Energy-efficient speed profile: an optimal approach with fixed running time

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Article Info

Article history:

Received Jan 15, 2021
Revised Mar 21, 2022
Accepted Mar 30, 2022

Keywords:

Energy-efficient operation methodology
Energy-saving
Metro system
Pontryagin's maximum principle
Timetable optimization

ABSTRACT

Tracking the optimal speed profile in electric train operation has been proposed as an efficient and feasible solution for not only reducing energy consumption, but also no at costs to upgrading the existing railway systems. This paper focuses on finding the optimal speed profile based on Pontryagin's maximum principle (PMP) while ensuring the fixed running time, and comparing energy saving levels in the cases of applying or not applying PMP. The way to determine the fixed running time also differs from works published is to calculate the total trip time equal to scheduled timetable exactly. Calculating accelerating time t_a, coasting time t_c, braking time t_b via values of maximum speed v_h, braking speed v_b of optimal speed profile. The other hands, v_h and v_b are determined by solving nonlinear equations with constraint condition: the running time equal to the demand time. Simulation results with data collected from electrified trains of Cat Linh-Ha Dong metro line, Vietnam show that energy reduction for the entire route when PMP utilization is up to 8.7% and running time complied with scheduled timetables.

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1. INTRODUCTION

High levels of air pollution and congestion are knotting issues related to transport in big cities. Under the circumstance, urban railway is considered a great potential to be efficient, reliable, and environmentally friendly means of transportation. However, its major disadvantage lies in consuming a large amount of electric energy in operation while the energy prices are steadily rising [1], [2]. Therefore, energy saving in train operation has become a hot topic to draw more attention from researchers worldwide. There are some energy efficiency strategies having been proposed. The first approach is related to enhance regenerative braking energy recuperation which can reduce the net energy consumption approximately 30% [1], [3], [4] by onboard/stationary energy storage systems [5]-[8], equipping the traction substations with reversible converters or active rectifiers so as to pump back the regenerative braking energy into utility source, as a result, all regenerated energy can be recovered [9], [10], and optimizing scheduled timetables makes the regenerated energy among trains interchange easier [11], [12]; the second one is energy-efficient traction systems, determining optimal position for substations by reducing energy losses in the power supply network, in on-board traction equipment [4]; the third one is energy - efficient driving by applying optimal theory is to minimize operating energy [13]-[18]. Among above approaches, the energy-efficient driving solution does not need to invest in equipment or infrastructure of existing metro lines, so this one is most suitable for metro lines in Vietnam to have just been installed. The researchers of South Australia University – Albrecht *et al* [19] and Howlett *et al.* [20] outlined an energy-efficient driving strategy for a train journey on a track with uphill and downhill gradients by designing control laws to calculate location of optimal switching points, and then determining the optimal speed profile; Baranov *et al*. [21] proposed a solution to minimize energy consumption and consider fixed trip time by supplementing Lagrange multi-plier λ in objective function, but calculating λ to find the actual time equal to demand time is not easy. In addition, some researchers have also applied intelligent train operation algorithms, such as: using fuzzy logic, neural networks, genetic algorithms, and particle swarm. To improve its performance; Lu *et al*. [22] applied three optimization algorithms (colony optimization algorithm, dynamic programming, genetic algorithm) to search for the optimal speed trajectory, Açikba and Söylemez [23] utilized artificial neural networks and genetic algorithms to determine the optimal coasting point. However, these studies also did not mention fixed running time. For the continuous control problem, the Pontryagin’s maximum principle (PMP) is used to find necessary conditions for an optimal strategy. It is shown that these conditions yield key equations that determine the optimal switching points. Therefore, in this paper, PMP has been presented to determine optimal speed profile for Cat Linh-Ha Dong metro line in Vietnam, ensuring fixed - running time as well. Simulation results with different scenarios are showed the effectiveness of optimal control method - PMP in saving energy of train operation without any changes about infrastructure, facilities.

2. TRAIN MODELING

The urban electric train often operates in three motion regimes: accelerating, coasting, braking as in Figure 1. To build-up the kinematic equation of the train, it is necessary to analyze the forces acting on the train, surveying the longitudinal profile of the train. Thereby, its kinematic equation can be represented by the following continuous - space model [24], [25].

$$\begin{cases} \frac{dx}{dt} = v \\ mv\frac{dv}{dx} = F_{tr}(v) - F_{br}(v) - W_0(v) - F_{grad}(x) \end{cases} \tag{1}$$

Where v, t, x, m represent respectively train speed (m/s), operation time (s), train position (m), full load translating mass of train (tone) and $F_{tr}, F_{br}, W_0, F_{grad}$ are traction, electrical braking, resistance, gradient resistance forces applied on the train. Depending on the structure of the trains, different gauges will have different impact forces and influence coefficients. Forces used in this paper are calculated from Cat Linh-Ha Dong metro line.

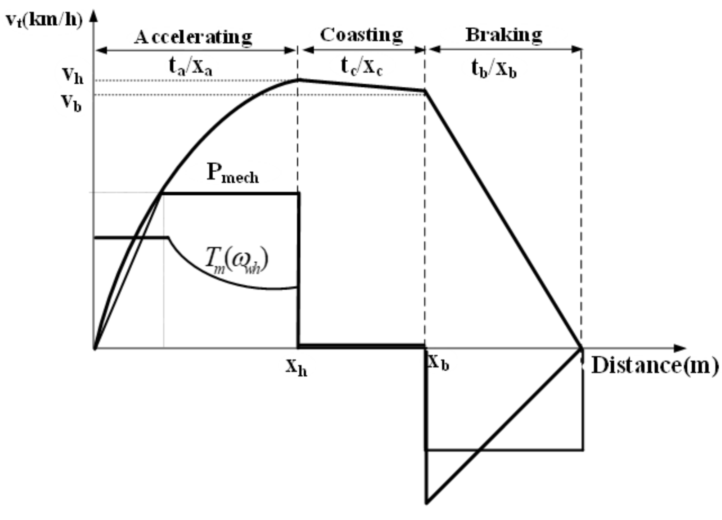

Figure 1. A typical speed profile with three motion phases for short inter-stations

Base on curves of traction force F_{tr}, braking force F_{br} given by manufacturers [26], using the identification method: least square method to find equivalent polynomials. The maximum traction and maximum braking forces corresponding to the speed v are:

$$F_{tr}=\begin{cases}13.2 & (0\le v\le 32)\\ -2.5\cdot 10^{-5}v^3+0.007\cdot v^2-0.66v+28.35 & (32<v\le 80)\end{cases} \quad (2)$$

$$F_{br}=\begin{cases}14.7(0\le v\le 65)\\ -0.254v+31.21(65<v\le 75)\\ -0.2027v+27.36(75<v\le 80)\end{cases} \quad (3)$$

The basic resistance$w_0$can be calculated by using Davis formula [27].

$$w_0=\frac{W_0}{m}=a+bv+cv^2 \quad (4)$$

Where a,b,c are coefficients of train's resistance. The gradient force F_{grad} caused by slope of road.

$$F_{grad}=mgsin\alpha \quad (5)$$

In which g,α are the gravity acceleration and the rail track slope respective.

3. PONTRYAGIN'S MAXIMUM PRINCIPLE APPLICATION FOR OPTIMAL SPEED PROFILE DETERMINATION AND FIXED RUNNING TIME

There are several optimal control theories applied to find the optimal speed profile. In this section, Pontryagin's maximal principle is applied to detect the optimal switching points, and then determining the optimal speed profile, ensuring fixed trip time. The main objective is to evaluate the energy savings when trains track the optimal speed profile compared with the original speed profile.

3.1. Speed profile optimality analysis based on PMP

Depending on the distance between stations, trains operate in four phases: accelerating, holding, coasting, braking. However, for urban metro lines, due to the short distance between stations, they only operate in 3 modes: accelerating, coasting, braking. From train's state equations with boundary conditions, and the objective function J, using PMP seeks the optimal switching points of these modes; thereby determining the speed, distance, and trip time in each phase.

3.1.1. Problem formulation

The motion of a train along a track is rewritten from the state equations (1).

$$\begin{cases}\frac{dt}{dx}=\frac{1}{v}\\ v\frac{dv}{dx}=u_{tr}f_{tr}(v)-u_{br}f_{br}(v)-w_0(v)-f_{grad}(x)\end{cases} \quad (6)$$

Where u_{tr},u_{br} are defined traction and braking control variables of train, both of which are restrained by: $u_{tr}\in[0,1]$; $u_{br}\in[0,1]$;$u_{mb}\in[0,1]$; f_{tr},f_{br},f_{grad} are forces per unit mass; traction force applied at the wheels, braking force, mechanical force, gradient force acting on the train.
Where:

$$f_{tr}=\frac{F_{tr}}{m};f_{br}=\frac{F_{br}}{m};f_{grad}=\frac{F_{grad}}{m} \quad (7)$$

Therefore, boundary conditions are given by:

$$\begin{cases}v(0)=0,v(X)=0,t(0)=0\\ 0\le v(x)\le V(x),0\le t(X)\le T,0\le x\le X\end{cases} \quad (8)$$

Where $V(x)$ is the maximum allowable speed, X is the terminal of the train operation; $v(0),v(X)$ are the speed at the beginning, at the end of the route; T is duration of the trip is also given by the timetable. From state equations (6), with boundary conditions (7), (8) how to find objective function in order to minimize the train's operation energy consumption as the train runs from location $x=0$ to location $x=X$ in time T by controlling the traction force, while ignoring electric braking force since regenerative braking energy is not recovered. The objective function is written as:

$$J=\int_0^X u_{tr}f_{tr}(v)dx\rightarrow \min \quad (9)$$

3.1.2. Solution

By PMP finding optimal solutions of an objective function is equivalent to maximizing its Hamiltonian equation. Based on (6), (8), a Hamilton function is formed as:

$$H = -u_{tr} f_{tr}(v) + p_1 \frac{1}{v} + p_2 \left(\frac{u_{tr} f_{tr}(v) - u_{br} f_{br}(v) - w_0(v) - f_{grad}(x)}{v} \right) \tag{10}$$

Where p_1, p_2 are adjoint variables.
Adjoint variable differential equations are reformed:

$$\frac{dp_1}{dx} = -\frac{\partial H}{\partial t} = 0 \tag{11}$$

$$\frac{dp_2}{dx} = -\frac{\partial H}{\partial v} = u_{tr} \frac{\partial f_{tr}}{\partial v} + p_1 \frac{1}{v^2} + \frac{p_2}{v^2} \left[u_{tr} f_{tr}(v) - u_{br} f_{br}(v) - w_0(v) - f_{grad}(x) \right] - \frac{p_2}{v} \left[u_{tr} \frac{\partial f_{tr}}{\partial v} - u_{br} \frac{\partial f_{br}}{\partial v} - \frac{\partial w_0}{\partial v} \right] \tag{12}$$

Define:

$$p = \frac{p_2}{v}, \ \text{ so} p \cdot v = p_2. \text{ Therefore } \frac{dp_2}{dx} = \frac{d(p \cdot v)}{dx} = p \frac{dv}{dx} + v \frac{dp}{dx} \tag{13}$$

$$\Rightarrow \frac{dp}{dx} = \frac{1}{v} \left(\frac{dp_2}{dx} - p \frac{dv}{dx} \right) \tag{14}$$

Given:

$$\frac{dv}{dx} = \frac{u_{tr} f_{tr}(v) - u_{br} f_{br}(v) - w_0(v) - f_{grad}(x)}{v} \tag{15}$$

Therefore, Hamiltonian function is rewritten.

$$H = (p-1) u_{tr} f_{tr} - p u_{br} f_{br} - p(w_0 + f_{grad}) + \frac{p_1}{v} \tag{16}$$

Hamiltonian function is maximized by the following values of u_{tr} and u_{br}:

$$\begin{cases} u_{tr} = 1 & if \quad p > 1 \\ u_{tr} \in (0,1) & if \quad p = 1 \\ u_{tr} = 0 & if \quad p < 1 \end{cases} and \begin{cases} u_{br} = 0 & if \quad 0 < p < 1 \\ u_{br} \in (0,1) & if \quad p = 0 \\ u_{br} = 1 & if \quad p < 0 \end{cases} \tag{17}$$

From the above analysis, five optimal control laws are designed:
- Full power (FP):$u_{tr} = 1, u_{br} = 0$ when $p > 1$
- Partial power (PP):$u_{tr} \in [0,1]$, $u_{br} = 0$ when $p = 1$
- Coasting (C): $u_{tr} = 0, u_{br} = 0$ when $0 < p < 1$
- Full braking (FB): $u_{tr} = 0, u_{br} = 1$ when $p < 0$
- Partial braking (PB):$u_{tr} = 0$, $u_{br} \in [0,1]$ when $p = 0$

Substitute (13), (12) in (14), finding the differential equation for $p(x)$

$$\frac{dp}{dx} = \frac{(1-p)}{v} u_{tr} f'_{tr}(v) + \frac{p}{v} u_{br} f'_{br}(v) + \frac{p}{v} w'_0(v) - \frac{p_1}{v^3} \tag{18}$$

From (11), easily, $p_1$is chosen by 0.
Full power mode: $p > 1, u_{br} = 0, u_{tr} = 1$, finding accelerating time t_a, accelerating distance x_a using (18).

$$\frac{dp}{dx} = \frac{(1-p)}{v} f'_{tr}(v) + \frac{p}{v} w'_0(v) \tag{19}$$

From (6) finding the differential equation to determine x_a, t_a, with initial conditions: $x(0) = 0, t(0) = 0$.

$$\begin{cases} \frac{dx}{dv} = \frac{v}{u_{tr} f_{tr}(v) - w_0(v) - f_{grad}(x)} \\ \frac{dt}{dv} = -\frac{1}{u_{tr} f_{tr}(v) - w_0(v) - f_{grad}(x)} \end{cases} \tag{20}$$

Coasting speed v_b is calculated as following [20].

$$v_b = \frac{\psi(v_h)}{\varphi'(v_h)} \tag{21}$$

Where v_h - hold speed:

$$\varphi(v) = v \cdot w_0(v) = v(a + bv + cv^2), \psi(v) = v^2 \cdot w_0'(v) = v^2(b + 2cv) \tag{22}$$

From (6) finding the differential equation to determine x_c, t_c, with $t(v = v_h) = t_a, x(v = v_h) = x_a$

$$\begin{cases} \frac{dx}{dv} = \frac{v}{-w_0(v) - f_{grad}(x)} \\ \frac{dt}{dv} = -\frac{1}{w_0(v) + f_{grad}(x)} \end{cases} \tag{23}$$

Full braking mode: $p < 0, u_{br} = 1, u_{tr} = 0$, finding braking time t_b, braking distance x_b using (14).

$$\frac{dp}{dx} = \frac{p}{v} f_{br}'(v) + \frac{p}{v} w_0'(v) \tag{24}$$

From (11) finding the differential equation with $t(v = v_b) = t_b, x(v = v_b) = x_b$

$$\begin{cases} \frac{dx}{dv} = \frac{v}{-u_{br} f_{br}(v) - w_0(v) - f_{grad}(x)} \\ \frac{dt}{dv} = \frac{1}{u_{br} f_{br}(v) - w_0(v) - f_{grad}(x)} \end{cases} \tag{25}$$

3.2. Fixed running time

To meet requirements of train's running time and stopping time at stations as the scheduled timetable when trains track the optimal speed profile. In this section, we will calculate t_a - the time of the accelerating phase, t_b - the time of coasting phase, t_c - the time of braking phase, and the total running time the of three phases must abide by the scheduled timetable. Calculating t_a, t_b, t_c is conducted by Maple software.

3.2.1. Accelerating phase

Equation motion of the train in optimal traction mode:

$$\frac{dv}{dt} = f_{tr}(v) - w_0(v) \tag{26}$$

Using the variable dissociation method, the running time in accelerating phase is expressed as:

$$\frac{dv}{f_{tr}(v) - w_0(v)} = dt \rightarrow \int_0^{v_h} \frac{dv}{f_{tr}(v) - w_0(v)} = \int_0^{t_a} dt \rightarrow \int_0^{v_h} \frac{dv}{f_{tr}(v) - w_0(v)} = t_a \tag{27}$$

Where: f_{tr} is calculated as (7).
From (27) the acceleration time may be employed:

$$\begin{aligned} t_a &= \int_0^{32} \frac{dv}{f_{tr}(v) - w_0(v)} + \int_{32}^{v_h} \frac{dv}{f_{tr}(v) - w_0(v)} \\ &= \int_0^{32} \frac{dv}{\frac{13.2}{247000} - (a + bv + cv^2)} + \int_{32}^{v_h} \frac{dv}{(-2.5 \cdot 10^{-5} v^3 + 0.007 \cdot v^2 - 0.66v + 28.35) - (a + bv + cv^2)} \\ &= -10^7 \frac{1}{\sqrt{10^{14}ac - 25 \cdot 10^{12} b^2 - 5.34 \cdot 10^9 c}} arctan\left(250 \cdot 10^{-9} \frac{2 \cdot 10^{13}(2cv + b)}{\sqrt{10^{14}ac - 25 \cdot 10^{12} b^2 - 5.34 \cdot 10^9 c}}\right)\Bigg|_0^{32} \\ &\quad \frac{-123.5 \cdot 10^6 arctan\left(\frac{5 \cdot 10^{-6}[2.0(12.35 \cdot 10^{12} c - 25.82 \cdot 10^9)v + 12.35 \cdot 10^{12} b + 1.52 \cdot 10^{12}]}{15.25 \cdot 10^{15} ac - 3.81 \cdot 10^{15} b^2 - 31.89 \cdot 10^{12} a - 936.16 \cdot 10^{12} b - 33.72 \cdot 10^{15} c + 13.04 \cdot 10^{12}}\right)\Bigg|_{32}^{v_h}}{\sqrt{15.25 \cdot 10^{15} ac - 3.81 \cdot 10^{15} b^2 - 31.89 \cdot 10^{12} a - 936.16 \cdot 10^{12} b - 33.72 \cdot 10^{15} c + 13.04 \cdot 10^{12}}} \end{aligned} \tag{28}$$

Using the Maple software tool, the acceleration time as a function of velocity. Acceleration distance is calculated as:

$$\frac{dx}{dt} = v \rightarrow dx = vdt = \frac{vdv}{f_{tr}(v) - w_0(v)} \rightarrow \int_0^{x_a} dx = \int_0^{v_h} \frac{vdv}{f_{tr}(v) - w_0(v)} \rightarrow x_a = \int_0^{v_h} \frac{vdv}{f_{tr}(v) - w_0(v)} \quad (29)$$

3.2.2. Coasting phase

Motion equation of the train in optimal coasting mode:

$$\frac{dv}{dt} = -w_0(v) \quad (30)$$

Using the variable dissociation method, the running time in coasting phase obtains:

$$\frac{dv}{-w_0(v)} = dt \rightarrow \int_{v_h}^{v_b} \frac{dv}{-w_0(v)} = \int_{t_a}^{t_a+t_c} dt \rightarrow t_c = \int_{v_b}^{v_h} \frac{dv}{a+bv+cv^2} = \frac{2}{\sqrt{4ac-b^2}} \arctan\left(\frac{2cv+b}{\sqrt{4ac-b^2}}\right) \quad (31)$$

In which braking velocity v_b is given as:

$$v_b = \frac{\psi(v_h)}{\varphi'(v_h)} with \varphi(v) = v \cdot w_0(v) = v(a+bv+cv^2), \psi(v) = v^2 \cdot w_0'(v) = v^2(b+2cv)$$

$$v_b = \frac{v_h}{1+\frac{w_0(v_h)}{v_h w_0'(v_h)}} = \frac{v_h}{1+\frac{a+v_h(b+cv_h)}{v_h(b+2cv_h)}} \quad (32)$$

Coasting distance is computed as: $\frac{dx}{dt} = v \rightarrow dx = vdt = \frac{vdv}{-w_0(v)}$

$$\int_{x_a}^{x_a+x_c} dx = \int_{v_h}^{v_b} \frac{vdv}{-w_0(v)} \rightarrow x_c = \int_{v_b}^{v_h} \frac{vdv}{a+bv+cv^2} \quad (33)$$

3.2.3. Braking phase

Motion equation of the train in optimal braking mode:

$$\frac{dv}{dt} = -w_0(v) - f_{br}(v) \quad (34)$$

Using the variable dissociation method, the running time in braking phase is given by:

$$\frac{dv}{-f_{br}(v) - w_0(v)} = dt \rightarrow \int_{v_b}^{0} \frac{dv}{-f_{br}(v) - w_0(v)} = \int_{t_a+t_c}^{t_a+t_c+t_b} dt \rightarrow \int_0^{v_b} \frac{dv}{f_{br}(v) + w_0(v)} = t_b \quad (35)$$

F_{br} is calculated as (7), from (35) the braking time can be written as:

$$t_b = \int_0^{v_b} \frac{dv}{f_{br\max} + (a+bv+cv^2)} = 2\frac{1}{\sqrt{4ac-b^2+4cf_{br\max}}} \arctan\left(\frac{2cv+b}{\sqrt{4ac-b^2+4cf_{br\max}}}\right) \quad (36)$$

The braking distance is calculated: from $\frac{dx}{dt} = v \rightarrow dx = vdt = \frac{vdv}{-f_{br}(v) - w_0(v)}$

$$\int_{x_a+x_c}^{x_a+x_c+x_b} dx = \int_{v_b}^{0} \frac{vdv}{-f_{br}(v) - w_0(v)} \rightarrow x_b = \int_0^{b_b} \frac{vdv}{f_{br}(v) + w_0(v)} \quad (37)$$

From the above calculations, total trip time is equal to $t_a + t_b + t_c$

4. SIMULATION RESULTS

The simulation is based on the data of Cat Linh-Ha Dong metro line, Vietnam. There are 12 stations, 1 depot, 6 traction substations, and two-side power supply mode. In this paper, simulation results are performed for the first Cat Linh station to the 12th Yen Nghia station with 12.661 km in length [28]. Simulation parameters are shown in Table 1 and Table 2. Cat Linh - Ha Dong metro line is short, operation modes of electrified train are comprised of three regimes: accelerating → coasting → braking.

Table 1. Simulation parameters of train

Parameters of metro train	Unit	Value
Train grand-up	2M2T	
Full load translating mass	kg	246700
Number of electrical traction units		08
Max speed	km/h	80
Base speed	km/h	40
Dwell time	s	30
Max acceleration/braking rates	m/s^2	0.94/1

Table 2. David's coefficients of train's resistance

Parameters	Value
$\boldsymbol{a}$	$1.19 \cdot 10^{-2}$
$\boldsymbol{b}$	$2.56 \cdot 10^{-3}$
$\boldsymbol{c}$	$1.54 \cdot 10^{-4}$

Figure 2 shows the responses of the optimal speed curve over the whole route in red when applying Pontryagin's maximum principle, and the speed curve without applying this principle in blue. The optimal speed profile has lower accelerating speeds, and longer coasting distances. Therefore, less power is consumed on the optimal speed profile. The responses in Figure 3 also indicate that the optimal speed profile and the original speed profile ensure the same running distance between stations and the fixed trip time according to the schedule.

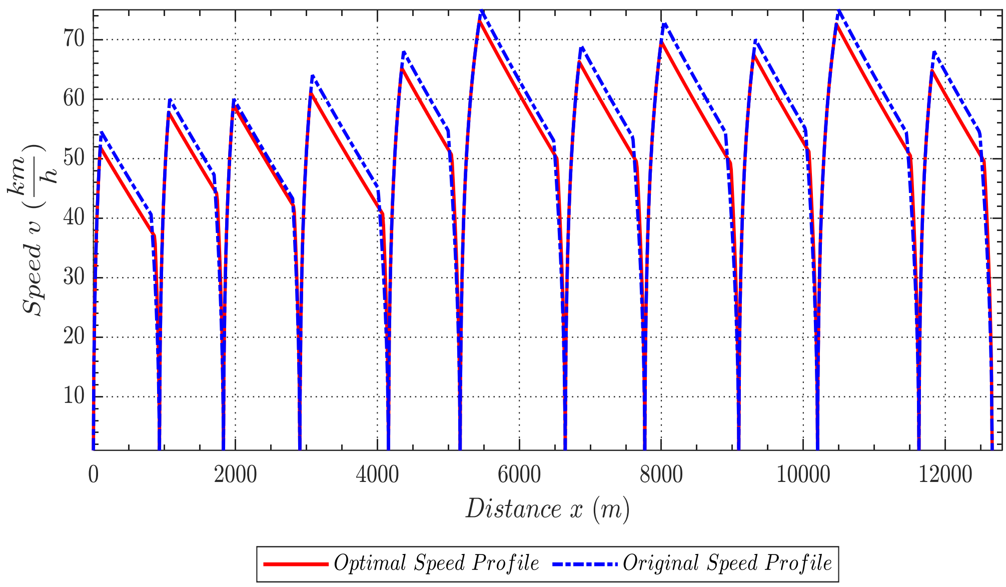

Figure 2. A comparison of optimal speed profile and original speed profile

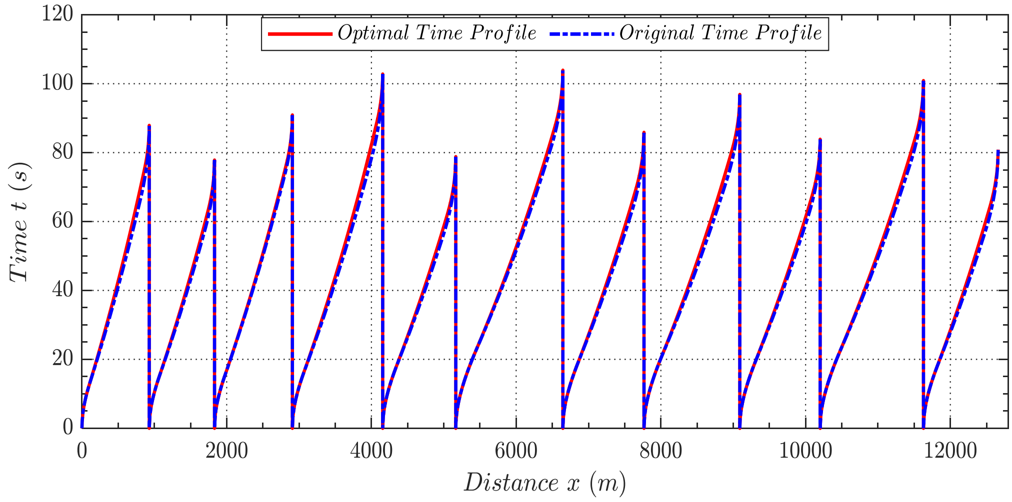

Figure 3. A comparison of optimal time profile and original time profile

Table 3. Results of a comparison of energy consumption with / without energy optimal strategy

Inter-station length	Distance (m)	Trip time (s)	Maximum speed in original speed profile (km/h)	Practical energy consumption (kWh)	Maximum speed in optimal speed profile (km/h)	Optimal energy consumption (kWh)	Energy saving (%)
Cat Linh - La Thanh	931	88	54.5	8.31	51.92	7.50	9.75
La Thanh - Thai Ha	902	78	60	10.20	57.75	9.40	7.84
Thai Ha - Lang	1076	91	60	10.20	59.06	9.86	3.33
Lang - Thuong Dinh	1248	103	64	11.73	61	10.60	9.63
Thuong Dinh - Ring Road 3	1010	79	68	13.41	65	12.23	8.80
Ring Road 3 - Phung Khoang	1480	104	75	16.75	73	15.82	5.55
Phung Khoang - Van Quan	1121	86	69	13.85	66.27	12.66	8.59
Van Quan - Ha Dong	1324	97	73	15.74	69.7	14.17	9.97
Ha Dong - La Khe	1110	84	70	14.30	66.27	13.18	7.83
La Khe - Van Khe	1428	101	75	16.75	72.58	15.53	7.28
Van Khe - Yen Nghia	1032	81	68	13.40	64.77	12.04	10.15
Total energy consumption				144.64		132.99	8.7

Regarding as track conditions, constraints, the speed from a station to another station is different. Results show in Table 3 the slowest speed at 54.5 km/h, the highest speed at 75 km/h, and being always smaller than limit speed 80 km/h while trip time complied with scheduled timetable. Table 3 also demonstrated that the lowest saving energy is 3.33% while the highest saving energy is to 10.15%. Therefore, saving energy of the whole route is 8.7%.

5. CONCLUSION

Simulation results by applying Pontryagin's maximum principle to find the optimal speed profile with fixed trip time for 12 stations of metro line Cat Linh - Ha Dong, Vietnam with three operation phases: accelerating, coasting, braking showed saving energy percent is up to 8.7% comparison with the original speed profile while the running time is unchanged. This research has not only brought remarkably economic benefits in energy consumption reduction without any costs for infrastructure of existing metro lines, but also created a foundation for designing the optimal speed profile applied to automation train operation (ATO).

ACKNOWLEDGMENTS

This research is funded by University of Transport and Communications (UTC) under grant number T2022-DT-007

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