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Comparative and analysis of evaluation of several direction of arrival estimation methods

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ABSTRACT

Modern-day, global, and clever array systems have several distinguished approaches with regard to a wireless device. In this paper, a comparative analysis has been carried out amongst nine direction of arrival (DOA) algorithms: Capon, multiple signal classification (MUSIC), Bartlett, Pisarenko, linear prediction, maximum entropies, and min-norm, root-MUSIC, and estimation of signal parameters via rotational invariance technique (ESPRIT). in this paper reduces the number of antennas and snaps necessary to observe six target swithout interference for several DOA estimation methods. After comparing and analyzing all the algorithms, it has been shown that the music algorithm is the best algorithm that reduces interference and detects the desired sources. MATLAB R2019b was used in the simulation process.

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1. INTRODUCTION

Wireless networks faced ever-changing needs over their spectrum resources, such as increasing the users in the network, capacity-intensive applications. The constant increase in global wireless subscribers becomes a challenge for the wireless providers to discover a practical path To reduce wireless congestion in a single network [1], [2].To increase the users' capacity and reduce the interference, a device that transmits a low capacity must be used, and that deals with the same frequencies within the same network. Consequently, it is necessary to keep up with the latest technologies that provide the system with higher capacity and data rate. Subsequently, the invention of smart antenna techniques is the solution for such needs [3], [4].

Space division multiple access (SDMA) is a technology for developing smart antennas. The SDMA technology divides the space, which is determined with the same frequencies for a group of users located in the same geographical area [5], [6]. A spatially separating technique approves Intracellular channel reuse based on specific angle decisiveness. Digital signal processing technology is combined with several antenna elements to form smart antennas to automatically amelioration the radiation modality.

Depending on the environment, the signal is responded to [7], [8]. Algorithms process the system formation of a group of the antenna array to form a beam. The necessary work of the algorithm is to develop a beam to improve the beam diagram so that the strong radiating force in the direction of the desired signal generated, and the null is generated in the order of the unwanted signal as shown in Figure 1 [9], [10].

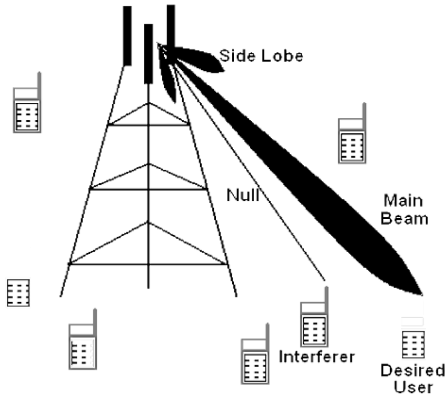

Figure 1. A smart antenna principle [11]

According to the beamforming technique, the directions of sources need to be tracked using a (DOA) estimation algorithms that compute the arrival angle of the incident signals. In general, the DOA estimation algorithms are classified into two groups:
– Conventional algorithms (e.g. Bartlett method), and
– Subspace algorithms (e.g. MUSIC, root-MUSIC, and ESPRIT) [11], [12].

Both types are studied and analyzed in this paper.
conduct an in-depth comparison and analysis between several DOA algorithms to estimate their behavior as explained in the following:
– the Experience of the algorithm's ability to recognize and track the number of signals with high accuracy and contain several multiple, overlapping, and AOA.
– Analyzing results from a combination of antenna elements and snapshots of the algorithms' behavior, which of them has the most influence on the accuracy of the algorithm.
– Recognize the algorithms achieve minimum requirements and the best performance overall.

2. LITERARURE REVIEW

In a study conducted by Khedekar and Mukhopadhyay [13], a three-dimensional approach was used to compare and analyze the three algorithms estimation of signal parameters via rotational invariance technique (ESPRIT), multiple signal classification (MUSIC), and mechanical alteration of direction of arrival (DOA). The results showed that the mechanical algorithm is better than other algorithms in terms of accuracy and durability. Using uniform linear array (ULA) structure [14], the three algorithms MUSIC, ESPRIT, and maximum-likelihood estimation (MLE) are compared to measure their effectiveness and performance in a study conducted by Ihedrane and Bri. The MUSIC algorithm showed progress over other algorithms in terms of having the highest efficiency, and with an error rate of only 0.8%.

Kwizera [15] used the MUSIC algorithm to estimate the DOA of “the uniform linear array and non-uniform linear array”. The algorithm was analysed in terms of accuracy and efficiency. The results showed that MUSIC performance in non-uniform arrays is much better than its performance in uniform linear arrays as it was more accurate and efficient. In Ganage and Ravinder [16] ‘in this study of various DOA techniques’, minimum variance distortionless response (MVDR), MUSIC, root–MUSIC and ESPRIT algorithms were presented, and the results were compared based on performance, resolution, accuracy, sample, and number of snapshots for uniform linear array and were implemented using MATLAB. The result show that in comparison with Among all the algorithms, the Music algorithm is better than any other algorithm.

3. DOA ESTIMATION

The signals received by one element array from distinct DOA $\theta 1$, $\theta 2$, $\theta 3 \ldots\ \theta M$ are presented as shown in Figure 2. The output signal can take (1).

$$\boldsymbol{x}(t) = \boldsymbol{A}(\theta)\boldsymbol{s}(t) + \boldsymbol{n}(t). \quad (1)$$

where =[$\boldsymbol{a}(\theta 1)$, $\boldsymbol{a}(\theta 2) \ldots \boldsymbol{a}(\theta M)$] (L×M) steering matrix
(t)=[$s1(t)$, $s2(t) \ldots\ sM(t)$] T signal vector
(t) an additive white noise

As listed earlier in this paper, DOA algorithms are divided into conventional algorithms, and subspace algorithms are classified into two parts. Conventional algorithms are known to be traditional and include the Capon and Bartlett algorithms. However, a significant flaw of the conventional algorithms is that they do not facilitate access to the angle of arrival. This flaw was resolved by utilizing subspace algorithms. Subspace algorithms are much better than conventional because they depend on noise as well as the signal subspace. Subspace algorithms are used to determine the antenna's spatial spectrum, allowing for estimation. of DOA [17].

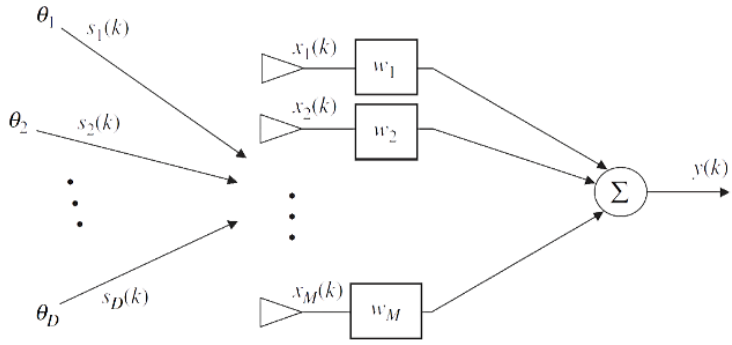

Figure 2. System model of signals received by one element array from a distinct DOA

4. ANGLES-OF-ARRIVAL ESTIMATION ALGORITHMS

4.1. Bartlett AOA Estimation

This method relies on directing the antenna radiation in one direction and calculating the output power, the weights are adjusted to maximize The signal-to-noise ratio [18]. The Bartlett can be determined using (2).

$$P_B(\Theta) = \overline{a}^H(\Theta).\ \overline{R}_{xx}.\overline{a}(\Theta). \quad (2)$$

4.2. Capon MVDR AOA estimation

Capon uses the most 'likelihood' technique to resolve a minimum difference problem, and hence the name minimum variance distortionless response (MVDR), the signal-to- interferes ratio (SIR) was maximized while the phase and capacity of the desired signal are maintained [19]. The Capon pseudospectrum is given as (3).

$$P_B(\Theta) = \frac{1}{\overline{a}^H(\Theta)\ \overline{R}_{xx}^{\ -1}\overline{a}(\Theta)} \quad (3)$$

4.3. Linear prediction AOA estimation

The aim of this approach is to avoid the problem of inaccuracy caused by delays. It is based on the uniform linear array's 'concept of minimizing the mean output signal power of the array elements subject to the constraint that the weight on a selected element' (ULA). The Estimated failure for both real output and output matrix sensor output is predicted using a linear process. from the ninth column $\overline{u}_m\ from$ matrix (M X M) can then be calculated using (4) [20]:

$$P_{LP_m}(\Theta) = \frac{\overline{u}^T{}_m\ \overline{R}_{xx}^{\ -1}\overline{u}_m}{\left|\overline{u}^T{}_m\,\overline{R}_{xx}^{\ -1}\,\overline{a}(\Theta)\right|^2} \quad (4)$$

4.4. Maximum entropies AOA estimate

The search for machine learning (ML) based methods is evolving rapidly. For example, it proposes an alternative projection (AP) method to solve the optimal solution for the probability function with less mathematical complexity [21].

The ML pseudo spectrum is given by (5).

$$P_{ME_j}(\Theta) = \frac{1}{\overline{a}^H(\Theta)\ \overline{C}_j\,\overline{C}_j^{\ H}\ \overline{a}(\Theta)} \quad (5)$$

where $\overline{C}_j$ Cartesian base equation, the inverse array correlation's jth column matrix $\overline{R}_{xx}^{\ -1}$.

4.5. Pisarenko harmonic AOA estimate

This method was proposed in the seventies and aims to analyse and estimate arrival angel and signal strength interferes with white noise [13]. Pisarenko harmonic decomposition (PHD) pseudospectrum is given by (6).

$$P_{PHD}(\Theta) = \frac{1}{\left|\bar{a}^H(\Theta)\,\bar{e}_1\right|^2} \tag{6}$$

4.6. Min-norm AOA estimate

The min-norm method optimizes the weight vector of the antenna array by solving the optimization problems. It is only relevant for ULA [22]. The Pseudospectrum of the min-norm method is given by (7).

$$P_{MN}(\Theta) = \frac{1}{\left|\bar{a}^H(\Theta)\,\bar{E}_N\,\bar{E}^H{}_N\,\bar{u}_1\right|^2} \tag{7}$$

where: $\bar{u}_1$= the first column of the (M×M) identity matrix and is equal to $[1\ 0\ 0\ 0\ 0\ ]^T$.
$\bar{E}_N$ = M− D noise eigenvectors.
$\bar{a}(\Theta)$= array steering vector.

4.7. The MUSIC AOA estimate

The MUSIC is widely used in the field of adaptive antennas because of the high efficiency in determining the angle of arrival, it can be calculated as (8) [22].

$$P_{MU}(\Theta) = \frac{1}{\left|\bar{a}^H(\Theta)\ \bar{E}_N\,\bar{E}^H{}_N\bar{a}(\Theta)\right|} \tag{8}$$

4.8. Root-MUSIC AOA estimate

Using polynomial roots to discover the angle of arrival, root-MUSIC the name of this system, and it can be written as (9) and (10) [23].

$$\bar{C} = \bar{E}_N\,\bar{E}^H{}_N \tag{9}$$

$$P_{RMU}(\theta) = \frac{1}{\left|\bar{a}^H(\Theta)\ \bar{C}\,\bar{a}(\Theta)\right|} \tag{10}$$

The measure in (10) can be expressed as (11).

$$\bar{a}^H(\Theta)\ \bar{C}\,\bar{a}(\Theta) = \sum_{l=-M+1}^{M+1} c_l\, e^{jkdsin\Theta} \tag{11}$$

where c_l is the total of diametrical elements of $\bar{C}$

$$c_l = \sum_{n-m=l} C_{mn} \tag{12}$$

$$D(z) = \sum_{l=-M+1}^{M+1} c_l\, z^l \tag{13}$$

where z =$e^{jkdsin\Theta}$

From (11), the angles of arrival (AOA) of the desired user can be calculated as (14) [24], [25].

$$\Theta_i = -\sin^{-1}\left(\frac{1}{kd}\arg(z_i)\right) \tag{14}$$

where z_i denotes the i root that is nearest to the unit circle

4.9. Root-min-norm AOA estimate

The equal basics utilized to the roots-MUSIC algorithm can additionally be utilized to the min-norm algorithm to make a root–min-norm algorithm as (15) [26].

$$P_{RMN}(\Theta) = \frac{1}{\left|\bar{a}^H(\Theta)\ \bar{c}_1\,\bar{c}_1{}^H\,\bar{a}(\Theta)\right|} \tag{15}$$

where $\overline{c}_1$ indicates the first column of $(\overline{C} = \overline{E}_N \overline{E}^H_N)$, M*M Hermitian matrix.

$$D(z) = \sum_{l=-M+1}^{M+1} c_l\, z^l \quad (16)$$

where $\overline{c}_1\, \overline{c}_1^{\,H}$ are the summation of the diagonal elements and it's denoted by c_l.

4.10. ESPRIT AOA estimate

The ESPRIT algorithm is very efficient and has many features such as it is easy to implement, and its signals can be accessed directly, which does not work in the complex implementation. The antenna array consists of 6 elements divided into two sub-arrays [27]. These arrays must be displaced translationally and not rotationally as shown in Figure 3.

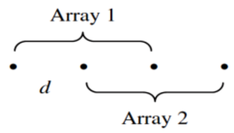

Figure 3. Doublet consisting of two similar displaced arrays

The outputs of the two arrays are obtained using (17) and (18).

$$\overline{x_1}(k) = \overline{A_1}.\overline{S}(k) + \overline{n_1}(k) \quad (17)$$

$$\overline{X}_2(k) = \overline{A_1}\phi.\overline{S}(k) + \overline{n_2}(k) \quad (18)$$

where $\overline{\Phi}$ = diag $(e^{jkdsin\Theta_1}, e^{jkdsin\Theta_2}, \dots \dots, e^{jkdsin\Theta_D})$
(D X D) diagonal unitary matrix with phase shifts between the subarrays for each AOA.

$$\overline{x}(k) = \begin{bmatrix}\overline{x}_1(k)\\ \overline{x}_2(k)\end{bmatrix} = \begin{bmatrix}\overline{A}_1\\ \overline{A}_1.\ \overline{\Phi}\end{bmatrix}.\overline{s}(k) + \begin{bmatrix}\overline{n}_1(k)\\ \overline{n}_2(k)\end{bmatrix} \quad (19)$$

The correlation - matrices for the two doublets are given by (20) and (21).

$$\overline{R}_{11} = E\left[\overline{x}_1 . \overline{x}^H{}_1\right] = \overline{A}\,\overline{R}_{ss}\overline{A}^{\,H} + \delta_n{}^2\overline{I} \quad (20)$$

$$\overline{R}_{22} = E\left[\overline{x}_2 . \overline{x}^H{}_2\right] = \overline{A\Phi}\,\overline{R}_{ss}\overline{\Phi}^H\,\overline{A}^{\,H} + \delta_n{}^2\,\overline{I} \quad (21)$$

Two signal subspaces $(\overline{E}_1, \overline{E}_2)$ are produced and they are related by means of a special non-singular transformation matrix.$\overline{\Psi}$ as in (22).

$$\overline{E}_1\overline{\Psi} = \overline{E}_2 \quad (22)$$

If the two subspaces $\overline{E}_1$and $\overline{E}_2$ are equally noisy, the rotation operator $\overline{\Psi}$ can be estimated using the total least-squares (TLS), this procedure is summarized as follows:

- Estimate $\overline{R}_{11}, \overline{R}_{22}$ from the data samples.
- Calculate the sum signals by large eigenvalues in either$\overline{R}_{22}$or $\overline{R}_{11}$.
- Calculate the signal subspaces $\overline{E}_1$and $\overline{E}_2$.
- Generate (2D $*$ 2D) matrix $\overline{C}$ from the signal subspaces such that;

$$\overline{C} = \begin{bmatrix}\overline{E}_1{}^H\\ \overline{E}_2{}^H\end{bmatrix}\left[\overline{E}_1\ \ \overline{E}_2\right] = \overline{E}_C\, \overline{\Lambda}\, \overline{E}_C{}^H \quad (23)$$

- Partition $\overline{E}_C$ to four D×D submatrices:

$$\overline{E}_C = \begin{bmatrix}\overline{E}_{11} & \overline{E}_{12}\\ \overline{E}_{21} & \overline{E}_{22}\end{bmatrix} \quad (24)$$

– Calculate the rotation operator $\overline{\Psi}$ such that

$$\overline{\Psi} = -\overline{E}_{12}\,\overline{E}_{22}^{\;-1} \tag{25}$$

– Calculate the eigenvalues of $\overline{\Psi}$.
– Calculate the (AOA) such that [28].

$$\Theta_i = \sin^{-1}\left(\frac{\arg(\lambda_i)}{kd}\right)\; i = 1,2,..D. \tag{26}$$

5. EXPERIMENTAL RESULTS

To study and analyze algorithms in the direction of arrival methods, every algorithm's decision capability is tested. The minimum that the array can are sensor and snapshots applicable to analyses is six signals, together with an angular distance over (10°, 20°, and 30°). Six sources are impinging on a uniform linear array with half-wavelength spacing. Initially, selected seven antenna elements with 100 snapshots, and this was varied until found all of the sources and simulations were run using MATLAB R2019b. The noise is an indiscriminate method developed using a MATLAB function, and the signals are deceptive to be double signals of capacity 1 with finite samples. The diagrams below show some of the simulations that run. Every algorithm's answer is determined by two parameters: Snap and amount of antenna elements both during after and before resolution.

In Figure 4, the Bartlett was used to estimate the AOA, using seven antenna elements with 100 snapshots; it identified only two sources. After increasing server antennas to 23 elements and increasing the number of snapshots to 1000, discovered all the sources.

For large antenna arrays, the precision is equal. Therefore, two sources are discovered if the separation angle between them is higher than the array resolution. Bias, however, is generated, causing the maximum values to deviate from the real AOA values. This bias can be reduced by increasing the length of the array. Figure 5 shows the Capon AOA estimate. At seven antenna elements and 100 snapshots, the Capon has the same performance as the Bartlett algorithm, with increased antenna elements. It is apparent that the Capon algorithm has a much more significant decision than the Bartlett algorithm; it needs 15 elements and 500 snapshots to resolve six sources successfully. In reality, the signals are highly correlated, and the Capon algorithm becomes worse. If a variety of signals is able to be kept and viewed as interfering, As Rayleigh amplitude and uniform phase are combined, the uncorrelated force is proportional, and the Capon approach performs well. Preliminary information and experience in some specific statistical properties are not required since these algorithms are non-parametric options-Capon and Bartlett algorithms' primary interest.

In Figure 6, the maximum signal power with the AOA information is presented. The linear predictive algorithm provides the most reliable overall performance over each of the Bartlett and the Capon AOA algorithms. Using seven antennas, identified four sources were, and when the number of antennas increased to 14 elements with 500 snapshots, the array was able to discover all the sources. The efficiency of the system depends on the choice of the antenna element. In this case, chose the central element and the corresponding vector.

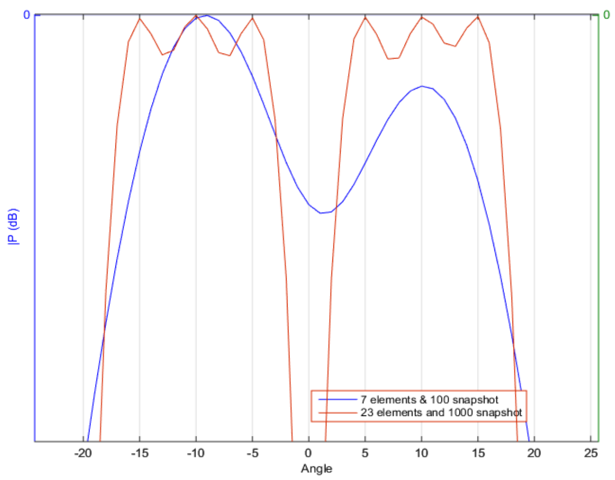

Figure 4. The Bartlett algorathim

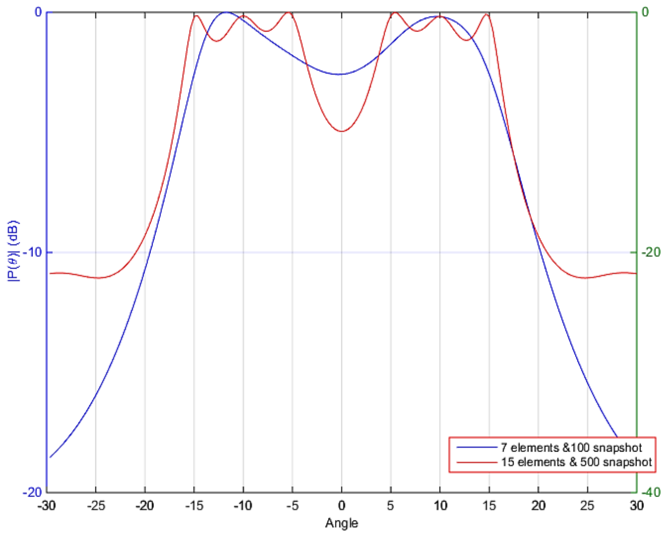

Figure 5. The Capon algorathim

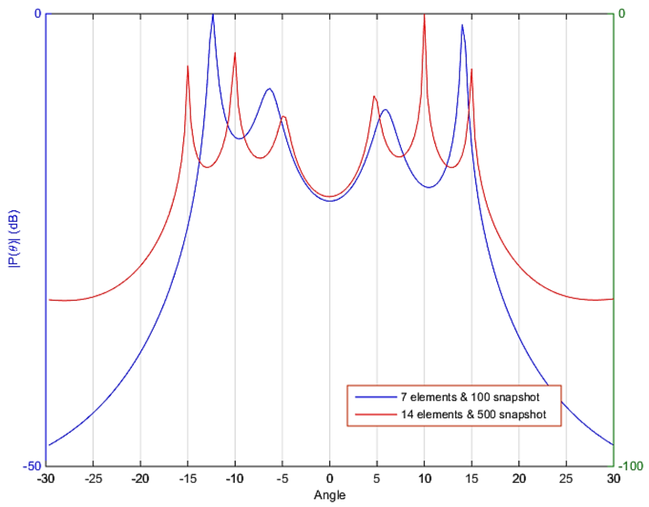

Figure 6. The linear predictive algorithim

Thirteen antenna and 401 snapshots were needed to identify six sources in the maximum entropy method, as shown in Figure 7. The resolution of the algorithm is greatly affected by the C_J column, where the center antenna element exhibits better performance at the conditions supposed in the research. Moreover, when selecting the center element, the maximum entropy algorithm provides the same performance as the linear predictive algorithm.

In Figure 8, the Pisarenko method is better than the previous algorithms in terms of accuracy; it only required 1000 snapshots and 12 antennas to separate 6 sources. The pseudo-spectrum from the min-norm algorithm is near similar to the PHD pseudo-spectrum, as shown in Figure 9. It is a collection of all noise eigenvectors, whereas in the PHD algorithm, only one noise eigenvector is used in the estimation process.

In Figure 10, the best results and accuracy were obtained by the MUSIC algorithm, where 11 antennas with 700 snapshots are needed to separate 6 sources. The locations of the root nodes near the unit circle are plotted in Figure 11. However, they do not exactly reflect the exact position of six angles of arrival of the AOA.

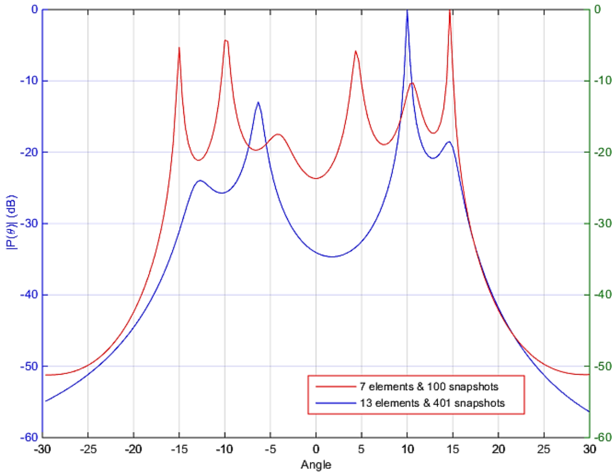

Figure 7. The maximum entropy AOA spectrum

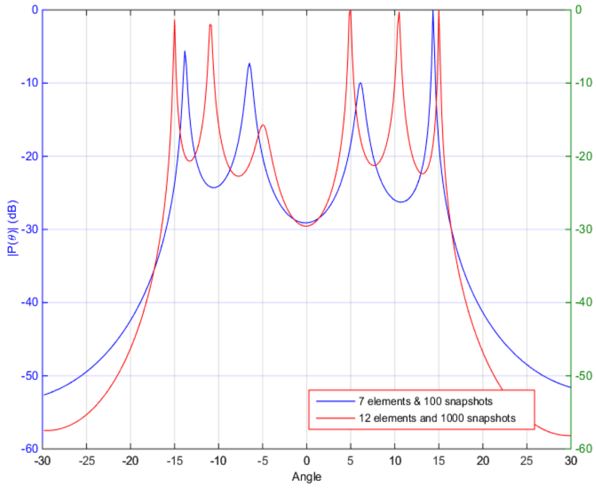

Figure 8. The Pisarenko AOA spectrum

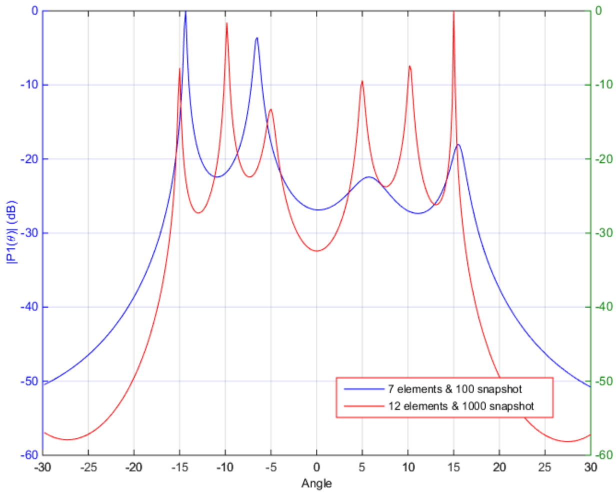

Figure 9. The min-norm AOA spectrum

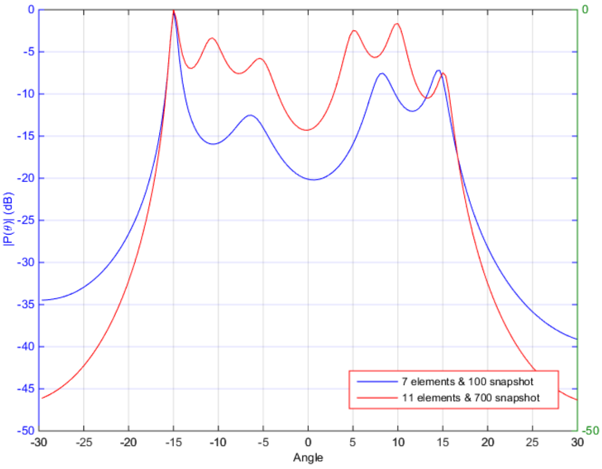

Figure 10. The MUSIC AOA spectrum

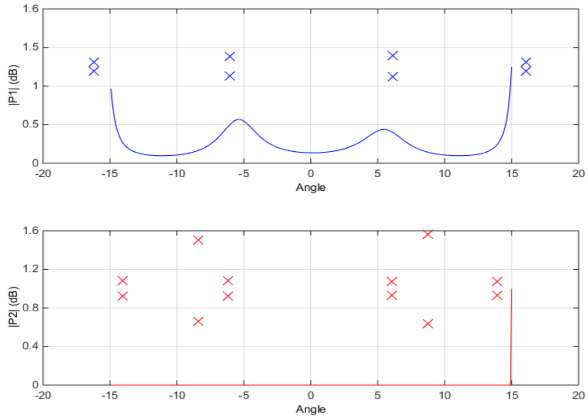

Figure 11. The root-MUSIC AOA spectrum

Root-min-norm algorithm allows giving an exposition indicator of the position of AOA, as shown in Figure 12. It no longer accurately indicates the specific position of AOA. Instead, it indicates six angles of entry. There is an error in finding the right root location since the incoming signals are partly corrected where the matrix correlation is calculated to be averaged over time, and the signal to noise ratio (SNR) is comparatively low. Figure 13 shows that the ESPRIT algorithm needs 17 elements with 1001 snapshots to resolve 6 signals. This algorithm provides less performance when increasing the number of sources.

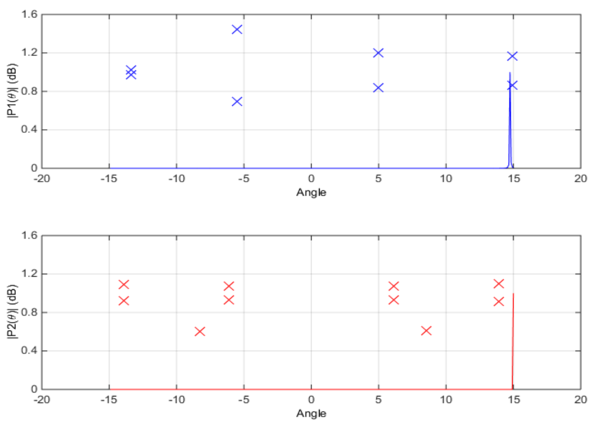

Figure 12. The root min-norm AOA spectrum

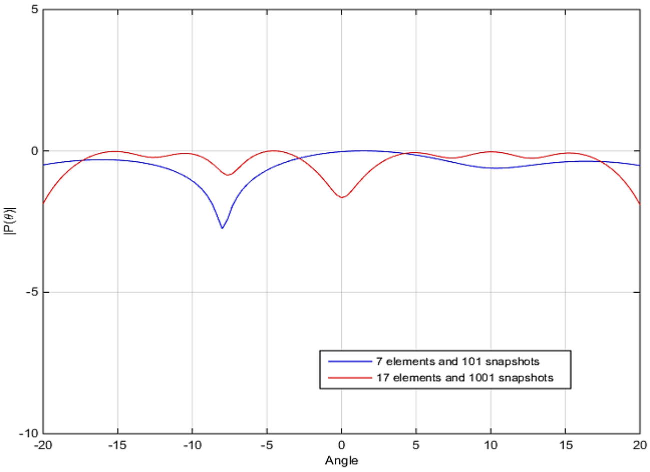

Figure 13. The ESPRIT AOA spectrum

6. CONCLUSION

In this paper, all DOA algorithms have been analyzed in terms of their performance based on server antennas and snapshots necessary to detect several sources. The best algorithm is the one that required the minimum number of antennas and snapshots to see the spaced sources. The results show that two of the algorithms, namely Pisarenko and the minimum-norm algorithms, exhibit the same performance for the same number of antennas and snapshots. The results also show that the snapshots are less effective than the antenna elements in identifying the signals. The MUSIC algorithm has proven to be the best in DOA estimations. It needs the minimum number of antennas and snapshots to resolve the angles of arrival of the received signals successfully.

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