

Decoding of the extended Golay code by the simplified successive-cancellation list decoder adapted to multi-kernel polar codes

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Article Info

Article history:

Received Feb 15, 2022

Revised Nov 10, 2022

Accepted Nov 26, 2022

Keywords:

Binary linear block code

Coding theory

Error-correcting code

Golay code

Multi-kernel polar code

ABSTRACT

This paper describes an adaptation of a polar code decoding technique in favor of the extended Golay code. Based on the bridge provided by a permutation matrix between the code words of these two classes of codes, the Golay code can be decoded by any polar code technique. Contrary to the successive-cancellation list technique which is characterized by a serial estimation of the bits, we propose in this work an adaptation of the simplified successive-cancellation list technique to polar codes equivalent to the Golay code. The simulations have achieved the performance of a maximum likelihood decoding, with the low decoding complexity of polar codes, compared to one of the universal decoders of linear codes most known in the literature.

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1. INTRODUCTION

One of the pioneers of existing error correction theory is the Golay code, which first appeared in the late 1940s [1]. It has a multitude of uses in communication, storage, and imaging systems [2]–[4]. In literature, a multitude of algebraic [5] and soft decision decoders [6]–[8] have been proposed. Golay code has gotten a lot of attention since its discovery because of its attractive structure and practical characteristics, and several construction methods have been presented over the years [9].

On the other hand, a class of error-correcting codes was discovered in 2009 by Arikan [10] called polar codes, allowing to reach, in a proven way, the channel capacity as it was described by Shannon [11] in his founder article. In addition, these codes have low encoding/decoding complexity using successive-cancellation (SC) decoding technique, which has made them a competitive coding scheme in modern wireless communication systems. Their constructions are inspired by the phenomenon of polarization using the Arikan kernel $T_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, which consists in transforming the channels into two categories: extremely unreliable channels used to forward the redundant part frozen at 0 and extremely reliable channels used to forward information. Their construction has been the source of inspiration for new construction of good binary linear codes [12], [13].

To improve binary linear block codes decoding by utilizing low complexity polar code decoding algorithms, some works in the literature have focused on the bridge between linear codes and the structure of polar codes. In Lin *et al.* [14] and Khebbou *et al.* [15] have proposed a transformation of some binary linear block codes,

namely extended Bose, Ray-Chaudhuri and Hocquenghem (EBCH), low density parity check (LDPC), and Golay codes, into polar codes with dynamic frozen bits (PCDF) [16], [17] for decoding them by the successive-cancellation list (SCL) technique, but the latter is hampered by SC decoding’s serial structure, i.e., the estimation of a bit depends on the estimation of the bits that precede it. To remedy this problem, an adaptation of the simplified-SCL technique to PCDF equivalent to EBCH and LDPC codes has been proposed in [18], but the proposed adaptation is limited to binary linear block codes whose length is $N = 2^m$ where m is positive integer. In this work, to validate the simplified-SCL technique for binary linear codes of arbitrary length, an adaptation of the simplified-SCL decoding technique to the benefit of the extended [24,12,8] Golay code whose length $N \neq 2^m$, by transforming it into a polar code with frozen dynamic bits based on multi-kernels.

The rest of this paper is structured as: section 2 introduces the preliminary concepts of this work. In section 3, a description of the proposed decoding method. Before concluding, in section 4 simulation and an interpretation of the results obtained using the method described in section 3 are presented.

2. PRELIMINARIES

2.1. Golay code

Golay initially introduced the Golay code as [23,12,7] code, which is a perfect code with length 23, dimension $k = 12$, and minimum distance $d_{min} = 7$. By extending the code, a [24,12,8] quasi-perfect self-dual binary linear code (extended Golay code) will be obtained; it encodes 12 bits of information in a 24-bit codeword. In Berlekamp [19], it has been revealed that the Golay code is a quadratic residue code in addition to being a cyclic Bose-Chaudhuri-Hocquenghem (BCH) code. By definition of a BCH code, we have the inequality $2t + 1 \leq t$, where t is the number of altered symbols in a codeword. So that any error of three bits can be rectified.

2.2. Polar code

The notation $P(N, K, F)$ denotes a polar code with length 2^n where $n \geq 1$, dimension K and a set of frozen bits F . The elements of F represent the positions of the $N - K$ less reliable channels, and the remaining K bits comprise the more reliable bit channels. When the positions of the frozen bits are fixed to 0, and the source information bits are organized in the remaining positions, to form the source vector u . The source codeword x is obtained via the encoding process applied on u by: $x = u \cdot G_p$, so that G_p is the kronecker product of order $\log_2(N)$ of the Arikan kernel, as a generator matrix of P .

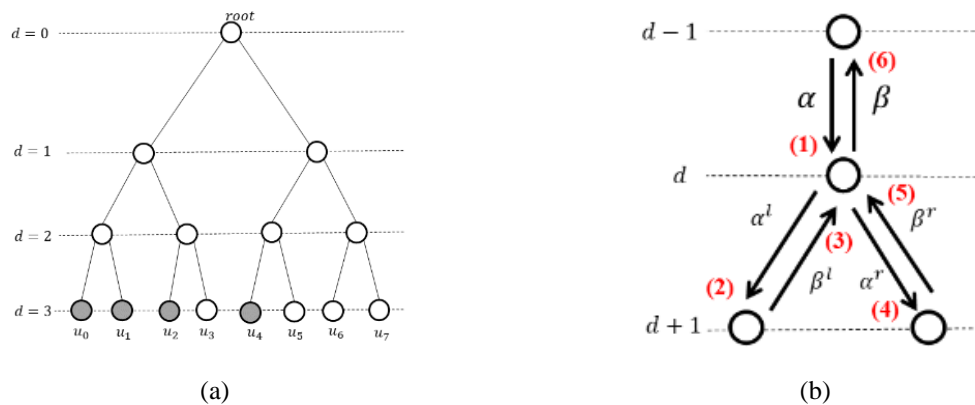


Figure 1. Decoding tree of $(8,4, \{0,1,2,4\})$ polar code: (a) decoding tree and (b) binary variable exchanging between nodes

The SC approach, which can be described as a binary tree, as the first polar decoding algorithm developed. Figure 1(a) depicts the binary tree of the $(8,4, \{0,1,2,4\})$ polar code, whereas Figure 1(b) depicts the different variables exchanged between nodes. At each node of a stage d , the number of bits processed by each node is $N_d = 2^{N-d-1}$ i.e., let’s use $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_{N_d-1}]$, by traversing the tree in the order indicated in the Figure 1(b), α^l and α^r are obtained by the functions:

$$\alpha^l_i = \text{sign}(\alpha_i) \cdot \text{sign}(\alpha_{i+\frac{N_d}{2}}) \cdot \min(|\alpha_i|, |\alpha_{i+\frac{N_d}{2}}|) \tag{1}$$

$$\alpha^{\rightarrow r}_i = (1 - 2\beta^{\rightarrow l}_i) \cdot \alpha_i + \alpha_{i+\frac{Nd}{2}} \quad (2)$$

And at the leaf node, β^l and β^r are obtained by applying the binary qualification function h (3):

$$\beta^l_v = h(\alpha^l_v) = \begin{cases} 0 & \text{if } \alpha^l_v \geq 0 \text{ or is frozen} \\ 1 & \text{otherwise} \end{cases} \quad \text{and} \quad \beta^r_v = h(\alpha^r_v) = \begin{cases} 0 & \text{if } \alpha^r_v \geq 0 \text{ or is frozen} \\ 1 & \text{otherwise} \end{cases} \quad (3)$$

The SC decoder can demonstrably reach a channel when $N \rightarrow \infty$. However, the error correction performance is poor for small lengths when compared to existing codes like Turbo codes and LDPC codes. The successive-cancellation list (SCL) decoder comes into the picture at this point [20]. So instead of making a hard decision by (3), the key concept is to make a probabilistic estimate for each possibility, by considering both 1 and 0, each one with a certain probability metric PM^i calculated by:

$$PM^i = \begin{cases} PM^{i-1} & \text{if } \beta = h(\alpha_i) \\ PM^{i-1} + |\alpha_i| & \text{otherwise} \end{cases} \quad (4)$$

At any leaf node, the number of possibilities is limited by list size L , by pruning the list each time to consider the best L possibilities.

2.3. Simplified-SCL decoding for polar codes

Despite their low complexity, the SC and SCL decoders are characterized by their serial nature. In other words, a bit's decoding is dependent on the decoding of the bits before it. Thus, at each instant, a bit can be decoded. To remedy these problems, a simplification of this technique simplified-SCL has been proposed in [21], to break the serial nature for some special nodes on the one hand, and to estimate all the leaf bits of the special node at one time on the other hand. These special nodes are described in the Figure 2.

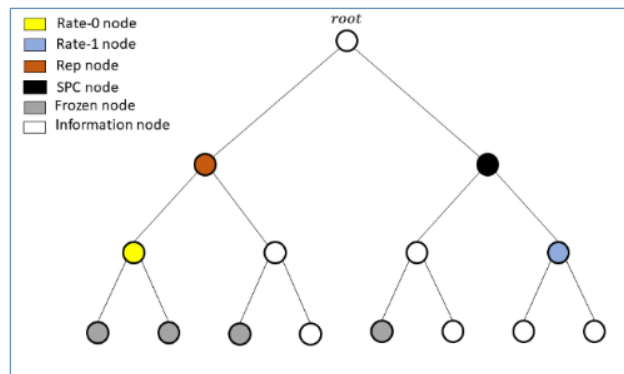


Figure 2. Labelled special nodes in binary tree for (8,4) polar code

- Rate-0 node: it is the root node of a subtree whose leaf nodes are entirely frozen bit nodes as illustrated in Figure 2. It is not necessary to go to the leaf node for such types of nodes, and the appropriate bits can be set to 0.
- Rate-1 node: it is the root node of a subtree whose leaf nodes are entirely information bit nodes as illustrated in Figure 2. Because no frozen bit nodes are providing prior information, traversing subtree yields no additional information. Hence it is sufficient to estimate the leaf bits at the current node.
- Repetition (REP) node: it is the root node of a subtree, with all leaf nodes, are frozen except the last node which is an information bit node as illustrated in Figure 2.
- Single parity check (SPC) node: the root node of a subtree whose leaf nodes are all information bit nodes except the first, which is a frozen bit node corresponds to the SPC node as illustrated in Figure 2.

When encountering one of these types of special nodes, an estimation of all leaf nodes is performed, without having to traverse the subtree in the special node which is root. The formulas for estimating the different variables mentioned in Figure 1 at each special node are presented in [21], as well as the formula for updating the the path metrics (PM) metrics. The simplified-SCL decoder significantly reduces latency and complexity while achieving the error correcting performance of a SCL.

2.4. Multi-kernel polar code

Gabry *et al.* [22] develops a method for constructing polar codes with arbitrary lengths $N \neq 2^n$. Instead of using the Arikan kernel to generate the generator matrix, the authors recommended polar codes based on a multi-kernel construction. I.e.,: $G_p = T_{n_1} \otimes T_{n_2} \otimes \dots \otimes T_{n_d}$ is the generator matrix of multi-kernel polar code and its length is $N = n_1 \times n_2 \times \dots \times n_d$.

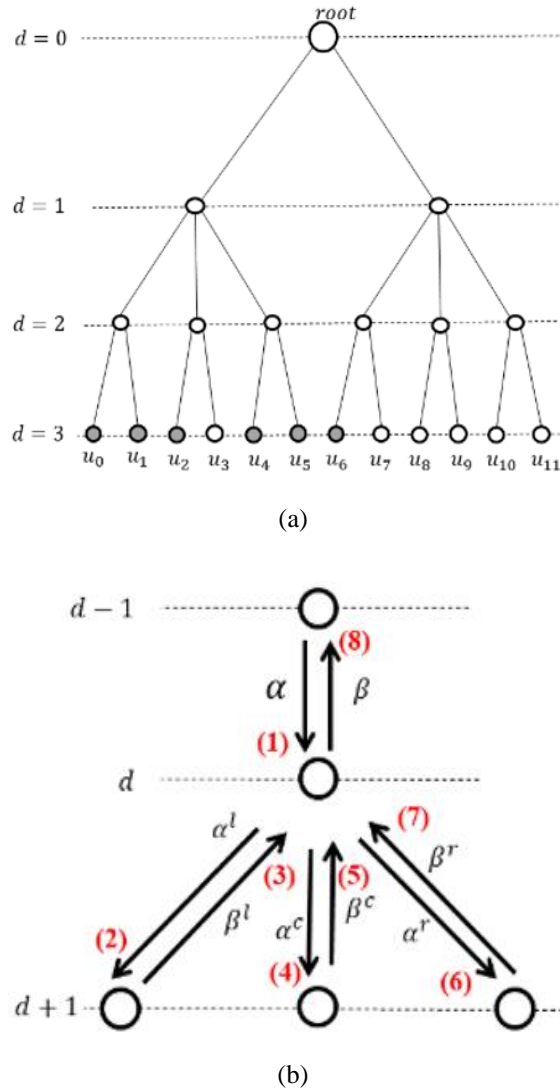


Figure 3. Decoding tree of (12,6) multi-kernel polar code: (a) decoding tree and (b) binary variable exchanging between nodes of the ternary kernel

As a regular polar code, the SC decoding technique, adapted to multi-kernel polar code (12,6) is illustrated in the Figure 3(a) its generator matrix is $G_p = T_2 \otimes T_3 \otimes T_2$, based on the Arikan kernel T_2 and the ternary kernel T_3 (5).

$$T_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} ; \quad T_3 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{5}$$

The decoding of the bits corresponding to the Arikan kernel T_2 is carried out, in the same way, described previously. Then for the ternary kernel T_3 , the decoding is carried out in the order mentioned in Figure 3(b).

$$\alpha \xrightarrow{l} \alpha_i = \text{sign}(\alpha_i) \cdot \text{sign}(\alpha_{i+\frac{N_d}{3}}) \cdot \text{sign}(\alpha_{i+\frac{2N_d}{3}}) \cdot \min(|\alpha_i|, |\alpha_{i+\frac{N_d}{3}}|, |\alpha_{i+\frac{2N_d}{3}}|) \tag{6}$$

$$\alpha \leftrightarrow_i^c = (1 - 2\beta \leftrightarrow_i^l) \cdot \alpha_i + \text{sign}(\alpha_{i+\frac{N_d}{3}}) \cdot \text{sign}(\alpha_{i+\frac{2N_d}{3}}) \cdot \min\left(\left|\alpha_{i+\frac{N_d}{3}}\right|, \left|\alpha_{i+\frac{2N_d}{3}}\right|\right) \quad (7)$$

$$\alpha \leftrightarrow_i^r = (1 - 2\beta \leftrightarrow_i^l) \cdot \alpha_{i+\frac{N_d}{3}} + (1 - 2\beta \leftrightarrow_i^l \oplus \beta_i^c) \cdot \alpha_{i+\frac{2N_d}{3}} \quad (8)$$

For β^l , β^c and β^r , the (3) is used to calculate their values, and the value of β returned to the parent node is obtained (9).

$$\beta_i = [\beta_i^l \oplus \beta_i^c, \beta_i^l \oplus \beta_i^r, \beta_i^l \oplus \beta_i^c \oplus \beta_i^r] \quad (9)$$

3. ADAPTED SIMPLIFIED-SCL DECODING FOR EXTENDED GOLAY CODE

Lin *et al.* [14] and Khebbou *et al.* [18], was the beginning of the decoding of binary linear block codes by polar code decoding techniques. The idea is to use a permutation matrix π to ensure a mapping between the codewords of a polar code and those of a binary linear block code. The polar codes obtained after permutation are PCDF. These were the first to be introduced into the literature as a solution to the problem of short minimum Hamming distance in short length regimes. As opposed to a regular polar code, some positions of the frozen bits are not fixed at 0 for PCDF, their values are obtained by linear functions of the information bits which precede them. In [16], an example is described using an extended BCH code.

3.1. Golay transformation to polar code

The transformation proposed in [14] applied on the extended Golay code is used in this work, it consists in identifying the permutation matrix π that optimizes the Bhattacharyya parameter [23], [24], which allows to classify the sets information bits and of frozen bits. The following notations are used:

- T_2 and T_3 : the kernels mentioned in (5).
- G_p : generator matrix of polar code based on multi-kernel with $N = 24$ obtained by $G_p = T_3 \otimes T_2 \otimes T_2 \otimes T_2$.
- G_{Golay} : generator matrix of extended [24,12,8] Golay code.
- H_{Golay} : parity check matrix of extended [24,12,8] Golay code.
- E : elimination matrix which ensures the transformation into a reduced row echelon form.
- E' : invertible elimination matrix which ensures the transformation into a backward reduced row echelon form.

The deduction of the polar code with frozen dynamic bits, equivalent to the Golay code can be done in two ways. The first one, by using G_{Golay} of the extended Golay code using (10).

$$R = E \cdot G_{Golay} \cdot \pi^{-1} \cdot G_p^{-1} \quad (10)$$

The set of the statically frozen bits are represented by indexes of the columns of R with values of 0 (their values are set to 0), the set of the information bits are represented by the columns of R with a single value of 1, and the remaining columns of R are for dynamic frozen bits (their values are linear functions of the information bits). The second one, by using H_{Golay} of the extended Golay code using (11).

$$R' = G_p \cdot \pi \cdot H_{Golay} \cdot E' \quad (11)$$

The position of dynamic frozen bits in R' is determined by the row index of the last non-zero components of multiple 1 columns, while the zero frozen bits correspond to the column index of non-zero components of single 1 columns of R' , and the remainders are information bits. In this work, we will transform the extended Golay code using the second way. To do this, let's take the matrices.

The matrix H_{Golay} is part of the (12), so $u \cdot R' = 0$ by definition of parity check matrix. So by observing the result R' , we can notice that the indices of the columns {4, 5, 6, 7, 8, 9, 10, 11, 12} are static frozen positions. The indices of the columns {1, 2, 3} have multiple 1 must also satisfy $u \cdot R' = 0$ and they represent dynamic frozen positions, whose system below allows to define their values according to the information bits. The remaining positions are information bits:

$$\begin{cases} u_{22} = u_{17} + u_{18} \\ u_{21} = u_{18} \\ u_{20} = u_{17} + u_{18} + u_{19} \end{cases} \quad (12)$$

And PM update for the information bits is performed (16):

$$PM_l = PM_l + \text{sign}(\alpha_i) \cdot \alpha_i - (1 - 2\beta_i) \cdot \alpha_i \quad (16)$$

For the parity check bit, it is updated (17):

$$PM_l = \begin{cases} PM_l + |\alpha_i| & \text{if } \beta_i = h(\alpha_i) \\ PM_l & \text{otherwise} \end{cases} \quad (17)$$

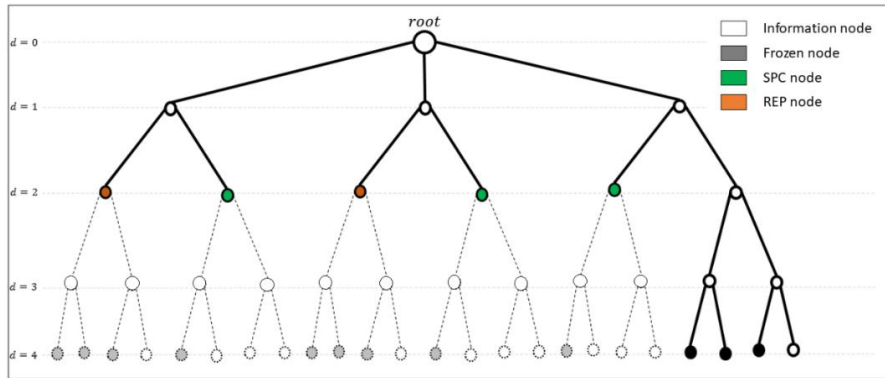


Figure 4. Adapted simplified successive-cancellation list decoding tree for polar code equivalent to extended Golay code [24,12,8] with dynamic frozen bits based on multi-kernel

4. RESULTS AND INTERPRETATIONS

To compare the technique proposed in this work with existing techniques in the literature, this section illustrates the decoding of the Golay code by three decoding methods, i.e., ordered statistics decoding (OSD), maximum likelihood decoding (MLD), and the simplified-SCL adapted to multi-kernel polar decoder proposed in this paper. Although MLD of forwarding error correction codes is known to be the most accurate, it is rarely employed in practice because it is too difficult to implement efficiently, and MLD of arbitrary linear codes was established as an NP-hard problem [25]. The OSD is a decoder that was proposed in 1995 as an approximation to the MLD for linear block codes in order to reduce decoding complexity to $O(nk^2 + 2^r nk)$, with r is the order used for OSD [26]. Figure 5 is a simulation of frame-error rate (FER) for the extended Golay code over the additive white gaussian noise (AWGN) channel, using three techniques.

Although both the adapted simplified-SCL with list size $L = 64$ and the first-order OSD achieve the performance of an MLD, but the OSD requires additional effort to control its decoding flow based on the results of sorting symbol reliabilities, compared to the simplified-SCL complexity. Since the proposed technique is a derivative of SC, its decoding complexity is $O(LN \log(N))$, which is a logarithmic complexity as opposed to the OSD having an exponential complexity $O(NK^2 + 2^r NK)$. Note that order $r = 1$ represents the order that allows achieving the saturation of an OSD. In other words, whatever the chosen order, the performances will remain the same.

Lin *et al.* [14] the decoding of binary linear block codes is treated by the SCL method, which consists in traversing the whole decoding tree. But it is characterized by its serial nature and the estimation of each bit is done based on the bit that precedes it. This work is the continuation of [18] which is limited to the adaptation of the simplified-SCL to codes of length is $N = 2^m$ where m positive integer, in order to generalize the simplified-SCL for PCDF based on multi-kernel, to take advantage of a binary linear block codes whose size $N \neq 2^m$.

By definition, the construction of polar codes is done explicitly; i.e., the code of size N is obtained starting from a code of size $N/2$. Thus each sub-tree of such polar code is also a polar code. For the multi-kernel polar code equivalent to the extended Golay code proposed in this work, we can notice from Figure 4 that the technique of decoding technique proposed in this paper, makes it possible to reduce the time steps and latency by those of 5 polar codes of size $N = 4$. As illustrated in [27, Table 1], the time steps required for decoding an SPC node is $(N_v + 1)$ for the SCL. However for the same node, the proposed technique in this work require only $(N_v - 2 \log(N_v) - 2)$. As well as for the REP node, the time steps required for decoding is $(2N_v - 1)$ versus 2 times steps for the proposed simplified-SCL in this work.

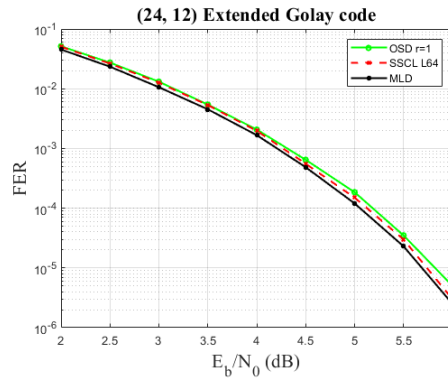


Figure 5. Frame-error rate for (24,12) extended Golay code

5. CONCLUSION

In this work, a less complex Golay code decoding technique has been shown compared to existing methods. It implements the simplified successive-cancellation list decoder adapted to multi-kernel polar code with frozen dynamic bits. With an appropriate selection of the permutation matrix, these algorithms can provide a maximum likelihood decoding for [24,12,8] extended Golay code, with a minimization of complexity and latency, by puncturing the tree traverse of the SCL principle. The MLD is considered as the most powerful decoding method but it is considered as an NP-hard problem, we compared the method described in this work with OSD, one of the universal decoder for linear codes known by performances close to those of MLD, but it has an exponential complexity. The method that we proposed is characterized by a logarithmic complexity that allows to reach the same performances of OSD. The constraint of the definition of the most optimal permutation matrix π for an eventual binary linear block code remains a direction of future reflections, since currently the identification of this matrix is done by a random search between the $N!$ possibilities. For that, the use of evolutionary algorithms as genetic algorithms can be an alternative to the direct search.





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