambda \tag{33}$$

where $H(\lambda) \triangleq \left(\lambda I_n - \tilde{A}\right)^{-1}\tilde{B}_f$ and $\Lambda(\Phi, \Psi)$ is defined in (11), also:

$$\Theta = \begin{bmatrix} -C^T C & -C^T D_f \\ -D_f^T C & D_f^T D_f - \beta^2 I \end{bmatrix} \tag{34}$$

then according to Lemma 3, the last part of (29) is also equivalent to the statement that $\exists\ P_2,\ Q_2 \in H_n,\ P_2 > 0$ and $Q_1 > 0$ such that the LMI (29) holds.

$$\begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_f \end{bmatrix}^T (\Phi \otimes P_2 + \Psi \otimes Q_2) \begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_f \end{bmatrix} + \Theta < 0 \tag{35}$$

similar to [54],

$$\begin{aligned} \Phi &= \begin{bmatrix} 0 & \bar{r} \\ r & 0 \end{bmatrix} \\ \Psi &= \begin{bmatrix} 0 & r \\ \bar{r} & 0 \end{bmatrix} \end{aligned} \tag{36}$$

now the inequality (29) can be reformulated as $N_U^T Z N_U < 0$ where N_U and Z are given by:

$$\begin{aligned} Z &= \begin{bmatrix} -C^T C & \bar{r}P_2 + rQ_2 & -C^T D_f \\ rP_1 + \bar{r}Q_1 & 0 & 0 \\ -D_f^T C & 0 & D_f^T D_f - \beta^2 I \end{bmatrix} \\ N_U &= \begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_f \\ 0 & I \end{bmatrix} \end{aligned} \tag{37}$$

by defining the matrices N_V and V as (38).

$$N_V = \begin{bmatrix} \lambda I & 0 \\ -I & 0 \\ 0 & I \end{bmatrix} \rightarrow V = \begin{bmatrix} I & \lambda I & 0 \end{bmatrix} \tag{38}$$

It can be obtained by Lemma 4 that inequality $N_U^T Z N_U < 0$ is equivalent to:

$$Z + \begin{bmatrix} \tilde{A}^T \\ -I \\ \tilde{B}_f^T \end{bmatrix} \begin{bmatrix} X & \lambda X & 0 \end{bmatrix} + \begin{bmatrix} X^T \\ \lambda X^T \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{A}^T & -I & \tilde{B}_f^T \end{bmatrix} < 0 \tag{39}$$

Now by substituting $N = X^T L$ inequality (31) is obtained, and the proof is completed.
Corollary 1. Solving the following convex optimization problem, results a feasible solution to the multi-objective H_-/H_∞ problem (fault detection problem) for Given:

$$\begin{aligned} &\max \quad \beta \\ &X.P_1, P_2.Q_1.Q_2.N \\ &s.t \quad (20).(31). \end{aligned} \tag{40}$$

Proof: By collecting the theorems 1 and 2, the proof is completed.
Remark 3: In this work, the residual evaluation function is defined as [52]:

$$J(t) = \left(\theta^{-1} \int_0^\theta r^T(s) r(s) ds\right)^{1/2} \tag{41}$$

where θ represents the detection time range. The upper threshold values are calculated as:

$$J_{th} = \sup_{\substack{f(t)=0 \\ d(t)\in L_2}} J(t) \tag{42}$$

4. SIMULATION EXAMPLES

The model considered for validating the results proved in this article is the testing bench that described in [24]. As can be seen in Figure 3, this type of test bench made up of two long aluminum rods which are glued together with the heat paste. The input of this model, which is a single input multi output system is voltage $V_r(t)$ and its outputs are $\theta_1,\ \theta_2, \theta_3\ and\ \theta_4$. Rod 1 thermal behavior is as (43) and (44).

$$\theta_1 = \mathcal{L}^{-1}[G_{13}] * \theta_3 \tag{43}$$

$$\theta_3 = \mathcal{L}^{-1}[G_{V3}] * V_r(t) \tag{44}$$

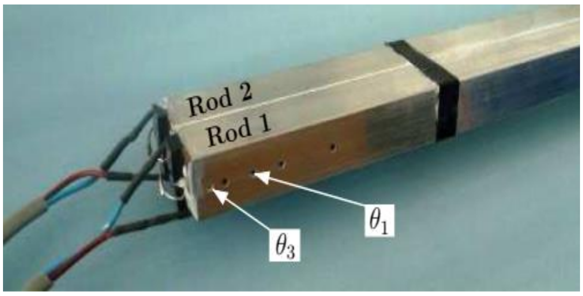

Figure 3. Two rods thermal bench

Rod 2 thermal behavior is as (45) and (46).

$$\theta_2 = \mathcal{L}^{-1}[G_{42}] * \theta_4 \tag{45}$$

$$\theta_4 = \mathcal{L}^{-1}[G_{V4}] * V_r(t) \tag{46}$$

Where numerical values of identified transfer functions are:

$$G_{V3}(s) = \frac{-348.5s^{0.5}+319.9}{1840s^{1.5}-130s+485.7s^{0.5}+1} = \frac{b_1^{V3}s^{0.5}+b_0^{V3}}{a_3^{V3}s^{1.5}+a_2^{V3}s+a_1^{V3}s^{0.5}+a_0^{V3}} \tag{47}$$

$$G_{31}(s) = \frac{1.1587}{2.367s^{0.5}+1} - 0.1599 = \frac{b_0^{31}}{a_1^{31}s^{0.5}+a_0^{31}} + D_{31} \tag{48}$$

$$G_{V4}(s) = \frac{-299.5s^{0.5}+260.7}{6472s^{1.5}-300.2s+453.2s^{0.5}+1} = \frac{b_1^{V4}s^{0.5}+b_0^{V4}}{a_3^{V4}s^{1.5}+a_2^{V4}s+a_1^{V4}s^{0.5}+a_0^{V4}} \tag{49}$$

$$G_{42}(s) = \frac{1.6361}{0.5415s^{0.5}+1} - 0.6372 = \frac{b_0^{42}}{a_1^{42}s^{0.5}+a_0^{42}} + D_{42} \tag{50}$$

Observability forms of fractional order transmittances (47)-(50) are:

$$\begin{cases} D^\alpha \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \theta_3(t) \end{bmatrix} = A_{V3} \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \theta_3(t) \end{bmatrix} + B_{V3}V_r(t) \\ \theta_3(t) = C_{V3} \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \theta_3(t) \end{bmatrix} \end{cases} \tag{51}$$

$$\begin{cases} D^\alpha \begin{bmatrix} \zeta_4(t) \\ \zeta_5(t) \\ \theta_4(t) \end{bmatrix} = A_{V4} \begin{bmatrix} \zeta_4(t) \\ \zeta_5(t) \\ \theta_4(t) \end{bmatrix} + B_{V4}V_r(t) \\ \theta_4(t) = C_{V4} \begin{bmatrix} \zeta_4(t) \\ \zeta_5(t) \\ \theta_4(t) \end{bmatrix} \end{cases} \tag{52}$$

$$\begin{cases} D^{\alpha}\zeta_3(t) = \frac{-a_0^{31}}{a_1^{31}}\zeta_3(t) + \frac{b_0^{31}}{a_1^{31}}\theta_3(t) = A_{31}\zeta_3(t) + B_{31}\theta_3(t) \\ \theta_1(t) = \zeta_3(t) + D_{31}\theta_3(t) \end{cases} \tag{53}$$

$$\begin{cases} D^{\alpha}\zeta_6(t) = \frac{-a_0^{42}}{a_1^{42}}\zeta_6(t) + \frac{b_0^{42}}{a_1^{42}}\theta_4(t) = A_{42}\zeta_6(t) + B_{42}\theta_4(t) \\ \theta_2(t) = \zeta_6(t) + D_{42}\theta_4(t) \end{cases} \tag{54}$$

where

$$A_{Vi} = \begin{bmatrix} 0 & 0 & \frac{-a_0^{Vi}}{a_3^{Vi}} \\ 1 & 0 & \frac{-a_1^{Vi}}{a_3^{Vi}} \\ 0 & 1 & \frac{-a_1^{Vi}}{a_3^{Vi}} \end{bmatrix}, \quad B_{Vi} = \begin{bmatrix} \frac{b_0^{Vi}}{b_3^{Vi}} \\ \frac{b_1^{Vi}}{b_3^{Vi}} \\ 0 \end{bmatrix}, \quad C_{Vi} = [0 \quad 0 \quad 1]; \qquad i = 3,4. \tag{55}$$

the pseudo state space description of this SIMO system is as (56):

$$\begin{cases} D^{\alpha}\, x(t) = A\, x(t) + B\, u(t) \\ y(t) = C\, x(t) + D\, u(t) \end{cases} \tag{56}$$

where

$$A = \begin{bmatrix} A_{V3} & 0 & 0 & 0 \\ B_{31}C_{V3} & A_{31} & 0 & 0 \\ 0 & 0 & A_{V4} & 0 \\ 0 & 0 & B_{42}C_{V4} & A_{42} \end{bmatrix}, \qquad B = \begin{bmatrix} B_{V3} \\ 0 \\ B_{V4} \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} D_{31}C_{V3} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C_{V3} & 0 & D_{41}C_{V4} & 1 \\ 0 & 0 & C_{V4} & 0 \end{bmatrix}, \quad D = 0.$$

$$x(t) = [\zeta_1(t)\ \ \zeta_2(t)\ \ \theta_3(t)\ \ \zeta_3(t)\ \ \zeta_4(t)\ \ \zeta_5(t)\ \ \theta_4(t)\ \ \zeta_6(t)]$$

$$y(t) = [\theta_1(t) \quad \theta_2(t) \quad \theta_3(t) \quad \theta_4(t)]$$

Consider the FOS (1) with the following parameters:

$$A = \begin{bmatrix} 0 & 0 & -0.0005 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -0.2640 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.0707 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4895 & -0.4225 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.0002 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -0.0700 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.0464 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3.0214 & -1.8467 \end{bmatrix}. B = 0, \quad B_d = \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$D = 0, \quad D_d = \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, D_f = \begin{bmatrix} 0.2 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \ C = \begin{bmatrix} 0 & 0 & -0.1599 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6372 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$d(t)$= be $0.5\exp(-0.4t)\cos(0.7\pi t)\,u(t)$.

The fault signal $f(t)$ is simulated as a square wave of unit amplitude from 40 to 60 steps. For a given $\gamma = 0.0328$, we solved the optimization problem Corollary 1 by YALMIP toolbox in Matlab and β is obtained as 71.8540. Furthermore, the observer gains were obtained as:

$$L = \begin{bmatrix} 1.0011 & 0 & 2.1735 & 0 \\ 0.0012 & 0 & 3.2501 & 0 \\ -0.0009 & 0 & 2.9924 & 0 \\ -0.0040 & 0 & 0.2422 & 0 \\ 0 & 0.0010 & 0 & 0.9972 \\ 0 & 0.0002 & 0 & 2.5353 \\ 0 & 0.0043 & 0 & 2.6462 \\ 0 & -0.8542 & 0 & 3.5680 \end{bmatrix}$$

Real and estimated outputs are represented in Figures 4-7. The threshold values of residual signal r_1 and r_3 computed by (42) as $J_{thr1} = 0.0908$ and $J_{thr3} = -0.00021620$. Residuals in the faulty cases are shown in Figures 8 and 9. It can be concluded that the robustness against disturbance and the fault sensitivity are both amplified, and the fault is well separated from disturbance.

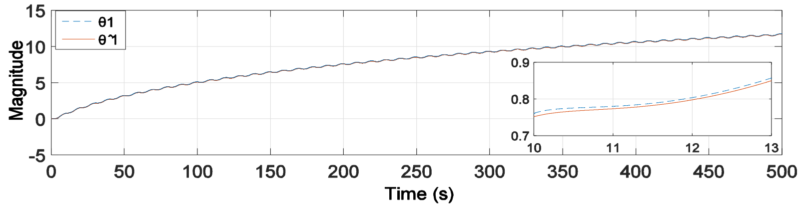

Figure 4. The output $\boldsymbol{\theta_1}$ and its estimate

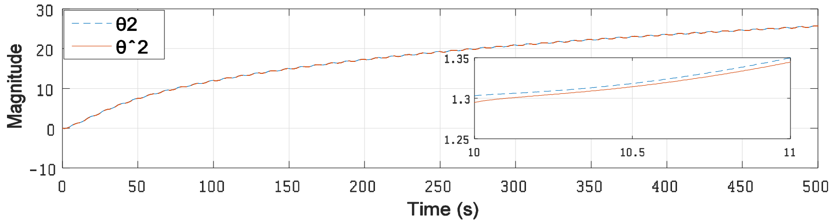

Figure 5. The output θ_2 and its estimate

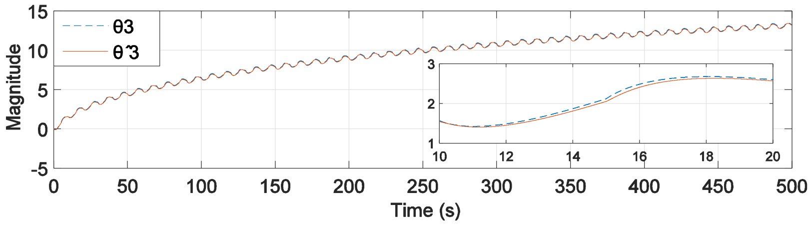

Figure 6. The output $\boldsymbol{\theta_3}$ and its estimate

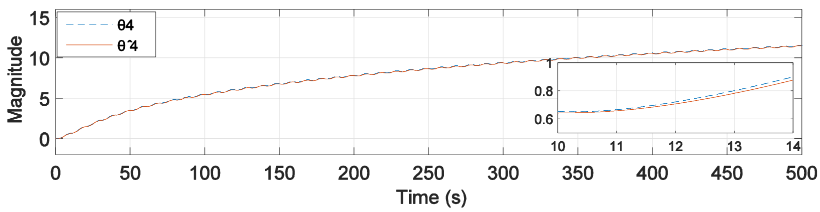

Figure 7. The output $\boldsymbol{\theta_4}$ and its estimate

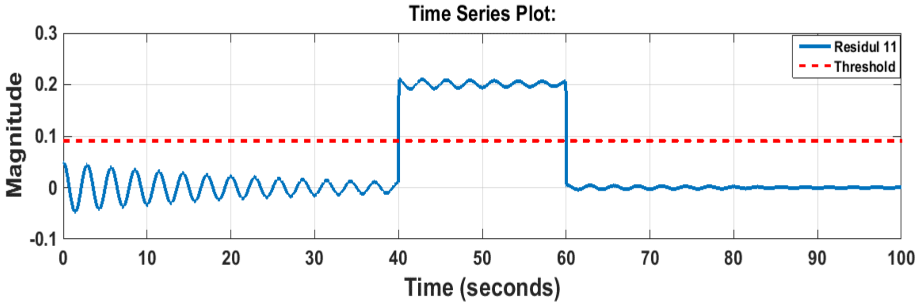

Figure 8. Residual signal $\boldsymbol{r_1}$

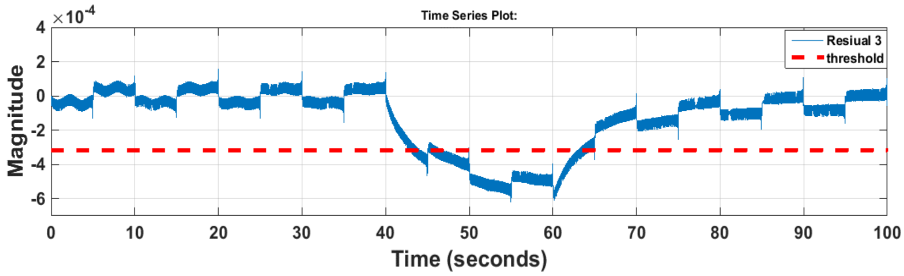

Figure 9. Residual signal $\boldsymbol{r_3}$

Aribi *et al.* [24] performed the simulation with the Luenberger diagnosis observer method without considering the effect of disturbance. Also, the overshot at the beginning is a big drawback of that method. Their other methods, generalized dynamic parity space, are very good, although they do not introduce a disturbance signal.

5. CONCLUSION

In this work, the robust fault detection problem has been investigated for a linear time-invariant fractional-order system in the simultaneous presence of sensor, actuator and process faults as well as input and output disturbances. The core of this study is the formulation of the FD design problem as the mixed H_∞/H_- robust optimization problem to satisfy the fault sensitivity and disturbance attenuation. Furthermore, the linear matrix inequality approach has been introduced to warrant stability and the two multi-objective H_∞/H_- performances. Finally, the effectiveness of the proposed theory is validated via numerical results. In future work, this problem will be solved by considering other algorithms.

REFERENCES

[1] K. B. Oldham and J. Spanier, *The Fractional Calculus: Integrations and Differentiations of Arbitrary Order*, New York: Academic Press, 1974.

[2] M. W. Hasan and N. H. Abbas, "An improved swarm intelligence algorithms-based nonlinear fractional order-PID controller for a trajectory tracking of underwater vehicles," *TELKOMNIKA Telecommunication Computing Electronics and Control*, vol. 18, no. 6, pp. 3173-3183, 2020, doi: 10.12928/telkomnika. v18i6.16282.

[3] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, vol. 204, 1st ed., Elsevier B.V. 2006.

[4] M. Moussa, M. Hamila, and A. Douik, "Face recognition using fractional coefficients and discrete cosine transform tool," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 11, no. 1, pp. 892-899, February 2021, doi: 10.11591/ijece.v11i1.pp892-899.

[5] Z. Nawawi *et al.*, "Fractal analysis of electrical tree grown in silicone rubber nanocomposites," *TELKOMNIKA Telecommunication Computing Electronics and Control*, vol. 18, no. 3, pp. 1573-1581, June 2020, doi: 10.12928/telkomnika.v18i3.13389.

[6] G. Besançon, G. Becq, and A. Voda, "Fractional-Order Modeling and Identification for a Phantom EEG System," in *IEEE Transactions on Control Systems Technology*, vol. 28, no. 1, pp. 130-138, Jan. 2020, doi: 10.1109/TCST.2019.2891621.

[7] Z. Gu, F. Li, F. Fang, and G. Zhang, "A Novel Retinex-Based Fractional-Order Variational Model for Images with Severely Low Light," in *IEEE Transactions on Image Processing*, vol. 29, pp. 3239-3253, 2020, doi: 10.1109/TIP.2019.2958144.

[8] M. F. Farayola, S. Shafie, F. M. Siam, and I. Khan, "Mathematical modeling of radiotherapy cancer treatment using Caputo fractional derivative," *Computer Methods and Programs in Biomedicine*, vol. 188, May 2020, doi: 10.1016/j.cmpb.2019.105306.

[9] R. Trivedi and P. K. Padhy, "Design of Indirect Fractional Order IMC Controller for Fractional Order Processes," in *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 68, no. 3, pp. 968-972, March 2021, doi: 10.1109/TCSII.2020.3013404.

[10] X. Su, W. Xu, W. Chen, and H. Yang, "Fractional creep and relaxation models of viscoelastic materials via a non-Newtonian time-varying viscosity: physical interpretation," *Mechanics of Materials*, vol. 140, January 2020, doi: 10.1016/j.mechmat.2019.103222.

[11] C. Yang, N. Verbeek, F. Xia, Y. Wang, and K. Youcef-Toumi, "Modeling and Control of Piezoelectric Hysteresis: A Polynomial-Based Fractional Order Disturbance Compensation Approach," in *IEEE Transactions on Industrial Electronics*, vol. 68, no. 4, pp. 3348-3358, April 2021, doi: 10.1109/TIE.2020.2977567.

[12] M. D. Paola and M. Zingales, "Exact mechanical models of fractional hereditary materials," *Journal of Rheology* vol. 56, no. 5, 2012, doi: 10.1122/1.4717492.

[13] W.-T. Geng, C. Lin, and B. Chen, "Observer-based stabilizing control for fractional-order systems with input delay," *ISA Transactions*, vol. 100, pp. 103-108, May 2020, doi: 10.1016/j.isatra.2019.11.026.

[14] Y. Gong, G. Wen, Z. Peng, T. Huang, and Y. Chen, "Observer-Based Time-Varying Formation Control of Fractional-Order Multi-Agent Systems with General Linear Dynamics," in *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 1, pp. 82-86, Jan. 2020, doi: 10.1109/TCSII.2019.2899063.

[15] S. S. Majidabad, H. T. Shandiz, A. Hajizadeh, and H. Tohidi, "Robust Block Control of Fractional-order Systems via Nonlinear Sliding Mode Techniques," *Journal of Control Engineering and Applied Informatics*, vol. 17, no. 1, pp. 31-40, 2015.

[16] C. Komathi and M. G. Umamaheswari, "Design of Gray Wolf Optimizer Algorithm-Based Fractional Order PI Controller for Power Factor Correction in SMPS Applications," in *IEEE Transactions on Power Electronics*, vol. 35, no. 2, pp. 2100-2118, Feb. 2020, doi: 10.1109/TPEL.2019.2920971.

[17] V. A. Kumar and A. Mouttou, "Improved performance with fractional order control for asymmetrical cascaded H-bridge multilevel inverter," *Bulletin of Electrical Engineering and Informatics*, vol. 9, no. 4, pp. 1335-1344, 2020, doi: 10.11591/eei.v9i4.1885.

[18] M. W. Hasan and N. H. Abbas, "An improved swarm intelligence algorithms-based nonlinear fractional order-PID controller for a trajectory tracking of underwater vehicles," *TELKOMNIKA Telecommunication Computing Electronics and Control*, vol. 18, no. 6, pp. 3173-3183, December 2020, doi: 10.12928/telkomnika.v18i6.16282.

[19] B. Aguiar, T. González, and M. Bernal, "A Way to Exploit the Fractional Stability Domain for Robust Chaos Suppression and Synchronization via LMIs," in *IEEE Transactions on Automatic Control*, vol. 61, no. 10, pp. 2796-2807, Oct. 2016, doi: 10.1109/TAC.2015.2499963.

[20] H. T. Tuan and H. Trinh, "A Linearized Stability Theorem for Nonlinear Delay Fractional Differential Equations," in *IEEE Transactions on Automatic Control*, vol. 63, no. 9, pp. 3180-3186, Sept. 2018, doi: 10.1109/TAC.2018.2791485.

[21] S. S. Majidabad, H. T. Shandiz, and A. Hajizadeh, "Nonlinear fractional-order power system stabilizer for multi-machine power systems based on sliding mode technique," *International Journal of Robust And Nonlinear Control*, vol. 25, no. 10, pp. 1548-1568, 2014, doi: 10.1002/rnc.3159

[22] N. Sene, "Stability analysis of the generalized fractional differential equations with and without exogenous inputs," *Journal of Nonlinear Sciences and Applications (JNSA)*, vol. 12, no. 9, pp. 562-572**,** April 12, 2019, doi: 10.22436/jnsa.012.09.01

[23] A. Aribi, M. Aoun, S. Najar, and M. N. Abdelkrim, "On the Fractional Systems' Fault Detection: Evaluation of Fractional Residual," *International Journal of Computer Applications*, vol. 45, no. 17, pp. 37-43, May 2012, doi: 10.5120/7005-9574. [Online]. Available: https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.674.9944&rep=rep1 &type=pdf

[24] A. Aribi, C. Farges, M. Aoun, P. Melchior, S. Najar, and M. N. Abdelkrim, "Fault detection based on fractional order models: Application to diagnosis of thermal systems," *Communications in Nonlinear Science Numerical Simulation*, vol. 19, no. 10, pp. 3679-3693, 2014, doi: 10.1016/j.cnsns.2014.03.006.

[25] H. Shen, X. Song, and Z. Wang, "Robust fault-tolerant control of uncertain fractional-order systems against actuator faults," *IET Control Theory and Applications*, vol. 7, no. 9, pp. 1233-1241, 2013, doi: 10.1049/iet-cta.2012.0822.

[26] W. Zhong, J. Lu, and Y. Miao, "Fault detection observer design for fractional-order systems," *2017 29th Chinese Control and Decision Conference (CCDC)*, 2017, pp. 2796-2801, doi: 10.1109/CCDC.2017.7978988.

[27] D. Mohapatra, B. Subudhi, and R. Daniel, "Real-time sensor fault detection in Tokamak using different machine learning algorithms," *Fusion Engineering and Design*, vol. 151, February 2020, doi: 10.1016/ j.fusengdes.2019.111401.

[28] S. Thakral and D. Bansal, "Novel high functionality fault tolerant ALU," *TELKOMNIKA Telecommunication Computing Electronics and Control*, vol. 18, no. 1, pp. 234-239, February 2020, doi: 10.12928/ telkomnika.v18i1.12645.

[29] C. D. Tran, P. Brandstetter, M. C. H. Nguyen, S. D. Ho, B. H. Dinh, and P. N. Pham, "A robust diagnosis method for speed sensor fault based on stator currents in the RFOC induction motor drive," *International Journal of*

Electrical and Computer Engineering (IJECE), vol. 10, no. 3, pp. 3035-3046, June 2020, doi: 10.11591/ijece.v10i3.pp3035-3046.
[30] J. Hang, H. Wu, S. Ding, W. Hua, and Q. Wang, "A DC-Flux-Injection Method for Fault Diagnosis of High-Resistance Connection in Direct-Torque-Controlled PMSM Drive System," in *IEEE Transactions on Power Electronics*, vol. 35, no. 3, pp. 3029-3042, March 2020, doi: 10.1109/TPEL.2019.2924929.
[31] E. Khoudry, A. Belfqih, T. Ouaderhman, J. Boukherouaa, and F. Elmariami, "A real-time fault diagnosis system for high-speed power system protection based on machine learning algorithm," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 10, no. 6, pp. 6122-6138, 2020, doi: 10.11591/ ijece.v10i6.pp6122-6138.
[32] Y. Li and S. Zhou, "A New Instantaneous Wavelet Bicoherence for Local Fault Detection of Rotating Machinery," in *IEEE Transactions on Instrumentation and Measurement*, vol. 69, no. 1, pp. 135-143, Jan. 2020, doi: 10.1109/TIM.2019.2895483.
[33] C. Jittawiriyanukoon, "Approximation of regression-based fault minimization for network traffic," *TELKOMNIKA Telecommunication Computing Electronics and Control*, vol. 18, no. 4, pp. 1802-1808, August 2020, doi: http://dx.doi.org/10.12928/telkomnika.v18i4.13192.
[34] Y. Zafari, A. H. Mazinan, and S. S. Majidabad, "Demagnetization Fault Detection for Five-Phase IPMSM Through Integral Terminal Sliding Mode Flux-Linkage Observer," *IETE Journal of Research*, vol. 65, no. 4, pp. 473-486, 2019, doi: 10.1080/03772063.2018.1436474.
[35] A. Andriamamonjy, D. Saelens, and R. Klein, "An auto-deployed model-based fault detection and diagnosis approach for Air Handling Units using BIM and Modelica," *Automation in Construction*, vol. 96, pp. 508-526, December 2018, doi: 10.1016/j.autcon.2018.09.016.
[36] J. L. Wang, G. H. Yang, and J. Liu, "An LMI approach to H- index and mixed H-/H∞ fault detection observer design," *Automatica (Journal of IFAC)*, vol. 43, no. 9, pp. 1656-1665, 2007, doi: 10.1016/j.automatica.2007.02.019.
[37] C. Yan, J. Chen, H. Liu, and H. Lu, "Model-Based Fault Tolerant Control for the Thermal Management of PEMFC Systems," in *IEEE Transactions on Industrial Electronics*, vol. 67, no. 4, pp. 2875-2884, April 2020, doi: 10.1109/TIE.2019.2912772.
[38] M. Zhong, S. X. Ding, J. Lam, and H. Wang, "An LMI approach to design robust fault detection filter for uncertain LTI systems," *Automatica*, vol. 39, no. 3, pp. 543-550, March 2003, doi: 10.1016/S0005-1098(02)00269-8.
[39] Z. Li, E. Mazars, Z. Zhang, and I. Jaimoukha, "State-space solution to the H_∞/H_-fault detection problem," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 3, pp. 282-299, 2011, doi: 10.1002/rnc.1690.
[40] X. Wei and M. Verhaegen, "Mixed H2212/H∞ fault detection observer design for LPV systems," *2008 47th IEEE Conference on Decision and Control,* 2008, pp. 1073-1078, doi: 10.1109/CDC.2008.4739360.
[41] M. Davoodi, N. Meskin, and K. Khorasani, "Simultaneous fault detection and consensus control design for a network of multi-agent systems," *Automatica,* vol. 66, pp. 185-194, April 2016, doi: 10.1016/j.automatica.2015.12.027.
[42] H. S.-Nejad, A. R. Ghiasi, and M. A. Badamchizadeh, "Robust simultaneous finite-time control and fault detection for uncertain linear switched systems with time-varying delay," *IET Control Theory and Applications*, vol. 11, no. 7, pp. 1041-1052, 2017, doi: 10.1049/iet-cta.2016.1177.
[43] Y. Wang, R. Wang, X. Xie, and H. Zhang "Observer-based H∞ fuzzy control for modified repetitive control systems," *Neurocomputing*, vol. 286, pp. 141-149, 2018, doi: 10.1016/j.neucom.2018.01.064.
[44] N. A. Sofianos and O. I. Kosmidou, "Guaranteed cost LMI-based fuzzy controller design for discrete-time nonlinear systems with polytopic uncertainties," *18th Mediterranean Conference on Control and Automation, MED'10*, 2010, pp. 1383-1388, doi: 10.1109/MED.2010.5547855.
[45] M. Wakaiki, "An LMI Approach to Stability Analysis of Coupled Parabolic Systems," in *IEEE Transactions on Automatic Control*, vol. 65, no. 1, pp. 404-411, Jan. 2020, doi: 10.1109/TAC.2019.2916534.
[46] H. Jafari, J. Poshtan, and S. Shamaghdari, "LMI-based robust fault detection and isolation in linear stochastic systems," *IET Science, Measurement and Technology*, vol. 14, no. 5, pp. 593-599, 2020, doi: 10.1049/iet-smt.2019.0108.
[47] S. X. Ding, *Model-Based Fault Diagnosis Techniques, Design Schemes, Algorithms and Tools*, Second Edition, London, UK: Springer-Verlag London 2008, 2013, doi: 10.1007/978-1-4471-4799-2.
[48] R. Gorenflo and F. Mainardi, *Fractional Calculus*, In: A. Carpinteri, F. Mainardi (eds), *Fractals and Fractional Calculus in Continuum Mechanics: International Centre for Mechanical Sciences (Courses and Lectures)*, vol. 378. Vienna: Springer, 1997, doi: 10.1007/978-3-7091-2664-6_5.
[49] K. B. Oldham and J. Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, vol. 111, New York, USA: Dover Books on Mathematics, 2006.
[50] J. Sabatier, M. Moze, and C. Farges, "LMI stability conditions for fractional order systems," *Computers and Mathematics with Applications,* vol. 59, no. 5, pp. 1594-1609, March 2010, doi: 10.1016/j.camwa.2009.08.003.
[51] P. M. Frank and X. Ding, "Survey of robust residual generation and evaluation methods in observer-based fault detection systems," *Journal of Process Control*, vol. 7, no. 6, pp. 403-424, December 1997, doi: 10.1016/S0959-1524(97)00016-4.
[52] M. Hou and R. J. Patton, "An LMI Approach to H_∞/H_-Fault Detection Observers," *UKACC International Conference on CONTROL'96, 2-5September 1996, venue, University of Exeter, UK (English)*, vol. 1, no. 427, 1996, pp. 305-310. {Online]. Available: https://www.tib.eu/en/search/id/BLCP:CN015407417/An-LMI-approach-to-H-H%EF%BF%BD-fault-detection-observers?cHash=cbb146808d100448c34c4988f7aee459
[53] M. Green and D. J. N. Limebeer, *Linear robust control*, New Jersey 07632, (NJ, USA), USA: Prentice-Hall, 1995.

[54] S. Liang, Y.-H. Wei, J.-W. Pan, Q. Gao, and Y. Wang, "Bounded real lemmas for fractional order systems," *International Journal of Automation and Computing*, vol. 12, pp. 192-198, 2015, doi: 10.1007/ s11633-014-0868-4.
[55] T. Iwasaki and S. Hara, "Generalized KYP lemma: unified frequency domain inequalities with design applications," in *IEEE Transactions on Automatic Control*, vol. 50, no. 1, pp. 41-59, Jan. 2005, doi: 10.1109/TAC.2004.840475.
[56] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H∞ control," *Int. J. Robust Nonlinear Control*, vol. 4, no. 4, pp. 421-448, 1994, doi: 10.1002/rnc.4590040403.
[57] D. Matignon, "Stability properties for generalized fractional differential systems," *ESAIM: Proceedings*, vol. 5, 1998, pp. 145-158, doi: 10.1051/proc:1998004.

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