TELKOMNIKA Telecommunication Computing Electronics and Control
Vol. 20, No. 3, June 2022, pp. 680~690
ISSN: 1693-6930, DOI: 10.12928/TELKOMNIKA.v20i3.23317

Attenuated-chattering adaptive second order variable structure controller for mismatched uncertain systems

Phan-Thanh Nguyen[1], Trieu Ton Ngoc[1,2], Cong-Trang Nguyen[3]
[1]Department of Industrial Electricity, Faculty of Electrical and Electronics Engineering,
Ho Chi Minh City University of Technology and Education, Ho Chi Minh City, Vietnam
[2]Department of Industrial Electricity, Faculty of Electrical and Electronics Engineering, Thu Duc College of Technology,
Ho Chi Minh City, Vietnam
[3]Power System Optimization Research Group, Faculty of Electrical and Electronics Engineering,
Ton Duc Thang University, Ho Chi Minh City, Vietnam

Article Info

Article history:

Received Aug 21, 2021
Revised Mar 23, 2022
Accepted Apr 01, 2022

Keywords:

Chattering removal
Mismatched uncertainty
Reduced-order estimator
Unknown disturbance
Variable structure control

ABSTRACT

In this paper, an attenuated-chattering adaptive second order variable structure controller (ACASOVSC) is proposed for mismatched uncertain systems using a Moore-Penrose inverse method. The key achievements of this study include three tasks: 1) influence of the chattering in control input is diminished, 2) finite-time convergence of system states is guaranteed, and 3) external disturbance is generally assumed to be unknown in advance. Firstly, a switching manifold which comprises only output information is defined. Secondly, a reduced-order variable structure estimator (ROVSE) with lower dimension is designed to reduce the computation burden and enhance the robustness. Thirdly, an adaptive approach is used to guess the upper bound of the unknown exogenous disturbance. Next, an ACASOVSC is investigated for attenuating the chattering phenomenon and stabilizing the system. Then, a novel linear matrix inequality (LMI) constraint by the Lyapunov technique is given such that the plant is entirely invariant to matched uncertainties and asymptotically stable. Finally, a mathematical illustration is simulated, which exhibits the usefulness and the feasible application of the proposed method.

Corresponding Author:

Cong-Trang Nguyen
Power System Optimization Research Group, Faculty of Electrical and Electronics Engineering
Ton Duc Thang University, Ho Chi Minh City, Vietnam
Email: nguyencongtrang@tdtu.edu.vn

NOMENCLATURE

A^T	transpose of the matrix A	$\|x(t)\|$	norm of the state vector $x(t)$
A^{-1}	inverse of the matrix A	$B^{\perp}$	perpendicular complement of matrix B
$x(t)$	states of the system	E^g	Moore-Penrose inverse of the matrix E
$y(t)$	output signal	λ_{max}	maximum eigenvalue
$u(t)$	sliding mode controller	$\beta_0, \beta_1, \beta_2$	unknown bounds of the uncertainties
$\bar{\varpi}(t)$	external disturbance	$\tilde{\beta}_0, \tilde{\beta}_1, \tilde{\beta}_2$	adaptive gain's estimation errors
$s(y(t), t)$	switching manifold function	$\zeta(x, t)$	matched non-linearity of system

1. INTRODUCTION

Variable structure control (VSC) with sliding mode, also called sliding mode control (SMC), is renowned for its robustness property to plant parameter fluctuations and the exogenous perturbations [1]. Owing to these advantages, many results associated with different types of SMC such as integral SMC, robust backstepping SMC, terminal SMC, and disturbance observer-based SMC [2]-[5]. Moreover, SMC has been successfully utilized to many practical engineering problems such as mechanical systems, wind turbines, power converters, induction motor drives, aircraft pitch control, fuel cells, anesthesia regulation, robotic manipulators, and vehicles [6]-[14]. Unfortunately, the main drawback of SMC is the unwanted high-frequency vibrations known as chattering phenomenon in control input. This phenomenon is generated by the discontinuous control signal and is very dangerous to the electromechanical plant's actuator [15]. Although the VSC in sliding mode has the remarkable achievements, in general, there are still three tasks that should be solved for SMC design. These involve: 1) chattering phenomenon removal: a novel SMC design not only ensures the finite-time stability of the plant but also suppresses the undesired chattering in control signal; 2) finite-time stability: state variables of the closed-loop plant should be converged to zero in finite-time; and 3) unknown external disturbances: in practical systems, an upper limit of the external perturbations is difficult to achieve advance knowledge.

For the above first task which should reduce the influence of chattering in the VSC systems, a boundary layer design which was developed in the paper [16] is undoubtedly the simplest technique. In this way, a discontinuous sign function is approximated by a smooth continuous function in an area named the margin layer around the switching surface. However, the margin layer design technique could not guarantee the asymptotic stability and the sacrifice of control exactness [17]. One solution to cope with the chattering is to replace the boundary layer design technique with well-known second-order sliding mode control (SOSMC) method. The concept of SOSMC methodology was first developed in the 1980s by [18]. The SOSMC technique and their applications have been widely applied in recent years, such as [19] and the reference therein. Cheng *et al.* [19] proposed controller for a class of uncertain dynamical structures with nonlinearities and known disturbance by using well-known linear matrix inequality (LMI) theory. Nevertheless, the state variables of the plant could not meet to zero in finite time.

For the above second task that must ensure the finite-time stabilization of the closed-loop plant, numerous applications of SOSMC have been reported in the published papers [20], [21]. Mathiyalagan and Sangeetha [20] presented a fractional switching surface based-SOSMC for nonlinear fractional-order systems satisfying the finite-time reachability condition. By creating a tangent-type barrier Lyapunov method, a novel SOSMC construction scheme was established in a step by step for handling the output constraint problem [21]. However, in these researches, the obtained results were not considered the external perturbations and a subset of the system's state information was assumed to be availability which cannot be measured directly or the available sensor prices are very expensive. In addition, a serious limitation of these controllers is that the chattering phenomenon is not reduced. These issues are also the third task which will be solved in this paper.

For the above third task which will solve unknown external perturbations, this problem has been investigated by researchers [22]-[24]. In [22], by using homogeneity approach, authors investigated the smooth super twisting algorithm that ensures the robustness and performance parameters for the linear time invariant systems. A state feedback controller was designed in [23] for perturbed dynamical structures that considers matched and unmatched exogenous perturbations. A controller was explored in the work [24] for attaining finite-time stability and reducing the chattering in control signals for nonlinear uncertain systems. However, the authors in these researches only consider the known disturbances which bounded by the output information or the state variables with the upper bound of the advance knowledge scalar. In practical control plants, an upper limit of the external perturbations is difficult to achieve advance knowledge.

Motivated by the aforementioned analysis, to the best of our wisdom, little consideration has been paid to achieving finite-time stability with reduced-chattering control input for the mismatched uncertain systems with unknown disturbances. This problem is still open in the literature. In this study, an adaptive second order variable structure controller (ACASOVSC) is suggested to lessen the chattering phenomenon and simultaneously delete the indispensability information requirement about the upper bound of exogenous perturbations via adaptive tuning law. In addition, utilizing the proposed approach, the state trajectories' convergence in the closed-loop plant is also ensured in finite time. Simulation is offered to validate the viability and efficiency of the suggested technique.

The arrangement of this work is as follows. The state-space model and preliminaries are pronounced in section 2. The key results of the work are derived in section 3 which includes a regular form of the plant, a novel reduced-order variable structure estimator (ROVSE), and the design of an ACASOVSC. The system's asymptotic stability is analyzed in section 4. The feasibility of the anticipated technique is clarified in section 5 with numerical illustration. To finish this work, a deduction is made in section 6.

2. PROBLEM STATEMENT AND PRELIMINARIES

We consider the systems with mismatched parameter uncertainties and unknown exogenous disturbances as below:

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + B[u(t) + \zeta(x,t)], y(t) = Cx(t) \quad (1)$$

Where:

$$x(t) \in R^n, u(t) \in R^m, y(t) \in R^p$$

Are respectively the vectors of system states, signals of control, and outputs of the plant. The indexes n, m, p are the number of the plant's state variables, control signals, and channels of output, respectively. The constant matrices A, B, C have suitable lengths. The vector $\zeta(x,t)$ is a matched non-linearity of the system. The term $\Delta A(t)$ are the mismatched uncertainties. Before moving with the main achievements in this paper, the standard assumptions are necessary for this work as:

- Assumption 1: the matrices B, C have full rank, $rank(CB) = m$, and $m \leq p < n$.
- Assumption 2: the pair (A, B) and (A, C) are entirely controllable and observable, respectively.
- Assumption 3: the symbol $\Delta A(t)$ indicates the mismatched parameter uncertainty of the system and satisfies the following form $\Delta A(t) = D\Sigma(x(t),t)E$, where the matrices D, E are constant and $\Sigma(x(t),t)$ is unknown matrix function but constrained as $\|\Sigma(x(t),t)\| \leq 1$ for all $(x,t) \in R^n \times R$.

To construct an attenuated-chattering adaptive controller in sliding mode control technique, we will definite a switching manifold function which includes the output information as:

$$s(y(t),t) = \dot{\sigma}(y(t),t) + \bar{X}\ \sigma(y(t),t) \quad (2)$$

Where:

$$\sigma(y,t) = Sx(t) = Fy(t) = FCx(t)$$

The matrix $F \in R^{m\times p}$ is scalar, and $S \in R^{m\times n}$ is a switching matrix. The matrix F should be chosen to guarantee $S = FC.\bar{X} \ \in R^{m\times m}$ is any diagonal matrix which given by designer.

3. MAIN RESULTS

3.1. Constructing a regular form of the plant

In this part, a regular form of the plant will be achieved by using the transformation matrix procedure and the published results in the paper [25]. Let us denote Γ an $n \times n$ symmetric matrix satiating $\Gamma = I - E^g E$ with $B^{\perp T} D \neq 0$, where I is an identity matrix. In other words, the system satisfies the mismatching condition. Now, the sliding matrix S in (2) can be stated as:

$$S = RB^T Q^{-1}$$

Where:
R is any $m \times m$ non-singular matrix.
$Q = \Gamma X \Gamma + BYB^T. X, Y$ are symmetric matrices which will be found in the LMI's solutions:

$$\Gamma X \Gamma + BYB^T > 0 \text{ and } B^{\perp T}(A\,\Gamma X \Gamma + \Gamma Y \Gamma A^T)B^{\perp} < 0$$

- Assumption 4: the sliding matrix S must be existed such that $S = RB^T Q^{-1}$ is solvable.

To transform the original system (1) into the regular form, the state transformation can be established as $[\vartheta\ \sigma]^T = Tx$, where $\vartheta \in R^{n-m}$ is unmeasurable reduced-order variables of the system and $\sigma \in R^p$ is measurable sliding matrix. A conversion matrix and its inverse form are $T = [B^{\perp T}\ RB^T Q^{-1}]^T$ and $T^{-1} = [QB^{\perp}(B^{\perp T} QB^{\perp})^{-1}\ B(SB)^{-1}]$, respectively. The matrix (SB) is supposed to be non-singular. Derivative of the state transformation, and employing the transformation matrix and its inverse form, the regular form of the original plant (1) can be gotten as:

$$\dot{\vartheta}(t) = [\bar{A}_{11} + \Delta\bar{A}_{11}(t)]\vartheta(t) + [\bar{A}_{12} + \Delta\bar{A}_{12}(t)]\sigma(t)$$

$$\dot{\sigma}(t) = [\bar{A}_{21} + \Delta\bar{A}_{21}(t)]\vartheta(t) + [\bar{A}_{22} + \Delta\bar{A}_{22}(t)]\sigma(t) + (SB)[u(t) + \zeta(x,t)] \quad (3)$$

Where:

$$\bar{A}_{11} + \Delta\bar{A}_{11} = B^{\perp T}[A + D\Sigma E]QB^{\perp}(B^{\perp T}QB^{\perp})^{-1}, \bar{A}_{12} + \Delta\bar{A}_{12} = B^{\perp T}[A + D\Sigma E]B(SB)^{-1}$$

$$\bar{A}_{21} + \Delta\bar{A}_{21} = RB^{T}Q^{-1} \times \, [A + D\Sigma E]QB^{\perp}(B^{\perp T}QB^{\perp})^{-1}$$

$$\bar{A}_{22} + \Delta\bar{A}_{22} = RB^{T}Q^{-1}[A + D\Sigma E]B(SB)^{-1}, \vartheta(t) = \mathrm{B}^{\perp T}x(t), \sigma(t) = Fy(t) = Sx(t)$$

By considering the achieved results in the paper [25], the (3) can be acquired:

$$\dot{\vartheta}(t) = \bar{A}_{11}\vartheta(t) + [\bar{A}_{12} + \Delta\bar{A}_{12}(t)]\sigma(t)$$

$$\dot{\sigma}(t) = \bar{A}_{21}\vartheta(t) + [\bar{A}_{22} + \Delta\bar{A}_{22}(t)]\sigma(t) + (SB)[u(t) + \zeta(x,t)] \tag{4}$$

3.2. Attenuated-chattering adaptive second order VSC design

In this part, an ACASOVSC based on the estimated variables and output signal will be investigated to alleviate the chattering and stabilize the mismatched uncertain systems (4). Firstly, a ROVSE will be designed to guess the immeasurable the plant's variables. A novel ROVSE including the estimation variable and measurable sliding function is designed as:

$$\dot{\hat{\vartheta}}(t) = \bar{A}_{11}\hat{\vartheta}(t) + \bar{A}_{12}\sigma(t) \tag{5}$$

By introducing the estimation error vector $\tilde{\vartheta}(t) = \hat{\vartheta}(t) - \vartheta(t)$, and combining the first equation of (4), the error dynamics of ROVSE is obtained by:

$$\dot{\tilde{\vartheta}}(t) = B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}\tilde{\vartheta}(t) - B^{\perp T}D\Sigma(x,t)EB(SB)^{-1}\sigma(t) \tag{6}$$

Now, we will propose the theorem to calculate the upper limit of the observer error.

- Theorem 1: consider the error dynamics equation (6), the matrix $B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}$ is stable matrix. The estimation error's norm is limited by the upper limit $\tilde{\psi}(t)$, which is the answer of the following (7).

$$\tilde{\psi}(t) = \lambda_{max} + \tilde{\eta}\|B^{\perp T}D\|\|EB(SB)^{-1}\|\|\sigma(t)\| \tag{7}$$

Where:
λ_{max} negative value and maximum eigenvalue of the matrix:

$$B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}$$

The sign $\tilde{\eta}$ is a positive scalar and an initial condition of the estimation error:

$$\tilde{\psi}(0) \geq \tilde{\eta}\|\tilde{\vartheta}(0)\|$$

- Proof of Theorem 1: from the published results [25], the matrix $B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}$ is stable. Thus, we get $\|exp[\,B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}t]\| \leq \tilde{\eta}\, exp(\,\lambda_{max}$ by solving the error dynamics (6) and multiplying the term $\exp(-\lambda\_max$ into two sides. After that, applying the lemma that proved in the publication [26], we can obtain:

$$\|\tilde{\vartheta}(t)\| \leq \tilde{\psi}(0)\, exp\left(\lambda_{max}()\int_0^{t\int} \tilde{\eta}\, exp\left[\lambda_{max}[][\|B^{\perp T}D\|\|EB(SB)^{-1}\|\sigma(\tau)]\right]\right) \tag{8}$$

$\leq \tilde{\psi}(t)$ if $\tilde{\psi}(0) \geq \tilde{\eta}\|\tilde{\vartheta}(0)\|$, where $\tilde{\vartheta}(0)$ and $\tilde{\psi}(0)$ are the initial values of the estimation error dynamics and the error upper bound, respectively. The upper bound $\tilde{\psi}(t)$ meets solutions of (7). Therefore, we can determine that $\|\tilde{\vartheta}(t)\| \leq \tilde{\psi}(t)$ for $t \geq 0$. That is, the proof of this theorem is proved completely.

Next, an ACASOVSC will be designed based on the obtained result in theorem 1 for solving the chattering and stabilizing the mismatched uncertain systems (1). To keep the plant's state trajectory moving along switching surface in finite time and decrease the unwanted chattering, the controller is constructed as:

$$u(t) = u(0) - (SB)^{-1}\int_0^t\{\tilde{k}_1[\|\hat{\vartheta}(t)\| + \tilde{\psi}(t)] + \tilde{k}_2\|\sigma(t)\| + \tilde{k}_3\|\dot{y}\| + \psi(t) + \alpha\|s(t)\|\}\, sign(s(t))dt, \tag{9}$$

Where:
$\tilde{k}_1, \tilde{k}_2, \tilde{k}_3$ are scalar gains which will be specified later, and

$$\psi(t) = \hat{\beta}_0 + \hat{\beta}_1\left[\left\|\hat{\vartheta}(t)\right\| + \tilde{\psi}(t)\right] + \hat{\beta}_2\|\sigma(t)\|,$$

the adaptive laws are:

$$\dot{\hat{\beta}}_0 = \bar{\gamma}\left[-\bar{v}_0\hat{\beta}_0 + 1\right] \dot{\hat{\beta}}_1(t) = \hat{\gamma}\{-\hat{v}_1\hat{\beta}_1(t) + [\left\|\hat{\vartheta}(t)\right\| + \tilde{\psi}(t)]\},\ \dot{\hat{\beta}}_2(t) = \tilde{\gamma}\left[-\tilde{v}_2\hat{\beta}_2(t) + \|\sigma(t)\|\right],$$

and the scalars $\alpha > 0, v_0 > 0, \hat{v}_1 > 0, \tilde{v}_2 > 0, \bar{\gamma} > 0, \hat{\gamma} > 0$, and $\tilde{\gamma} > 0$.

- Theorem 2: consider the systems with mismatched uncertainties and unknown perturbations (4) subject to assumptions 1-4. Concurrently, the switching matrix F gratifies the formula $S = FC$ and the error dynamics of the ROVSE (6) satisfies Theorem 1. Then, the state variable trajectories of the plant (1) are forced onto the switching manifold $s(y(t), t) = 0$ in finite time under the control signal (9) when the scalar gains satisfy the settings.

$$\tilde{k}_1 \geq \|SAQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}\|\|B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}\|$$

$$\tilde{k}_2 \geq \|SAQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}\|[\|B^{\perp T}AB(SB)^{-1}\| + \|B^{\perp T}D\|\|EB(SB)^{-1}\|]$$

$$\tilde{k}_3 \geq [\|SAB(SB)^{-1}\| + \|\bar{X}\|]\|F\|$$

- Proof of Theorem 2: in order to prove this theorem, the several statements should be noted. Firstly, by differencing the switching function and using (1), we have:

$$\dot{\sigma}(y,t) = SAQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}\vartheta(t) + SAB(SB)^{-1}\sigma(t) + (SB)u(t) + \varpi(t) \tag{10}$$

Where:

$$\varpi(t) = S\Delta A(t)QB^{\perp}(B^{\perp T}QB^{\perp})^{-1}\vartheta(t) + S\Delta A(t)B(SB)^{-1}\sigma(t) + (SB)\zeta(x,t)$$

The second order derivative of the switching function is expanded:

$$\ddot{\sigma}(y,t) = SAQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}\dot{\vartheta}(t) + SAB(SB)^{-1}\dot{\sigma}(t) + (SB)\dot{u}(t) + \dot{\varpi}(t)$$

Hence, the time derivative of the switching manifold (2) can be rewritten as:

$$\dot{s}(y(t),t) = SAQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}\dot{\vartheta}(t) + [SAB(SB)^{-1} + \bar{X}]\dot{\sigma}(y(t),t) + (SB)\dot{u}(t) + \dot{\varpi}(t \tag{11}$$

Secondly, the perturbation $\dot{\varpi}(t)$ in (11) is assumed to be satisfied the form:

$$\|\dot{\varpi}(t)\| \leq \beta_0 + \beta_1\|\vartheta(t)\| + \beta_2\|\sigma(t)\|$$

Where β_0, β_1, and β_2 are unknown bound of the uncertainties. However, in the practical control plants, the uncertainties' limits are unidentified in advance due to the complex of the systems. Thus, these bounds will be estimated by the adaptive tuning laws. Since $\tilde{\vartheta}(t) = \hat{\vartheta}(t) - \vartheta(t)$ and using theorem 1, the disturbances can be presented as:

$$\|\dot{\varpi}(t)\| \leq \beta_0 + \beta_1\left[\left\|\hat{\vartheta}(t)\right\| + \tilde{\psi}(t)\right] + \beta_2\|F\|\|y\|$$

Now, we will use the Lyapunov function $V(\sigma)$, which defined in the published research [27] as:

$$V(\sigma) = \|s(t)\| + 0.5\bar{\gamma}^{-1}\tilde{\beta}_0^2(t) + 0.5\hat{\gamma}^{-1}\tilde{\beta}_1^2(t) + 0.5\tilde{\gamma}^{-1}\tilde{\beta}_2^2(t)$$

Where $\tilde{\beta}_0, \tilde{\beta}_1, \tilde{\beta}_2$ are the adaptive gain's estimation errors. That is:

$$\tilde{\beta}_0 = \beta_0 - \hat{\beta}_0, \tilde{\beta}_1 = \beta_1 - \hat{\beta}_1, \tilde{\beta}_2 = \beta_2 - \hat{\beta}_2$$

By taking time derivative of $V(\sigma)$ and taking norm right hand side, and using the control signal (9), and the Lyapunov function, we obtain:

$$\dot{V}(\sigma) \leq -\alpha V(\sigma) + 0.5\alpha\bar{\gamma}^{-1}\tilde{\beta}_0^2(t) + 0.5\alpha\hat{\gamma}^{-1}\tilde{\beta}_1^2(t) + 0.5\alpha\tilde{\gamma}^{-1}\tilde{\beta}_2^2(t) + \bar{v}_0 + \hat{v}_1\left[-\hat{\beta}_1^2(t) + \hat{\beta}_1(t)\beta_1(t)\right] + \tilde{v}_2\left[-\hat{\beta}_2^2(t) + \hat{\beta}_2(t)\beta_2(t)\right] \quad (12)$$

By choosing $\bar{v}_0 \geq \frac{\alpha}{\bar{\gamma}}, \hat{v}_1 \geq \frac{\alpha}{\hat{\gamma}}, \tilde{v}_2 \geq \frac{\alpha}{\tilde{\gamma}}$, it can be shown that:

$$\dot{V}(\sigma) \leq -\alpha V(\sigma) + 0.5[\bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2]$$

The Lyapunov function $\dot{V}(\sigma)$ can be articulated as:

$$0 \leq V(t) \leq e^{-\alpha t}\{V(0) - 0.5\alpha^{-1}[\bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2]\} + 0.5\alpha^{-1}[\bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2)]$$

Now, we can see that:

$$\lim_{t\to\infty} V(t) \leq 0.5\alpha^{-1}[\bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2]$$

In addition, by using above Lyapunov function, it is easy to see that:

$$0 \leq \|s(t)\| + 0.5\bar{\gamma}^{-1}\tilde{\beta}_0^2 + 0.5\hat{\gamma}^{-1}\tilde{\beta}_1^2 + 0.5\tilde{\gamma}^{-1}\tilde{\beta}_2^2 \leq e^{-\alpha t}V(0) + 0.5[\bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2]\int_0^t e^{-\alpha(t-\tau)}d\tau$$

It follows that:

$$\|s(t)\| \leq e^{-\alpha t}\{\|s(0)\| + 0.5\bar{\gamma}^{-1}\tilde{\beta}_0^2(0) + 0.5\hat{\gamma}^{-1}\tilde{\beta}_1^2(0) + 0.5\tilde{\gamma}^{-1}\tilde{\beta}_2^2(0) - 0.5\alpha^{-1}[\bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2]\} + 0.5\alpha^{-1}[\bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2]$$

So that:

$$\lim_{t\to\infty}\|s(t)\| \leq 0.5\alpha^{-1}[\bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2] = 0.5\alpha^{-1}\theta(t)$$

Where:

$$\theta(t) = \bar{v}_0\beta_0^2 + \hat{v}_1\beta_1^2 + \tilde{v}_2\beta_2^2$$

It is clear that $\dot{V}(\sigma) < 0$ if $\|s(t)\| \leq \frac{0.5\theta(t)}{\alpha}$. The decrease of $V(\sigma)$ ultimately controls the state trajectories of the closed-loop plant into $\|s(t)\| \leq \frac{0.5\theta(t)}{\alpha}$ [28]. Thus, the trajectories of the plant are limited eventually as:

$$\lim_{t\to\infty} s(t) \in (\|s(t)\| \leq 0.5\alpha^{-1}\theta(t))$$

Which is a small set obtaining the origin of the closed-loop plant. To ensure constrained motion around the switching surface, the positive constant α is selected to be large enough so that $\dot{V}(\sigma) < 0$ when $V(\sigma)$ is out of the limited field that comprises an equilibrium point [28]. That is, the switching manifold $s(t)$ will approach to zero in finite time and the state trajectories of the plant are reached in finite time under the controller (9).

4. ASYMPTOTIC STABILITY IN SLIDING MODE

In this part, we will continue to analyze the asymptotic stability of the system (3) in the sliding mode via the LMI technique and Lyapunov method. Now, we establish the following LMI:

$$\begin{bmatrix} \hat{A}^TG + G^T\hat{A} + \varepsilon^{-1}G^T\hat{D}\hat{D}^TG & \hat{A}^TG - G^T + \Psi + \varepsilon^{-1}G^T\hat{D}\hat{D}^TG & \hat{E}^T \\ G^T\hat{A} - G + \Psi + \varepsilon^{-1}G^T\hat{D}\hat{D}^TG & -G - G^T + \varepsilon^{-1}G^T\hat{D}\hat{D}^TG & 0 \\ \hat{E} & 0 & -\varepsilon^{-1}I \end{bmatrix} < 0 \quad (13)$$

Where:

$$\tilde{A} = B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}, \tilde{D} = B^{\perp T}D, \tilde{E} = EQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}$$

A non-zero matrix $G \in R^{(n-m)\times(n-m)}$, a positive matrix $\Psi \in R^{(n-m)\times(n-m)}$, and $\varepsilon > 0$. To prove the asymptotic stability of the plant, the following theorem is created.

- Theorem 3: assume that the proposed LMI (13) has a feasible answer $\Psi > 0$, non-zero matrix G. The switching manifold function is defined in (2). Then, the motion dynamics in the first equation of the systems (3) restricted to the specified switching manifold $s(y(t), t) = 0$ is asymptotically stable.
- Proof of theorem 3: the first equation (3) in sliding mode can be presented by a switched plant.

$$\dot{\vartheta}(t) = \left[\hat{A} + \hat{D}\Sigma(x,t)\hat{E}\right]\vartheta(t)$$

Where:

$$\hat{A} = B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}, \hat{D} = B^{\perp T}D, \text{ and}$$

$$\hat{E} = EQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}$$

To prove the stability of the switching motion, let us cogitate the Lyapunov function as $V(t) = \vartheta^T\Psi\vartheta$.
Then, differentiating $V(t)$ with respect to the switched plant's trajectories, we have:

$$V^{\cdot}(t) = \vartheta\text{\^{}}T\ (t)\{\left[\hat{A} + \hat{D}\Sigma(x,t)\hat{E}\right]^T \times \Psi + \Psi[A\hat{} + D\hat{}\Sigma(x,t)E\hat{}]\}\vartheta(t)$$

Based on the Lyapunov function theory, the time derivative $\dot{V}(t) < 0$. By utilizing the Lemmas 1, 2 in the published paper [27], simultaneously applying the well-known Schur complement [29] to LMI (13), let us first specify:

$$W = [0\quad \Psi;\quad \Psi\quad 0]^T, \Pi = [I\ \ I]$$

$$\Xi = [(\hat{A} + \hat{D}\Sigma(x,t)\hat{E})^T - I]^T$$

$$\Xi^{\perp} = [I\quad \hat{A} + \hat{D}\Sigma(x,t)\hat{E}]^T$$

$$\Pi^T = [I\ \ I]^T, \Pi^{\perp T} = [I\ -I]^T$$

And G is the answer of the proposed LMI (13). This produces:

$$\Theta \le \begin{bmatrix} \hat{A}^TG + G^T\hat{A} + \varepsilon^{-1}G^T\hat{D}\hat{D}^TG & \hat{A}^TG - G^T + \Psi + \varepsilon^{-1}G^T\hat{D}\hat{D}^TG \\ G^T\hat{A} - G + \Psi + \varepsilon^{-1}G^T\hat{D}\hat{D}^TG & -G - G^T + \varepsilon^{-1}G^T\hat{D}\hat{D}^TG \end{bmatrix} + \varepsilon\begin{bmatrix}\hat{E}^T \\ 0\end{bmatrix}\begin{bmatrix}\hat{E} & 0\end{bmatrix} < 0, \quad (14)$$

Where ε is a positive scalar. By using the Lemma 2 of [27] and the (14), we have $\Xi^{\perp T}W\Xi^{\perp} < 0$.
Since,

$$W = [0\ \ \Psi;\ \ \Psi\ \ 0]^T,$$

$$\Xi^{\perp} = [I\quad \hat{A} + \hat{D}\boldsymbol{\Sigma}(\boldsymbol{x},t)\hat{E}]^T,$$

We obtain:

$$\Xi^{\perp T}W\Xi^{\perp} = [\hat{A} + \hat{D}\Sigma(x,t)\hat{E}]^T\Psi + \Psi[\hat{A} + \hat{D}\Sigma(x,t)\hat{E}] < 0$$

Now, we can conclude that $\dot{V}(t) < 0$. Thus, if the established LMI (13) is fulfilled, then the mismatched uncertain systems (1) is asymptotically stable. This theorem is entirely proved.

5. NUMERICAL EXAMPLE

For this part, the suggested technique will be executed to the detail example which will be simulated by the well-known Matlab software. Now, we consider the example that modified from [26]

$$\dot{x}(t) = \left\{\begin{bmatrix} -3 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 1 & -2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\Sigma(x(t),t)[0\ \ 1\ \ 0]\right\}x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}[u(t) + \zeta(x,t)]$$

$$y(t) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}x(t) \quad (15)$$

Where:

$x(t) \in R^3$,

$u(t) \in R^1$,

$y(t) \in R^2$,

and the parameter uncertainties:

$$\Sigma(x(t), t) = 0.14\, sin(\, 0.273t)$$

The disturbance $\dot{\varpi}(t)$ is assumed to satisfy.

$$\|\dot{\varpi}(t)\| \le \beta_0(t) + \beta_1(t) \left\| \begin{bmatrix} -0.7071 & 0.7071 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} [x_1(t)\ \ x_2(t)\ \ x_3(t)]^T \right\| + \beta_2(t)\|F\|\|y(t)\| \tag{16}$$

By using the LMI Toolbox of the Matlab software environment, we attain the asymmetric matrices as:

$$X = [0.3519\ \ 0\ \ -0.0607; 0\ \ 0\ \ 0; -0.0607\ \ 0\ \ 0.3691]$$

$$Y = [0.5400]$$

Next, we find the sliding matrix F by solving the equation:

$$FC = RB^T Q^{-1}$$

And select the diagonal matrix $\bar{X} = [1]$. The switching manifold (2) is calculated as :

$$s(y(t), t) = [1.852\ \ -1.852]\dot{y}(t) + [1.852\ \ -1.852]y(t) \tag{17}$$

After the calculating the matrices $\bar{A}_{11}$ and $\bar{A}_{12}$, the error dynamics (6) is found by:

$$\dot{\tilde{\vartheta}}(t) = [-4.0000\ \ 0.7071; 0.0000\ \ -2.0000]^T \tilde{\vartheta}(t) - [-0.7071\ \ -1.0000]^T \Sigma(x, t)[0.5400]\sigma(t) \tag{18}$$

Where the unknown function of the uncertainties:

$$\Sigma(x(t), t) = 0.14 \sin(\, 0.273t)$$

And the output switching function:

$$\sigma(t) = [1.852\ \ -1.852]y(t)$$

By selecting the scalar:

$$\tilde{\eta} = 0.016$$

The upper bound of estimation error is the answer of the form:

$$\dot{\tilde{\psi}}(t) = [-2.0000]\tilde{\psi}(t) + 0.0106\|\sigma(t)\|$$

According to (9), the ACASOVSC of the mismatched uncertain systems is computed as:

$$u(t) = -0.5400 \int_0^t \{\hat{\beta}_0(t) + [10.6897 + \hat{\beta}_1(t)][\|\hat{\vartheta}(t)\| + \tilde{\psi}(t)] + [7.8965 + \hat{\beta}_2(t)]\|\sigma(t)\| + 10.4765\|\dot{y}\| + 0.0150\|s(t)\|\} sign(s(t))dt \tag{19}$$

Where $\hat{\vartheta}(t)$ and $\tilde{\psi}(t)$ are answers of the ROVSE (5) and the observer error's upper bound (7), respectively. The estimation variables $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ are answers of the adaptive laws:

$$\dot{\hat{\beta}}_0 = -0.0121\hat{\beta}_0 + 1$$

$$\dot{\hat{\beta}}_1 = -0.0101\hat{\beta}_1 + [\|\hat{\vartheta}\| + \tilde{\psi}]$$

$$\dot{\hat{\beta}}_2 = \tilde{\gamma}[-0.0211\hat{\beta}_2 + \|\sigma\|]$$

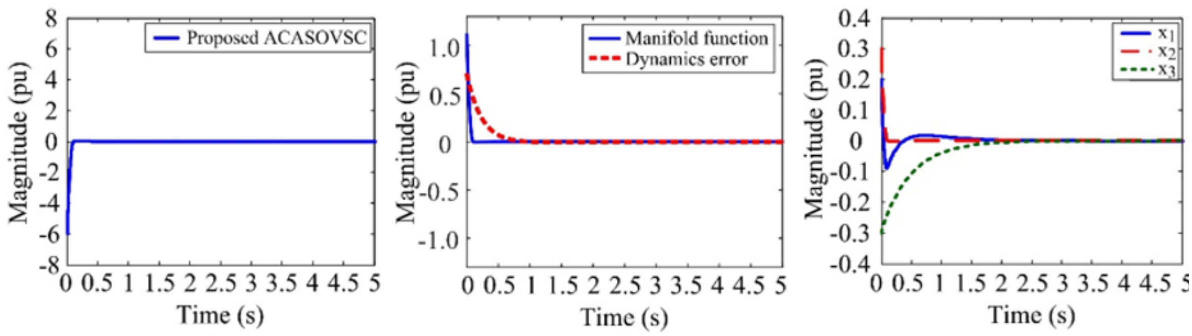
The initial conditions of system states, the ROVSE, and the error dynamics are selected to be:

$x(0) = [0.2 \;\; 0.3 \;\; -0.3]^T$,

$\hat{\vartheta}(0) = [-0.15 \;\; 0.1]^T$, and

$\tilde{\vartheta}(0) = [0.2 - 0.3]^T$, respectively.

The control signal, the switching manifold, the error dynamics, and the plant state trajectories of the mismatched uncertain systems (1) by applying the proposed ACASOVSC (19) are displayed in Figure 1.



(a) (b) (c)

Figure 1. The time histories of the system; (a) the control signal (b) the switching manifold and the error dynamics and (c) the system states

- Remark 1: the time response of the proposed reduced-chattering SOSMC signal (19) considering the unknown external disturbance (16) is plotted in Figure 1(a). The time history of the switching manifold function (17) and the ROVSE's error dynamics (18) are displayed in Figure 1(b). From these figures, we see that the suggested controller guarantees fast behavior and quite small amplitude. In addition, the chattering phenomenon in input is weakened. Therefore, these results lead to the proposed controller could not be applied in the published works [20], [21], [23]. This is the first and third main tasks of our work.
- Remark 2: the graph of the plant states variables is showed in Figure 1(c) under the proposed ACASOVSC (19). We can see that the plant states decline immediately to zero after 1.6 seconds. In order words, the states of the plant tend to the specified manifold $s(y(t), t) = 0$ in finite time which the published work [19] could not be performed the achievement. This is the second key task in this research.

From the above-mentioned examination of the obtained simulation results, the proposed approach does not need the availability of the state variables. It is concluded that the suggested technique is effective in dealing with the finite-time stabilization and the proposed chattering diminution problem. Moreover, this method is very useful and more realistic since it can be easily implemented in practice.

6. CONCLUSION

In this part, we have presented new results in designing the ACASOVSC for the mismatched uncertain systems affected by unknown external disturbances. The proposed controller can effectively reduce the undesired chattering in control input and stabilize the closed-loop plant with the unknown perturbations. In addition, the gain tuning mechanism in the adaptive laws guarantees that the gain is not overvalued regarding the real unknown value of the external disturbances. Further, the sufficient condition has been given by terms of strict LMIs such that the motion dynamics owns the property of asymptotical stability. Finally, in the obtained results of the simulation section, the suggested controller is demonstrated to be better in addressing chattering than the existing variable structure controllers.

ACKNOWLEDGEMENTS

The authors would like to thank the financial support provided by Ho Chi Minh City University of Technology and Education in Vietnam.

REFERENCES

[1] J. Y. Hung, W. Gao and J. C. Hung, "Variable structure control: a survey," in *IEEE Transactions on Industrial Electronics*, vol. 40, no. 1, pp. 2-22, Feb. 1993, doi: 10.1109/41.184817.

[2] A. Eltayeb, M. F. Rahmat, M. A. M. Basri, and M. S. Mahmoud, "An improved design of integral sliding mode controller for chattering attenuation and trajectory tracking of the quadrotor UAV," *Arabian Journal* for *Science* and *Engineering*, vol. 45, no. 8, pp. 6949-6961, 2020, doi: 10.1007/s13369-020-04569-5.

[3] A. R. Firdaus and M. O. Tokhi, "Design of set-point weighting-based dynamic integral sliding mode control with nonlinear full-order state observers for quadcopter UAVs," *International Journal of Modelling, Identification and Control*, vol. 32, no. 2, pp. 114-134, 2019, doi: 10.1504/IJMIC.2019.10024016.

[4] J. Huang, M. Zhang, S. Ri, C. Xiong, Z. Li, and Y. Kang, "High-Order Disturbance-Observer-Based Sliding Mode Control for Mobile Wheeled Inverted Pendulum Systems," in *IEEE Transactions on Industrial Electronics*, vol. 67, no. 3, pp. 2030-2041, March 2020, doi: 10.1109/TIE.2019.2903778.

[5] S. A. Ajwad, J. Iqbal, A. A. Khan, and A. Mehmood, "Disturbance-observer-based robust control of a serial-link robotic manipulator using SMC and PBC techniques," *Studies in Informatics and Control*, vol. 24, no. 4, pp. 401-408, Dec. 2015. [Online]. Available: https://www.researchgate.net/profile/Jamshed-Iqbal-2/publication/287198642_Disturbance-Observer-Based_Robust_Control_of_a_Serial-link_Robotic_Manipulator_Using_SMC_and_PBC_Techniques/links/567296ff08aeb8b21c70cee9/Disturbance-Observer-Based-Robust-Control-of-a-Serial-link-Robotic-Manipulator-Using-SMC-and-PBC-Techniques.pdf

[6] M. M. A. Hbib, O. Belghazi, M. R. Douiri, and M. Cherkaoui, "New methodology for asynchronous motor the adaptive-sliding-mode-control capable of high performance regulation," *Telecommunication Computing Electronics and Control (TELKOMNIKA)*, vol. 16, no. 2, pp. 558-566, Apr. 2018, doi: 10.12928/TELKOMNIKA.v16i2.7057.

[7] M. Derbeli, O. Barambones, M. Farhat, J. A. R-Hernanz, and L. Sbita, "Robust high order sliding mode control for performance improvement of PEM fuel cell power systems," *International Journal Hydrogen Energy*, vol. 45, no. 53, pp. 29222-29234, Oct. 2020, doi: 10.1016/j.ijhydene.2020.07.172.

[8] M. Ilyas, M. F. U. Butt, M. Bilal, K. Mahmood, A. Khaqan, and R. A. Riaz, "A review of modern control strategies for clinical evaluation of propofol anesthesia administration employing hypnosis level regulation," *BioMed Research International*, vol. 2017, pp. 1-12, 2017, doi: 10.1155/2017/7432310.

[9] X. Cheng, H. Liu, and W. Lu, "Chattering-suppressed sliding mode control for flexible-joint robot manipulators," *Actuators*, vol. 10, no. 11, pp. 1-24, 2021, doi: 10.3390/act10110288.

[10] S. Riache, M. Kidouche, and A. Rezoug, "Adaptive robust nonsingular terminal sliding mode design controller for quadrotor aerial manipulator," *Telecommunication Computing Electronics and Control (TELKOMNIKA)*, vol. 17, no. 3, pp. 1501-1512, 2019, doi: 10.12928/TELKOMNIKA.v17i3.10470.

[11] N. Ullah, Y. Mehmood, J. Aslam, A. Ali, and J. Iqbal, "UAVs-UGV Leader Follower Formation Using Adaptive Non-Singular Terminal Super Twisting Sliding Mode Control," in *IEEE Access*, vol. 9, pp. 74385-74405, 2021, doi: 10.1109/ACCESS.2021.3081483.

[12] A. Khaqan, Q. U. Hasan, S. A. Malik, M. Bilal, M. F. U. Butt, and R. A. Riaz, "Comparison of two nonlinear control strategies for hypnosis regulation," *Arabian Journal for Science and Engineering*, vol. 42, no. 12, pp. 5165-5178, 2017, doi: 10.1007/s13369-017-2610-3.

[13] T. N. Truong, A. T. Vo, and H. -J. Kang, "A Backstepping Global Fast Terminal Sliding Mode Control for Trajectory Tracking Control of Industrial Robotic Manipulators," in IEEE Access, vol. 9, pp. 31921-31931, 2021, doi: 10.1109/ACCESS.2021.3060115.

[14] O. Mechali, L. Xu, Y. Huang, M. Shi, and X. Xie, "Observer-based fixedtime continuous nonsingular terminal sliding mode control of quadrotor aircraft under uncertainties and disturbances for robust trajectory tracking: Theory and experiment," *Control Engineering Practice***,** vol. 111, pp. 1-23, Jun. 2021, doi: 10.1016/j.conengprac.2021.104806.

[15] V. Utkin, "On Convergence Time and Disturbance Rejection of Super-Twisting Control," in *IEEE Transactions on Automatic Control*, vol. 58, no. 8, pp. 2013-2017, Aug. 2013, doi: 10.1109/TAC.2013.2251812.

[16] J. A. Burton and A. S. I. Zinober, "Continuous approximation of variable structure control," *International Journal of System Science*, vol. 17, no. 6, pp. 875-885, 1986, doi: 10.1080/00207728608926853.

[17] R. A. DeCarlo, S. H. Zak, and G. P. Matthews, "Variable structure control of nonlinear multivariable systems: a tutorial," in *Proceedings of the IEEE*, 1988, vol. 76, no. 3, pp. 212-232, doi: 10.1109/5.4400.

[18] A. Levant, "Sliding order and sliding accuracy in sliding mode control," *International Journal of Control*, vol. 58, no. 6, pp. 1247-1263, 1993, doi: 10.1080/00207179308923053.

[19] M. Jafari and S. Mobayen, "Second-order sliding set design for a class of uncertain nonlinear systems with disturbances: An LMI approach," *Mathematics and Computers in Simulation*, vol. 156, pp. 110-125, Feb. 2019, doi: 10.1016/j.matcom.2018.06.015.

[20] K. Mathiyalagan and G. Sangeetha, “Second-order sliding mode control for nonlinear fractional-order systems,” *Applied Mathematics and Computation*, vol. 383, 2020, doi: 10.1016/j.amc.2020.125264.
[21] S. Ding, J. H. Park, and C.-C. Chen, “Second-order sliding mode controller design with output constraint,” *Automatica*, vol. 112, Feb. 2020, doi: 10.1016/j.automatica.2019.108704.
[22] I. Khan, A. I. Bhatti, A. Arshad, and Q. Khan, “Robustness and performance parameterization of smooth second order sliding mode control,” *International Journal of Control Automation and System*, vol. 14, no. 3, pp. 681-690, 2016, doi: 10.1007/s12555-014-0181-6.
[23] S. Mobayen and F. Tchier, “Robust global second-order sliding mode control with adaptive parameter-tuning law for perturbed dynamical systems,” *Transactions of the Institute Measurement and Control*, vol. 40, no. 9, pp. 2855-2867, 2018, doi: 10.1177/0142331217708832.
[24] B. Zhang, S. Gao, X. Yang, D. Zhao, P. Zhang, and Y. Zeng, “A chattering-free adaptive second-order nonsingular fast terminal sliding mode control scheme for a class of nonlinear uncertain systems,” *International Journal of Modelling, Identification and Control*, vol. 29, no. 3, pp. 255-265, Apr. 2018, doi: 10.1504/ijmic.2018.091248.
[25] C. T. Nguyen and Y.-W. Tsai, “Finite-time output feedback controller based on observer for the time-varying delayed systems: A Moore-Penrose Inverse approach,” *Mathematical Problems in Engineering*, vol. 2017, 2017, doi: 10.1155/2017/2808094.
[26] J. Lian, G. M. Dimirovski, and J. Zhao, “Stabilizability of uncertain switched systems via static/dynamic output feedback sliding mode control,” *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 8624-8629, 2008, doi: 10.3182/20080706-5-KR-1001.01458.
[27] J. Zhang and Y. Xia, “Design of Static Output Feedback Sliding Mode Control for Uncertain Linear Systems,” in *IEEE Transactions on Industrial Electronics*, vol. 57, no. 6, pp. 2161-2170, June 2010, doi: 10.1109/TIE.2009.2033485.
[28] F. Plestan, Y. Shtessel, V. Brégeault, and A. Poznyak, “New methodologies for adaptive sliding mode control,” *International Journal of Control*, vol. 83, no. 9, pp. 1907-1919, 2010, doi: 10.1080/00207179.2010.501385.
[29] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, Philadelphia, PA: SIAM, 1994. [Online]. Available: https://web.stanford.edu/~boyd/lmibook/lmibook.pdf

BIOGRAPHIES OF AUTHORS

Phan-Thanh Nguyen is currently a lecturer in the Faculty of Electronics and Electrical Engineering, Ho Chi Minh City University of Technology and Education. He obtained his Ph.D. in Electrical Engineering at Southern Taiwan University of Science and Technology, Taiwan, in 2016. His current research interests are Intelligent control systems, Automatic motion control and Electric drives. He can be contacted at email: thanhnp@hcmute.edu.vn.

Trieu Ton Ngoc was born in Quang Ngai, Vietnam, in 1981. He received the B.Eng. and M.Eng. degrees in electrical engineering from the Ho Chi Minh City University of Technology and Education, Ho Chi Minh City, Vietnam, in 2005 and 2009, respectively, where he is currently pursuing the Ph.D. degree with the Faculty of Electrical and Electronics Engineering. He works with the Faculty of Electrical and Electronics Engineering, Thu Duc College of Technology, Ho Chi Minh City. Since 2015, he has been working with the Power System and Renewable Energy Laboratory, Ho Chi Minh City University of Technology and Education. He can be contacted at email: trieutn.ncs@hcmute.edu.vn.

Cong-Trang Nguyen has completed the Ph.D. degree in automation and control from Da-Yeh University, Taiwan. He is currently a member of the Power System Optimization Research Group, Faculty of Electrical and Electronics Engineering, Ton Duc Thang University, Ho Chi Minh City, Vietnam. His current research interests include sliding mode control, optimization algorithm. He can be contacted at email: nguyencongtrang@tdtu.edu.vn.