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Adaptive Fuzzy Sliding Mode Control for a Class of Nonlinear System

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Abstract

For a class of nonlinear system with parameter perturbation and external disturbance, adaptive fuzzy control can be used to approach the system unknown functions to reduce the control input and the steady-state error. And an adaptive switch control gain whose adaptive law is decreasing function is designed to weaken the system chattering, the switch gain of estimate will increase on the basis of the original without decreasing with the elimination of interference.If system is interferenced many times. Against the shortcomings, this paper proposes an improved adaptive law that can weaken the system chattering effectively while maintaining the strong robustness. The simulation results by tests show that this method is correct and effective.

Keywords*: Adaptive Fuzzy Control, Integral Sliding Mode, Nonlinear System, Robustness*

1. Introduction

As a control method for systems which retain strong robustness with the characteristics of parametric uncertainties and disturbance, sliding mode control technology is highlighted by control research. In order to achieve excellent control effect, many researchers apply various control theories to sliding mode control, such as adaptive sliding mode control [1-3], fuzzy sliding mode control [4] and neural network sliding mode control [5]. Tractor steering angle controller is designed through adaptive sliding mode control method in quotation [6]. Outstanding trajectory tracking and handling stability is obtained and the effects of parameter perturbations and external disturbances on the system operability can be overcome effectively [7]. Adaptive integral sliding mode is applied to systems with uncertain parameters and the effects of control are achieved successfully in quotation [8]. Feedback linearization method and posture control holder combining with variable sliding mode structure are designed based on differential geometry for heavy equipment airdrop process motion model with the characteristics of strong coupling, strong nonlinearity and large disturbance [9-12].

However, there is a problem in the above researches. The traditional adaptive law is non decreasing function, the uncertainty is estimated to increase in the original basis, not with the estimated value of disturbance and the disappearance of gradually decreasing, buffeting increased with the time prolonging. The method of dead time characteristic and adaptive law nonlinear characteristics in combination, can better solve the problem. The basic idea of the method is, in reaching mode of the traditional sliding mode control, adaptive law guarantee system can quickly reach the sliding surface; when the switching function value reaches the set value, the adaptive law to change, the switching function as the value of the switching gain, the sliding mode control of switching gain decreased with the decrease of switching function, the final driven by the steady system enters the sliding mode.

This paper used fuzzy sliding mode control to solve a class of strongly coupled nonlinear system parameter perturbation and external disturbance by using the adaptive integral type. Through the design of integral type sliding surface based on the error, using adaptive to the unknown functions in the system of fuzzy approximation of the switching gain adaptive law design of switching function improved fuzzy inference system, weaken the chattering, and Lee

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Jaap Andrianof direct method to prove the stability of the system, the simulation results show that, the improved method effectively weakening the adaptive law is non decreasing control input chattering, so that the system has good dynamic performance and robustness.

2. System Description and Integral type Sliding Mode Surface Design

Considering the SISO nonlinear system

$$\ddot{x} = f(x,t) + g(x,t)u(t) + d(t) \tag{1}$$

In the above formula, $f(x,t)$, $g(x,t)$ are unknown nonlinear functions, and $g(x,t) > 0$, $d(t)$ external interference.Defining the system tracking error is $e(t) = x(t) - r(t)$, using the system tracking error feedback build sliding mode surface
$\dot{s}(t) = KE(t) = \ddot{e}(t) + k_1\dot{e}(t) + k_2 e(t)$

So:

$$s(t) = \dot{e} + \int_0^t \left[k_1\dot{e}(\tau) + k_2 e(\tau)\right]d\tau = \dot{x}(t) - \int_0^t \left[\ddot{r}(t) - k_1\dot{e}(\tau) - k_2 e(\tau)\right]d\tau \tag{2}$$

Can be seen from the type (2), the system has constructed the integral sliding mode surface, the system tracking error depends on the state feedback matrix $K = [1, k_1, k_2]$, by determining suitable k_1 and k_2, the tracking error $e(t)$ will be close to zero, and the system will have a good dynamic performance.

3. Design of an Adaptive Fuzzy Sliding Mode Controller

3.1. Algorithm Design

For type (1) nonlinear system, said if $f(x,t)$, $g(x,t)$ and $d(t)$ as is known, $s(t) = \dot{s}(t) = 0$ can be based on the sliding mode in the ideal state of the control law for the calculation of surface:

$$u^*(t) = g(x,t)^{-1}\left[-f(x,t) - d(t) + \ddot{r}(t) - k_1\dot{e}(t) - k_2 e(t)\right] \tag{3}$$

If *g*(*x,t*)and *d(t)* is unknown, $u^*(t)$ is difficult to achieve, and a fuzzy system approach $u^*(t)$ is used.
The switching function $s(t)$ is as the input of fuzzy controller, which forms a single input fuzzy approximation system, fuzzy rules, and the fuzzy controller for:

Rule i : IF s is F_s^i, THEN u is α_i (4)

And *i* =1, 2, 3, …, m, α_i and F_s^i are fuzzy sets; by centroid method to defuzzification the fuzzy controller output will be:

$$u_{fz}(s,\alpha) = \sum_{i=1}^{m} \mu_i \alpha_i / \sum_{i=1}^{m} \mu_i = \alpha^{T}\omega^{T} \tag{5}$$

And μ_i is the *i*-th rule's weight, $\alpha = [\alpha_1, \alpha_2, \alpha_3, ..., \alpha_m]$, $\omega = [\omega_1, \omega_2, \omega_3, ..., \omega_m]$, ω_i is defined to be:

$$\omega_i = \mu_i / \sum_{i=1}^{m} \mu_i \tag{6}$$

On the basis of fuzzy approximation theory, there is an optimal fuzzy system $u_{\mathrm{fz}}\left(s,\alpha^*\right)$ that can approach $u^*(t)$.

$$u^*(t)=u_{\mathrm{fz}}\left(s,\alpha^*\right)+\varepsilon=(\alpha^*)^{\mathrm{T}}\omega+\varepsilon \tag{7}$$

And the ε is approximate error that meets $|\varepsilon|<D$ .If fuzzy system u_{fz} approaches $u^*(t)$, so

$$u_{\mathrm{fz}}\left(s,\hat{\alpha}\right)=\hat{\alpha}^{\mathrm{T}}\omega \tag{8}$$

And $\hat{\alpha}$ is the estimated value of α^* .The switching control law to compensate the error between switching control law robustness, strong on the approximation error, defined as the switching control law:

$$u_{\mathrm{vs}}=-\eta(t)\operatorname{sgn}\left(s(t)\right) \tag{9}$$

So the total control rule of system (1) is :

$$u(t)=u_{\mathrm{fz}}+u_{\mathrm{vs}} \tag{10}$$

In the switching controller, because of the uncertain parameters of the system and the existence of interference, resulting in the switching gain $\eta(t)$ is difficult to determine, the actual control to determine, if $\eta(t)$ the value selected is too large, will produce buffeting larger, if too small, robust system decline and tend to be unstable. In order to reduce the amount of calculation can be used to control system from the law of use to design $\eta(t)$, definition:

$$u_{\mathrm{vs}}=-\hat{\eta}(t)\operatorname{sgn}\left(s(t)\right) \tag{11}$$

$$\dot{\hat{\eta}}(t)=\gamma_2\left|s(t)\right| \tag{12}$$

$\hat{\eta}(t)$ is the switch gain of estimate, γ_2 is the adaptive factor of the real number to characterizethe speed of the adaptive law with speed. The adaptive estimation error can be defined as:

$$\tilde{\eta}(t)=\hat{\eta}(t)-D \tag{13}$$

Define $\tilde{\alpha}=\hat{\alpha}-\alpha^*$, so formula (7) can be transformed as:

$$\tilde{u}_{\mathrm{fz}}=\hat{u}_{\mathrm{fz}}-u^*=\hat{u}_{\mathrm{fz}}-u_{\mathrm{fz}}^*-\varepsilon=\tilde{\alpha}^{\mathrm{T}}\omega-\varepsilon \tag{14}$$

Put formula (2) into formula (3), and can get:

$$\begin{aligned}u^*(t)&=g(x,t)^{-1}\left[-f(x,t)-d(t)+\ddot{r}(t)+\ddot{e}(t)-\dot{s}(t)\right]\\&=g(x,t)^{-1}\left[g(x,t)u(t)-\dot{s}(t)\right]\end{aligned} \tag{15}$$

So:

$$\dot{s}(t)=g(x,t)\left[u(t)-u^*(t)\right]=g(x,t)\left[u_{\mathrm{fz}}+u_{\mathrm{vs}}-u^*(t)\right] \tag{16}$$

3.2. The Stability Proof
Define Lyapunov function as:

$$V(t)=0.5s^2(t)+\frac{g(x,t)}{2\gamma_1}\tilde{\alpha}^T\tilde{\alpha}+\frac{g(x,t)}{2\gamma_2}\tilde{\eta}^2(t)$$

So:

$$\begin{aligned}\dot{V}(t)&=s(t)\dot{s}(t)+\frac{g(x,t)}{\gamma_1}\tilde{\alpha}^T\dot{\tilde{\alpha}}+\frac{g(x,t)}{\gamma_2}\tilde{\eta}(t)\dot{\tilde{\eta}}(t)\\&=s(t)g(x,t)\left(u_{\mathrm{fz}}+-u^*(t)\right)+\frac{g(x,t)}{\gamma_1}\tilde{\alpha}^T\dot{\tilde{\alpha}}+\frac{g(x,t)}{\gamma_2}\tilde{\eta}(t)\dot{\tilde{\eta}}(t)\\&=s(t)g(x,t)\left(\tilde{\alpha}^{\mathrm{T}}\omega\right)+\frac{g(x,t)}{\gamma_1}\tilde{\alpha}^T\dot{\tilde{\alpha}}+\frac{g(x,t)}{\gamma_2}\tilde{\eta}(t)\dot{\tilde{\eta}}(t)\\&=g(x,t)\tilde{\alpha}^{\mathrm{T}}\left(s(t)\omega+\frac{\dot{\tilde{\alpha}}}{\gamma_1}\right)+s(t)g(x,t)\left(u_{\mathrm{vs}}-\varepsilon\right)+\frac{g(x,t)}{\gamma_2}\left(\hat{\eta}(t)-D\right)\dot{\hat{\eta}}(t)\\&=g(x,t)\tilde{\alpha}^{\mathrm{T}}\left(s(t)\omega+\frac{\dot{\tilde{\alpha}}}{\gamma_1}\right)+\frac{g(x,t)}{\gamma_2}\left(\hat{\eta}(t)-D\right)\dot{\hat{\eta}}(t)-\hat{\eta}(t)\left|s(t)\right|g(x,t)-\varepsilon s(t)g(x,t)\end{aligned}\tag{17}$$

For system stability, fuzzy approximation coefficient estimation using the following algorithm

$$\dot{\tilde{\alpha}}=\dot{\hat{\alpha}}=-\gamma_1 s(t)\omega \tag{18}$$

Put the type (12) and (18) into type (17), available:

$$\begin{aligned}\dot{V}(t)&=-\hat{\eta}(t)\left|s(t)\right|g(x,t)-\varepsilon s(t)g(x,t)+\left(\hat{\eta}(t)-D\right)\left|s(t)\right|g(x,t)\\&=-\varepsilon s(t)g(x,t)-D\left|s(t)\right|g(x,t)\le\left|\varepsilon\right|\left|s(t)\right|g(x,t)-D\left|s(t)\right|g(x,t)\\&=-\left(D-\left|\varepsilon\right|\right)\left|s(t)\right|g(x,t)\le 0\end{aligned}\tag{19}$$

Because the type (12) characterization of the adaptive estimation law is non decreasing function, namely the adaptive law $\hat{\eta}(t)$ does not change with the weakening or disturbance, can only increase in the original basis, and the actual system subject to disturbances or parameter is variable, with the extension of time, the switching adaptive law is decided by the control will be more and more large, chattering will strengthen. In order to reduce buffeting strength system, the method of adaptive law is used.

3.3. Improved Adaptive Control Law
Here draw on the experience of method of the nonlinear characteristics of dead zone to design adaptive law to improve the formula (12) such as follows:

$$\begin{cases}\dot{\hat{\eta}}(t)=\gamma_2\left|s(t)\right|, & |s|\ge\delta\\ \hat{\eta}(t)=|s|+n, & |s|<\delta\end{cases}\tag{20}$$

Here $\delta>0$, $n>0$. The theoretical analysis is as follows:

(a) When the system state, far distance sliding surface, namely the approach section of sliding mode control, need to use the control law can guarantee the system stability and the larger the system to the sliding surface, then the adaptive law (12), which can guarantee the system's convergence in a large range, and the control input strong, can quickly force the system state into the sliding surface set. The festival has proved the stability.

(b) When the state of the system, has returned to the sliding surface, a modified adaptive law to control, can make the switch gain decreased with the decrease of the error. The stability proof:

The Lyapunov function

$$V(t)=0.5s^2(t)+\frac{g(x,t)}{2\gamma_1}\tilde{\alpha}^T\tilde{\alpha} \tag{21}$$

By formula (17) and (18) available:

$$\begin{aligned}\dot{V}(t)&=g(x,t)\tilde{\alpha}^{\mathrm{T}}\left(s(t)\omega+\dot{\tilde{\alpha}}/\gamma_1\right)+s(t)g(x,t)(u_{\mathrm{vs}}-\varepsilon)\\&=s(t)g(x,t)\left(-\hat{\eta}(t)\mathrm{sgn}(s(t))-\varepsilon\right)\\&=s(t)g(x,t)\left(-|s(t)|\mathrm{sgn}(s(t))-n\,\mathrm{sgn}(s(t))-\varepsilon\right)\\&=g(x,t)\left(-|s(t)|^2-n|s(t)|-\varepsilon s(t)\right)\end{aligned} \tag{22}$$

The approximation error goes to zero, so the type (22) with the following changes:

$$\dot{V}(t)=g(x,t)\left(-|s(t)|^2-n|s(t)|-\varepsilon s(t)\right)=-g(x,t)\left(|s(t)|^2+n|s(t)|\right)\le 0 \tag{23}$$

(c) Multiple disturbance by the system, changes in the system error performance in S. Error increase or decrease S will be in (a) and (b) the boundary line and switching of two state representation, eventually drove converge to zero, switching estimation value of control gain also decreased.

(d) In the adaptive law expression (20), determining the gain of switch control the speed of change, is beneficial for the system to uncertain interference suppression, on the other hand, takes a long time to suppress the disturbance. For a small positive number, can converge into the sliding mode to ensure system. As a boundary value.

4. Verification and Simulation Analysis

Based on the tracking error state feedback integral sliding mode surface, use the following 5 kinds of membership function of fuzzy:

$$\mu_{NM}(s)=\exp\left[-\left((s+\pi/6)/(\pi/24)\right)^2\right] \tag{24}$$

$$\mu_{NS}(s)=\exp\left[-\left((s+\pi/12)/(\pi/24)\right)^2\right] \tag{25}$$

$$\mu_{ZO}(s)=\exp\left[-\left(s/(\pi/24)\right)^2\right] \tag{26}$$

$$\mu_{PS}(s)=\exp\left[-\left((s-\pi/12)/(\pi/24)\right)^2\right] \tag{27}$$

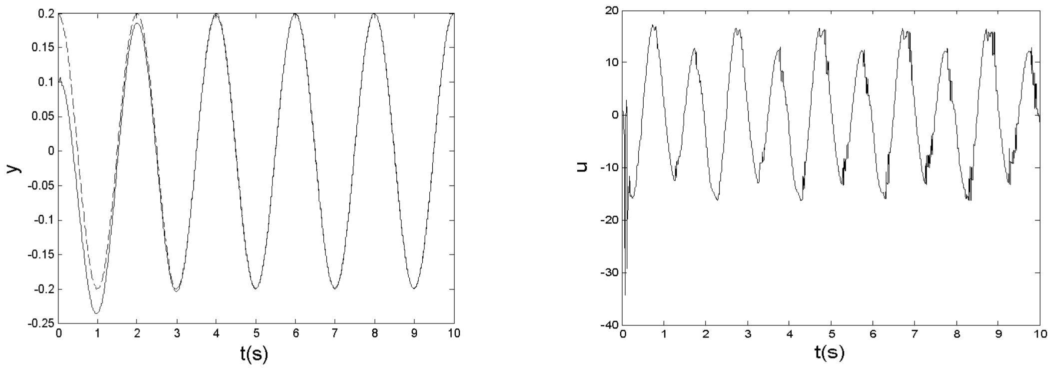
$$\mu_{PM}(s)=\exp\left[-\left((s-\pi/6)/(\pi/24)\right)^2\right] \tag{28}$$

Method to verify this, consider the inverted pendulum system, the following equation of state are:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{g\sin x_1 - mlx_2^2\cos x_1 \sin x_1 / \left(m_c + m\right)}{l\left(4/3 - m\cos^2 x_1 / \left(m_c + m\right)\right)} + \dfrac{\cos x_1 / \left(m_c + m\right)}{l\left(4/3 - m\cos^2 x_1 / \left(m_c + m\right)\right)} u\left(t\right) + d\left(t\right) \end{cases} \quad (29)$$

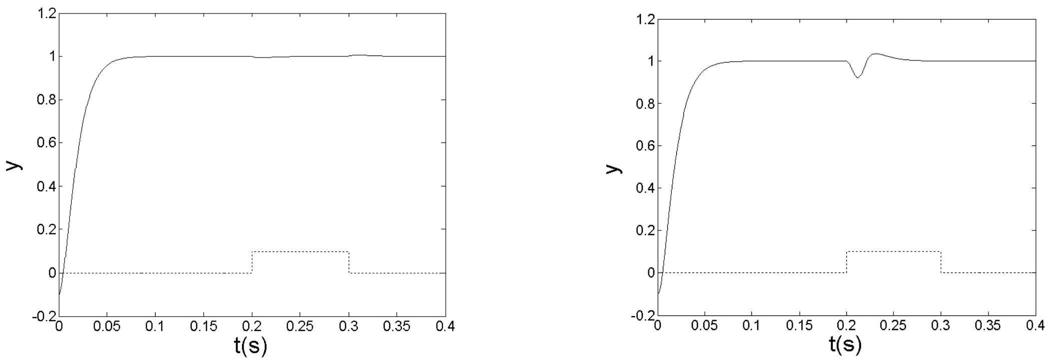
And in (29) x_1 is swing angle and x_2 is swing speed, $g = 9.8\mathrm{m}/s^2$, $m_c = 1\mathrm{k}g$ vehicle quality $m = 0.1\mathrm{k}g$ as the pendulum rod quality $l = 0.5\mathrm{m}$ half the length of pendulumas the control input. Follow the sinusoidal signal, the position command, the initial state of the system, when the system is: interference; follow the step signal, the initial state is applied for 0 seconds, the interference of 0.2 seconds.

The control law by type (10), (18) and (20), the controller parameters $\gamma_1 = 1000$, $\gamma_2 = 10$, $\delta = 0.01$, $n = 1$ the simulation curve as shown fromFigure 1 to 3.



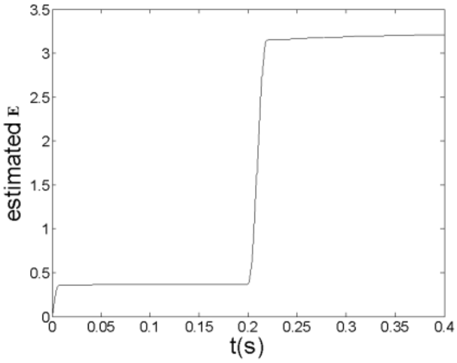
(a) Position tracking sine signal waveform (b) control input waveform

Figure 1. Sinusoidal signal system to follow and control the input waveform

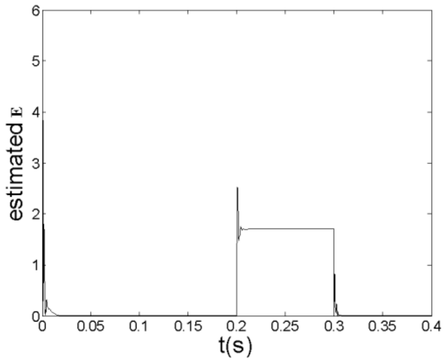


(a) The conventional method processing stepwaveform signal tracking (b) The improved method processing step waveform signal tracking

Figure 2. The system step with signal waveform



(a) Adaptive parameter conventional waveform



(b) Adaptive parameters improved waveform

Figure 3. Adaptive trajectory

5. Conclusion

Based on the analysis of the adaptive law of traditional adaptive fuzzy sliding mode control method is overcome, improving method is put forward. The nonlinear characteristics of applied to the adaptive law, effectively solve the adaptive law is non decreasing defects. Simulation results show that the improved method is correct and effective. While maintaining the original adaptive fuzzy sliding mode control based on advantage of suppressing disturbance, new method obviously weaken the chattering, reduces the control input, and achieve the adaptive law to estimate the parameters change with the disturbance changes, effectively solves the problems of the previous methods.

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