Moving-horizon estimation approach for nonlinear systems with measurement contaminated by outliers

Moath Awawdeh[1], Tarig Faisal[1], Anees Bashir[1], Abdel Ilah Nour Alshbatat[2], Rana T. H. Momani[3]
[1]Department of Electrical Engineering, Faculty of Engineering, Higher Colleges of Technology (HCT), Abu Dhabi 12389, UAE
[2]Department of Communication and Electronics Engineering, Faculty of Engineering, Tafila Technical University, Tafila, Jordan
[3]Department of Calssroom Teacher, Faculty of Educational Science, Zarqa University, Zarqa, Jordan

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ABSTRACT

An application of moving-horizon strategy for nonlinear systems with possible outliers in measurements is addressed. With the increased success of moving-horizon strategy in the state estimation for linear systems with outliers acting on the measurement, investigating the nonlinear approach is highly required. In this paper we applied the nonlinear version which has been presented in the literature in term of discrete-time linear time-invariant systems, where the applied strategy considers minimizing a least-squares functions in which each measure possibly contaminated by outlier is left out in turn and the lowest cost is propagated. The moving horizon filter effectiveness as compared with the extended Kalman filter is shown by means of simulation example and estimation error comparison. The moving horizon filter shows the feature of resisting outliers with robust estimation.

Corresponding Author:

Moath Awawdeh
Department of Electrical Engineering, Faculty of Engineering, Higher Colleges of Technology (HCT)
Applied Research Division - Fund Number 113128 Abu Dhabi 12389, United Arab Emirates (UAE)
Email: mawawdeh@hct.ac.ae

1. INTRODUCTION

One of the main serious challenges when dealing with measurement of state variables is the large noise or what is called outliers. Such gross errors might be due to many reasons, starting from the field device malfunction to simple large noise. These abnormal signals which are usually called as outliers or anomalies attracting attention in various fields, such as data quality in process control [1]–[3] which affect as well the accuracy of the predictive models, health care [4]–[6] network intrusion [7]–[10] where many approaches have been investigated and developed to defend against network attacks, environmental monitoring [11]–[15], positioning estimation [16], cloud management [17]–[19], and fault detection[20]–[22]. Hence, various methods have been investigated and proposed to detect outliers in different applications [23]–[25]. In this paper, we developed the nonlinear approach of well-established algorithm to deal with estimating state variables of a nonlinear system containing measures that are possibly corrupted by outliers. This estimation will be performed using a moving-horizon estimation approach, which has been set in the preliminary result by Alessandri and Awawdeh [26] for discrete-time linear systems.

The Kalman filter (KF) is considered to be the best estimator when quadratic estimation error needs to be reduced or minimized. The recurring estimator works on the principle of iterating the estimated update depending on the current residual to generate the new output. Checking for outliers is a crucial task to ensure estimator robustness [27]; where robustness is a key factor in designing of filters particularly improving the kalman filter robustness in the presence of outlier is required [28]. More investigation on the development of statistical tests and for the purpose if identification can be found on [29]–[32]. Moving-horizon estimation (MHE) utilizes most recent information by minimizing least squares function which is originally presented by [33] and MHE results were obtained initially for linear systems [34], [35] and then they were extended for nonlinear systems and large-scale systems [36]–[40]. Based on the preliminary results by Alessandri and Awawdeh [26], the stability scheme is driven from [37]. The approach of this paper focuses on moving-horizon estimator for a nonlinear systems with measures containing outliers by adopting the worst-case cost functions, the moving horizon estimator stability conditions for uncertain linear systems are reported in [41] and original linear version of such approach is addressed in [26]. The approach includes minimizing a set of least squares functions while the measurement that is possibly affected by outlier is excluded, which ensure the generating of a the lowest cost each time, hence propagating the the estimate to the next instant.

The organization of the paper is as follows: in section 2 the investigation of the moving-horizon estimation method is discussed. Finding of stability and robustness are briefed, in sections 3. Respectively, in sections 4 to 6 simulation results are described and discussed. Finally, the conclusion is drawn in section 7.

2. NONLINEAR MOVING-HORIZON ESTIMATION APPROACH

In this section, and following the success of the linear approach of moving horizon estimation in solving the problem of outliers in industrial systems, we propose the the nonlinear framework of such developed approach. Considering a nonlinear system, with $t = 0, 1, \dots$

$$x_{t+1} = f(x_t, u_t) + \xi_t \tag{1a}$$

$$y_t = h(x_t) + \eta_t \tag{1b}$$

Where $x_t \in X \subset \mathbb{R}^n$ is the continuous state, $u_t \in U \subset \mathbb{R}^m$ is the control vector, $y_t \in Y \subset \mathbb{R}^p$ is the output, $\xi_t \in W \subset \mathbb{R}^n$ is the disturbance of system, and $\eta_t \in V \subset \mathbb{R}^p$ is disturbance of measurement, assuming ξ_t and η_t as unknown deterministic variables. Since, in this paper we are introducing the nonlinear theme of our preliminary works mentioned earlier; so we refer the reader to [26], and [42] and reference therein for a complete understanding of the assumption and pre-definition.

The proposed approach (see Figure 1) can be summarize as for a given measurement batch $t = 1, 2, 3, \dots N + 1$, a cost function J_t is to be minimized with the constrains.

$$\hat{x}_{i+1|t} = f\left(\hat{x}_{i|t}, u_i\right), \quad i = t - N, \dots, t - 1. \tag{2}$$

The cost function is defined for two possible cases (assumption), as follow:

- Case 1: measurement is contaminated by an outlier ($k - thmeasure$), the function J_t which leaves out the contaminated measure is defined as:

$$J_t^k(\hat{x}_{t-N,t}) = \mu \parallel \hat{x}_{t-N,t} - \bar{x}_{t-N} \parallel^2 + \frac{1}{N} \sum_{\substack{i=t-N \\ i \neq t-N+k-1}}^{t} (y_i - h(\hat{x}_{i,t}))^2 \tag{3}$$

- Case 2: measurement is free of outlier, the function J_t is :

$$J_t^0(\hat{x}_{t-N,t}) = \mu \parallel \hat{x}_{t-N,t} - \bar{x}_{t-N} \parallel^2 + \frac{1}{N+1} \sum_{i=t-N}^{t} (y_i - h(\hat{x}_{i,t}))^2 \tag{4}$$

Where the tuning parameter $\mu > 0$ and J_t^k is minimized under (2). The approach is summarized as solving a problem as follow.

$$\min_{x \in \Re^n \text{ s.t. (2) holds}} J_t^k(x), \; k = 0, 1, \ldots, N+1$$

The estimate of x_{t-N} at time t is:

$$\hat{x}_{t-N|t} \in \arg\min_{x \in \Re^n \text{ s.t. (2) holds}} J_t(\hat{x})$$

and accordingly,

$$\hat{x}_{t-N}^k \in \arg\min_{x \in \Re^n \text{ s.t. (2) holds}} J_t^k(x)$$

$$k(t)^* \in \arg\min_{k=0,1,\ldots,N+1} J_t^k\left(\hat{x}_{t-N}^k\right)$$

In case, X is known, the cost can be minimized on X, i.e., $\hat{x} \in X$. By comparing case 1 and case 2, the optimal cost is propagated for $t+1, t+2, \ldots$. the estimate. Recalling that, a solution exists by assuming that X is compact and $f(., u)$, $h(.)$ and $J_t(.)$ are continuous. The sketch given in Figure 1, illustrate the approach of leave-one-out MHE strategy for nonlinear systems.

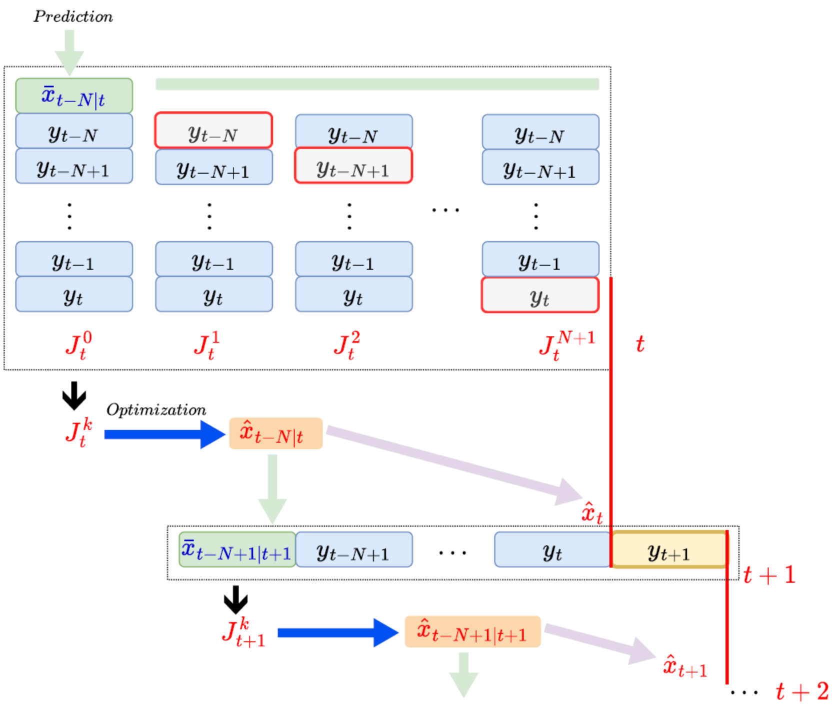

Figure 1. Scheme of MHE approach for nonlinear systems when moving from t to $t+1$

With reference to [41] and [43], (3) and (4) are considered to compare with $k(t^*)$. Concerning the extension of the proposed approach to the case with multiple outliers in the batch, the assumption of $N+1$ can adjusted with a more general setting. If k outliers affect the $N+1$ measurements, we consider all permutations of k measurements taken from the set of the $N+1$ ones in the batch. Thus, for example, we need to consider a number of:

$$n_2 = \binom{2}{N+1}$$

Costs to discriminate among the various cases with two outliers. Of course, we need to consider the "no outlier" setting and all the case that corresponds to "one outlier" in the batch. In general, we have to account for up to k outliers in the batch by using:

$$n_{0,1,\ldots,k} = \sum_{i=0}^{k} \binom{i}{N+1}$$

Costs to perform the comparisons, considering the estimate of x_{t-N} at t. The stability of estimation error is investigated, supposing to perform a perfect or approximate minimization with fact that error is bounded [37]. In next section, a brief of estimation error stability with necessary recall of assumptions which were investigated in the preliminary work [42] and original assumption proposed by Alessandri *et al.* [37].

3. ESTIMATION ERROR - RECALL OF STABILITY

With the assumption that X, U, W, and V are compact set (introduced in section 2), assume that $f(., u)$ and $h(.)$ are C^2 on X.

$$y_{t-N}^{t}|_k = H_k(x_{t-N}) + D_{\xi_k}(x_{t-N})\xi_{t-N}^{t-1}|_k + \eta_{t-N}^{t}|_k$$

where:

$$H_N\left(x_{t-N}, u_{t-N}^{t-1}\right) := \begin{pmatrix} h \circ f^{u_{t-1}} \circ \cdots \circ f^{u_{t-N}}(x_{t-N}) \\ h \circ f^{u_{t-2}} \circ \cdots \circ f^{u_{t-N}}(x_{t-N}) \\ \vdots \\ h \circ f^{u_{t-N}}(x_{t-N}) \\ h\left(x_{t-N}\right) \end{pmatrix} \tag{5}$$

$$D_{\xi}\left(x_{t-N}, u_{t-N}^{t-1}\right) := \begin{pmatrix} \frac{\partial h \circ f_{(1)}^{\xi_{t-N}}}{\partial \xi_{t-N}} & 0 & \cdots & 0 \\ \frac{\partial h \circ f_{(2)}^{\xi_{t-N}^{t-N+1}}}{\partial \xi_{t-N}} & \frac{\partial h \circ f_{(2)}^{\xi_{t-N}^{t-N+1}}}{\partial \xi_{t-N+1}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h \circ f_{(N)}^{\xi_{t-N}^{t-1}}}{\partial \xi_{t-N}} & \frac{\partial h \circ f_{(N)}^{\xi_{t-N}^{t-1}}}{\partial \xi_{t-N+1}} & \cdots & \frac{\partial h \circ f_{(N)}^{\xi_{t-N}^{t-1}}}{\partial \xi_{t-1}} \end{pmatrix} \tag{6}$$

Following the assumption that; while the system is X observable, there exists a K-function $\varphi(\cdot)$, such that:

$$\varphi\left(|x' - x''|\right) \leq \left|H_N\left(x', u\right) - H_N\left(x'', u\right)\right| \tag{7}$$

for all that $x' \in X$, $x'' \in X$, and $u \in U$. Moreover,

$$\delta := \inf_{x', x'' \in \mathcal{X};\, x' \neq x''} \frac{\varphi\left(|x' - x''|^2\right)}{|x' - x''|^2} > 0 \tag{8}$$

under perfect minimization/approximate minimization with bounded error, if $\mu \geq 0$ is chosen such that:

$$\frac{8\, L_f \mu}{\mu + \frac{\delta}{N+1}} < 1 \tag{9}$$

where L_f is the lipschitz constant (f, X).
The estimation error is exponentially bounded and the optimal cost can be:

$$J_t^*(\hat{x}_{t-N}) = \mu \parallel \hat{x}_{t-N} - \bar{x}_{t-N} \parallel^2 + \begin{cases} \frac{1}{N+1} \left\| y_{t-N}^t - H_0\left(\hat{x}_{t-N}\right) \right\|^2 \text{if } k^* = 0 \\ \frac{1}{N} \left\| y_{t-N}^t | k^* - H_{k^*}\left(\hat{x}_{t-N}\right) \right\|^2 \text{if } k^* \in \{1, \ldots, N+1\} \end{cases} \quad (10)$$

4. SIMULATION EXAMPLE

In this section we prove the approach robustness and efficiency by the meaning of simulation, considering an undamped oscillator with a pulsation equal to ω, the damping coefficient ζ is assumed to be unknown and a constant.

$$x_3(t) = \zeta$$

The discrete nonlinear system and linear observation equations are:

$$\begin{aligned} x_1^{t+1} &= x_1^t + T x_2^t \\ x_2^{t+1} &= -\omega^2 T x_1^t + (1 - 2\omega T x_3^t) x_2^t + 12T + T\xi_t \\ x_3^{t+1} &= x_3^t \\ y_t &= x_1^t + \eta_t \end{aligned}$$

where $T > 0$ is the sample time.

Specifically, we choose $\zeta = 0.2$, $\omega = 5$ rad/s, and $T = 0.01$ s. The distributions of initial state, system and measurement noises were taken according to (add Kalman book reference, example 5.3, page 173), i.e., they are zero-mean white Gaussian processes with covariances $P_{0|-1} = \text{diag}(2, 2, 2)$, $Q = \text{diag}(0, 0, 4.47)$, and $r = 0.01$ except in case of outliers, for which the covariance was chosen much larger than r, i.e., equal to 10.

Outliers were generated randomly with dispersion $\sigma = 10$, and randomly-positioned over 100 time steps. We will evaluate the performances of such filters by the root mean square error (RMSE):

$$RMSE(t) = \left(\sum_{i=1}^{M} \frac{\|e_{t,i}\|^2}{M} \right)^{1/2}$$

where $e_{t,i}$ is the estimation error at time t in the i-th simulation run, and M is the number of simulation runs.

The results of such tests with a MHF for different μ and a KF for different choices of threshold values σ_t are shown in Figures 3, 5, and 7. RMSE means of the MHF and KF for different choices of μ, σ_t, and r are shown in Figures 4, 6, and 8. In Table 5, the computational times for both estimators are reported. Though a convenient choice of μ makes the MHF performs better in term of RMSE, its computational effort is larger.

5. SIMULATION DISCUSSION

In the mentioned example we have tried to pick up the possible best parameter tuning for KF, extended Kalman filter (EKF), and MHF from different simulation setting for KF/EKF choosing σ_t equal to $\sqrt{S_t}$ or $5\sqrt{S_t}$ or $10\sqrt{S_t}$, and a MHF with μ equal to 0.1 or 0.5 or 1, in order to reduce the effect of outliers and get the minimum estimation error. More test result with a MHF with $\mu = 0.6$ and EKF with no threshold check are shown in Figure 9. It is clear that pure EKF is not able to resist outliers. In Figure 10 a tuning threshold value of KF has been adopted with $\sigma_t = 10\sqrt{S_t}$ which it has been chosen among different simulation test. RMSES of MHF and EKF for 100 runs with $r = 0.01$ and different parameters (μ and σ_t) are shown in Figure 11 for x_1, x_2 and x_3. Tables 2 to 4 show the RMSE means of the MHF and KF for different choices of μ, σ_t, and r. In Table 5, the computational times for both estimators are reported. However, it is more demanding from the computational point of view as compared with the extended Kalman filter.

6. SIMULATION-BASED DISCUSISON ON PREDICTIVE MOVING HORIZON ESTIMATION (TO LINEAR-BASED)

The proposed approach in this paper considers some complexity in computing the the different cost functions, obtained by excluding one measurement each time and some how computationally demanding. In this section we tried to consider an attempt to check for y_{t+1} if it is outlier by computing only the minima of two cost functions (one computed by neglecting y_{t+1} and one by accounting y_{t+1}). However, this approach is much less computationally demanding but without stability proof in general (the proof is based on the precise requirement that the estimation is associated with the best of the optimal). This is a serious drawback, as without such a property it seems difficult to prove robustness, which is satisfied in the proposed method. However, we may guess that both stability and hence robustness may be ensured in practice and somehow evaluated via simulations also to measure the expected computational savings. In this section will denote the attempt proposed as PMHF (predictive MHF), as it relies on the prediction on the outlier occurrence when proceeding with the new measure update. Simulation were performed with absolute value of the outliers over the rejection thresholds in such a way to ensure the better conditions of work for the PMHF (the modulus of the outliers are randomly chosen in the range $(\bar{r}_v, 10\bar{r}_v)$. A simulation run is depicted in Figure 2, while Table 1 summarizes the results obtained over 100 runs with initial states that were randomly generated with mean equal to $(1 \quad 1)^\top$ and covariance $P_0 = \mathrm{diag}(2, 2)$, and system and measurement noises given by zero-mean white Gaussian processes with covariances $Q = \mathrm{diag}(1, 1)$ and $r = 0.1$, respectively. For MHF and PMHF, we set $\rho = 0.001$.

The computational effort of the PMHF is about 35% less than that of the MHF, but of course this is paid with an increase of the RMSEs. In any case, also the PMHF performs much better than the KF in terms of precision because of the large RMSEs given with all the various thresholds. Summing up, at the moment the PMHF should be ranked as a "heuristic" approach beacause of the lack of guaranteed stability and robustness properties but, since it allows on to trade between computational burden and precision, it may be the subject of future investigation aimed at proving such properties.

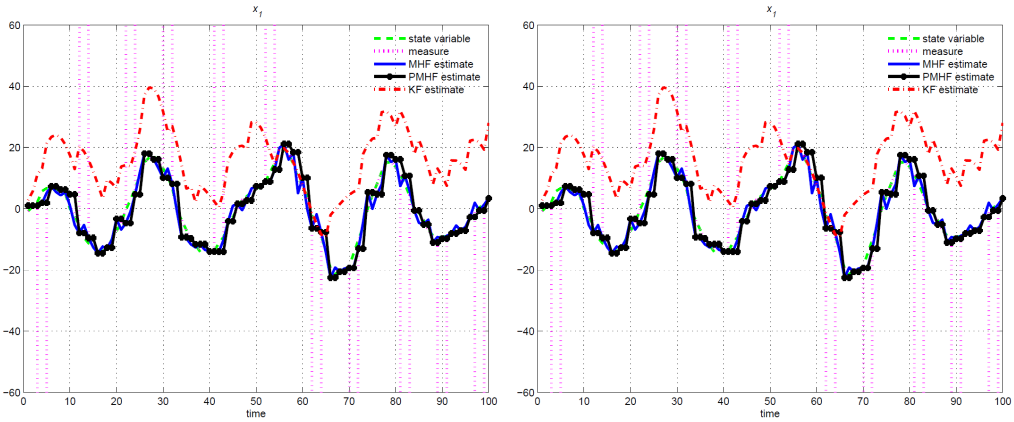

Figure 2. True state, measures, and estimates of x_1 and x_2 in a simulation run with zero-mean Gaussian measurement noises having $r = 0.1$ and using MHF and PMHF with $\rho = 10^{-3}$ and $N = 3$, and a KF with $\sigma_t = 2\sqrt{s_t}$

Table 1. MCT (in s) and RMSEs over 100 runs for MHF, PMHF, and KF

	MHF				PMHF			
	N=3	N=4	N=5	N=6	N=3	N=4	N=5	N=6
MCT	0.0273	0.0315	0.0380	0.0434	0.0209	0.0225	0.0261	0.0251
RMSE (x_1)	1.7120	1.5839	1.6698	1.7409	2.2165	2.1357	2.1965	2.2819
RMSE (x_2)	1.8337	1.7714	1.7985	1.8405	2.0471	1.9577	1.9318	1.9578

	KF			
	$\sigma_t = \sqrt{s_t}$	$\sigma_t = 2\sqrt{s_t}$	$\sigma_t = 5\sqrt{s_t}$	$\sigma_t = 10\sqrt{s_t}$
MCT	0.0070	0.0067	0.0066	0.0086
RMSE (x_1)	6.9472	6.4459	9.1978	18.666
RMSE (x_2)	3.5151	3.2509	3.6555	6.6224

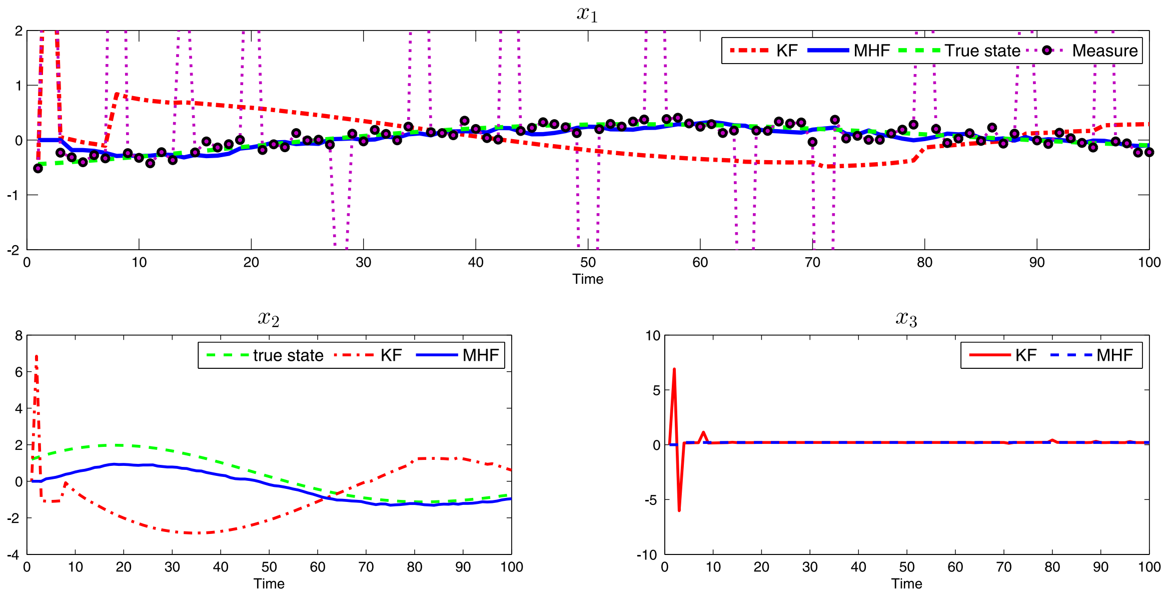

Figure 3. States estimation for MHF with $\mu = 0.1$ and KF without residual check

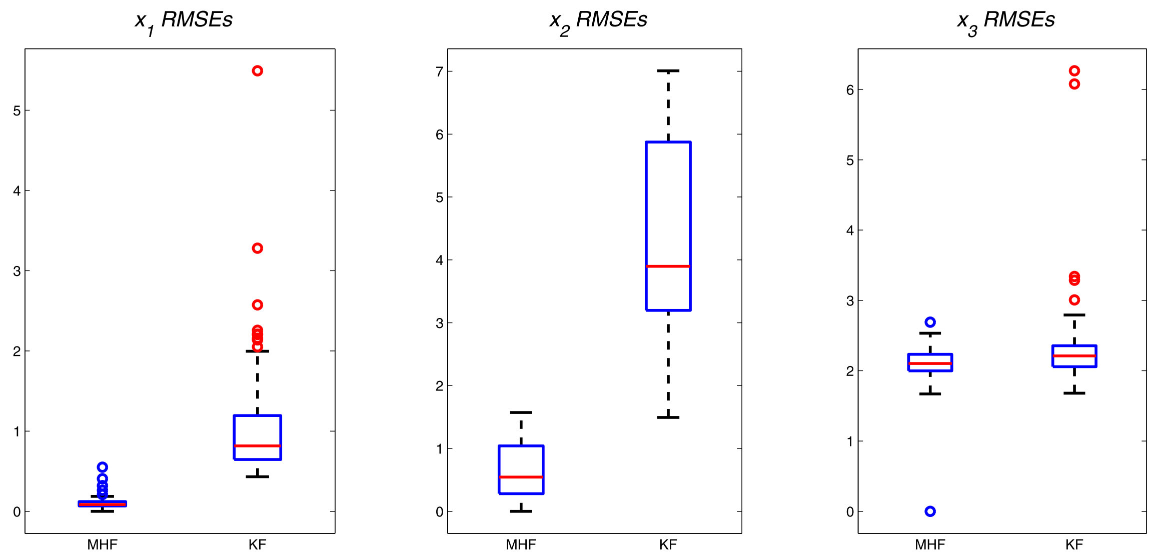

Figure 4. RMSES of MHF and KF for 100 runs with $\mu = 0.1$

Table 2. Means of RMSEs for MHF with $\mu = 0.1$ and KF without residual check

r	MHE			KF		
	x_1	x_2	x_3	x_1	x_2	x_3
0.01	0.1678	0.6852	2.0905	1.5170	4.3395	2.3006
0.10	0.2326	0.8642	2.0455	1.8418	3.3232	2.2107
1.00	0.4818	1.4177	2.0526	1.7744	3.8066	2.1922

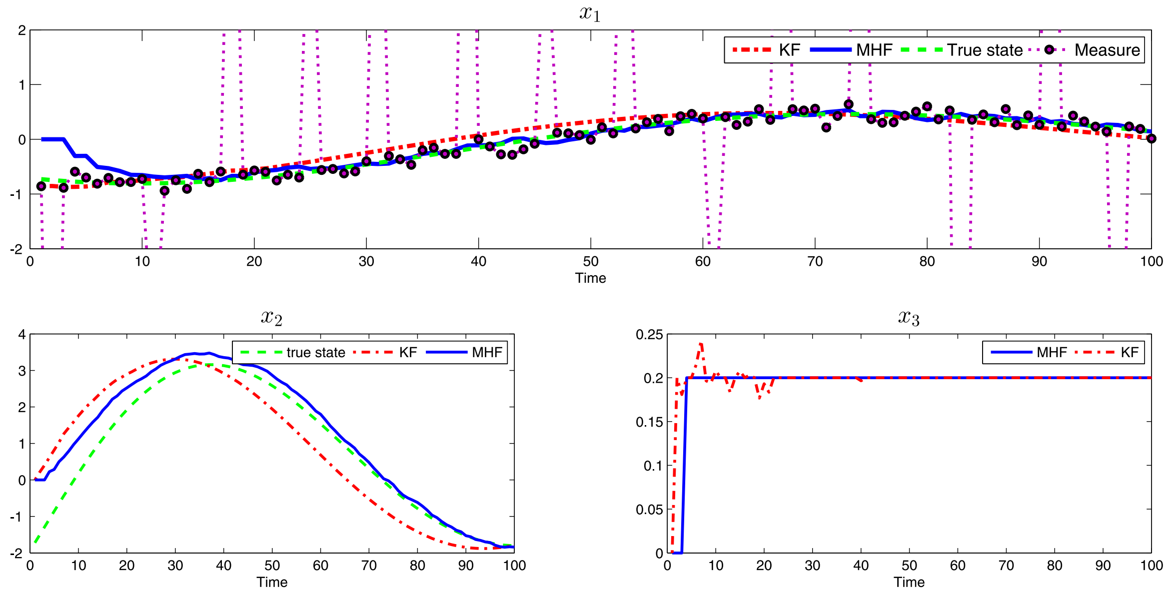

Figure 5. States estimation for MHF with $\mu = 0.5$ and KF with $\sigma_t = \sqrt{S_t}$

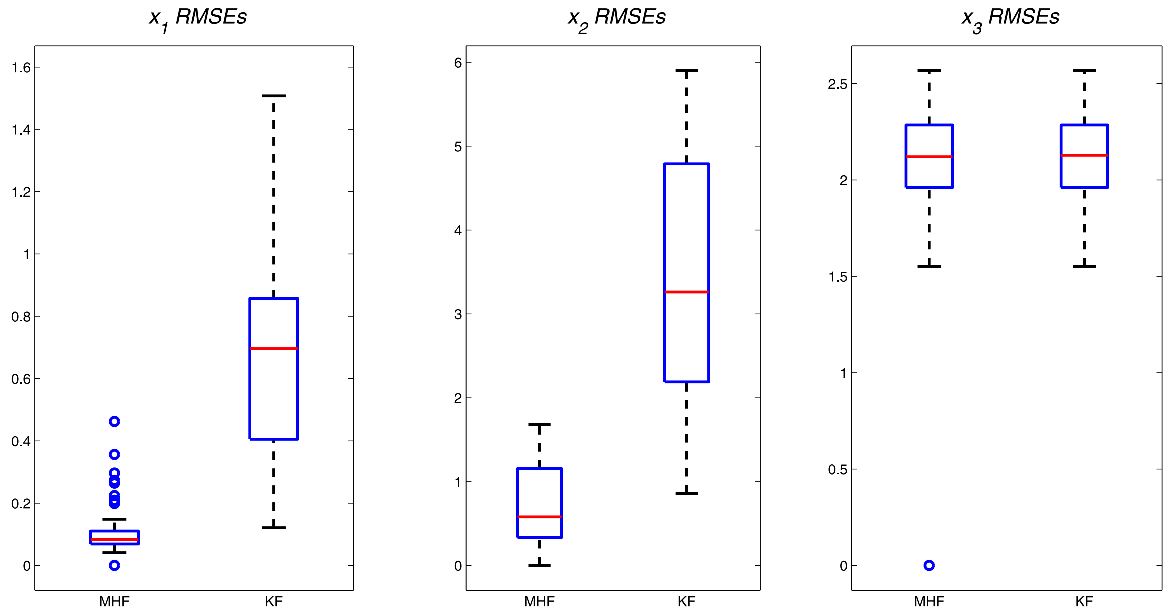

Figure 6. RMSES of MHF and KF for 100 runs with $r = 0.01$

Table 3. Means of RMSEs for MHF with $\mu = 0.5$ and KF with $\sigma_t = \sqrt{S_t}$

r	MHE			KF		
	x_1	x_2	x_3	x_1	x_2	x_3
0.01	0.1760	0.7584	2.0944	0.7144	3.4064	2.1153
0.10	0.2124	0.7262	2.0049	0.3343	1.6533	2.0343
1.00	0.5125	1.6230	2.0024	0.6088	2.8931	2.0204

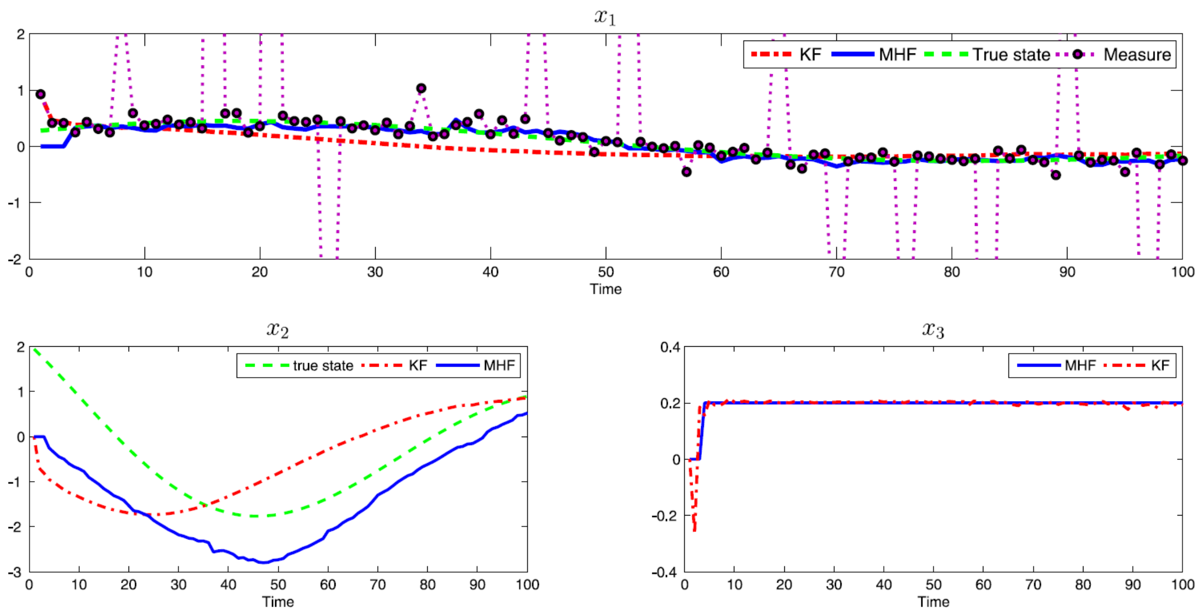

Figure 7. States estimation for MHF with $\mu = 1.0$ and KF with $\sigma_t = 10\sqrt{S_t}$

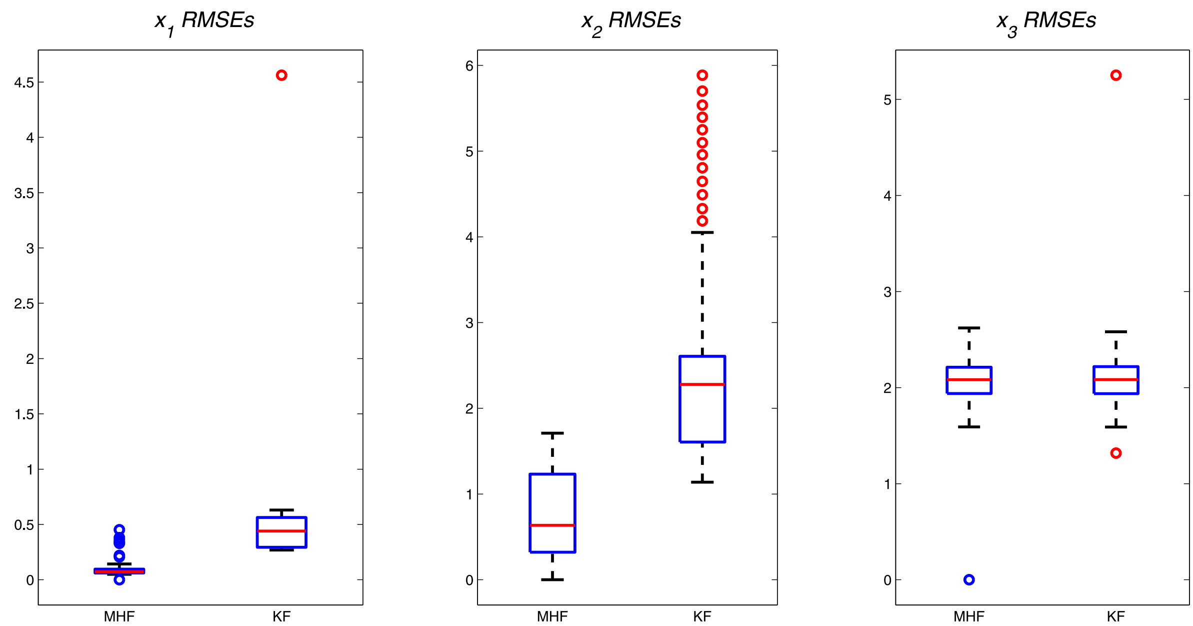

Figure 8. RMSES of MHF and KF for 100 runs with $r = 0.01$

Table 4. Means of RMSEs for MHF with $\mu = 1.0$ and KF with $\sigma_t = 10\sqrt{S_t}$

r	MHE			KF		
	x_1	x_2	x_3	x_1	x_2	x_3
0.01	0.1742	0.7840	2.0577	0.4608	2.4653	2.1062
0.10	0.2198	0.7199	2.0099	0.4423	1.6763	2.0488
1.00	0.4702	1.3058	2.0794	0.8392	3.1265	2.1289

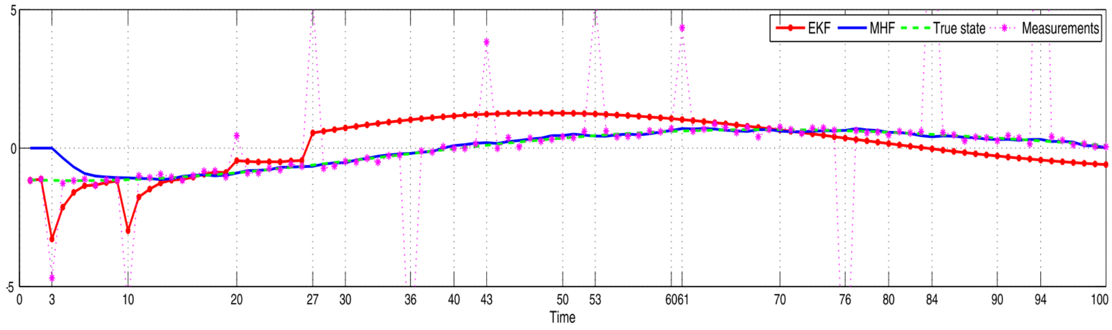

Figure 9. Simulation run over 100 monte-carlo with multiple randomly-positioned outliers, MHF with $\mu = 0.6$ and pure EKF (no residual check)

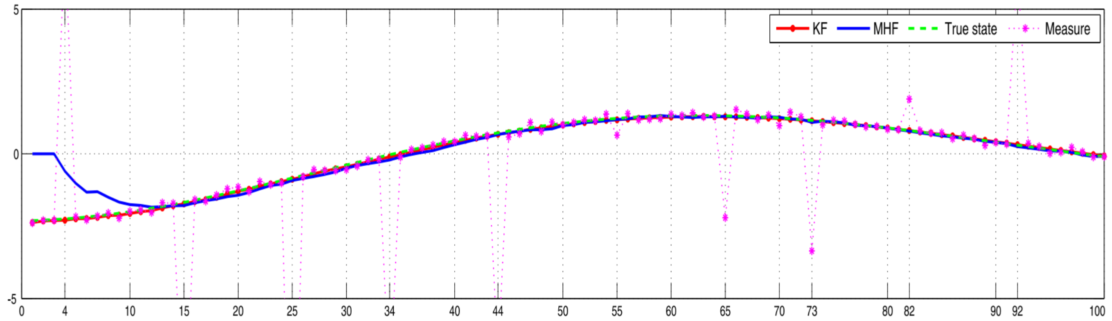

Figure 10. Simulation run over 100 monte-carlo with multiple randomly-positioned outliers, MHF with $\mu = 0.1$ and KF with residual check of $\sigma_t = 10\sqrt{S_t}$

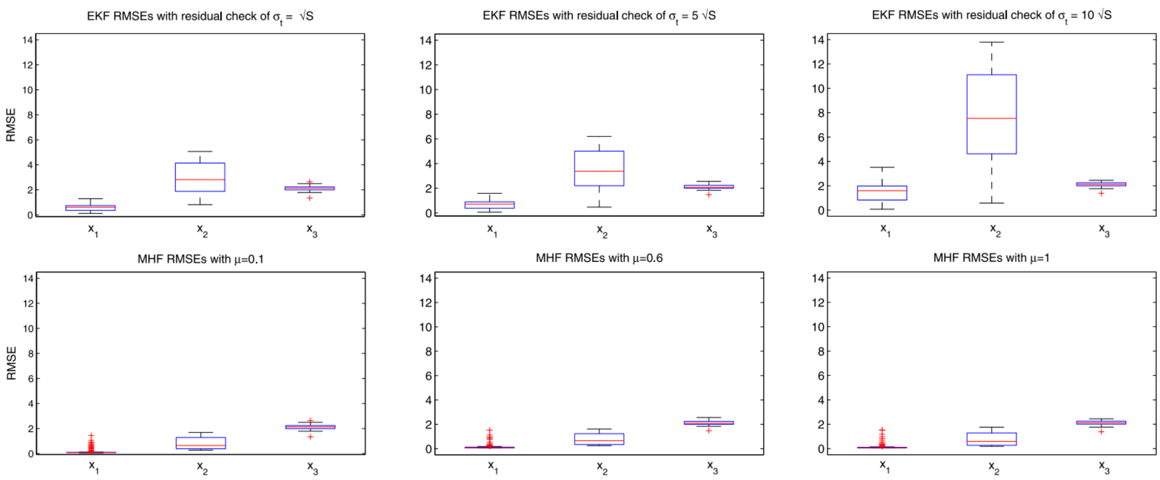

Figure 11. RMSES of MHF and EKF for 100 runs with $r = 0.01$ and different parameters (μ and σ_t)

Table 5. Means of computational time in seconds over 100 runs

r	MHE			EKF		
	$\mu = 0.1$	$\mu = 0.6$	$\mu = 1$	$\sigma_t = \sqrt{S_t}$	$\sigma_t = 5\sqrt{S_t}$	$\sigma_t = 10\sqrt{S_t}$
0.01	6.4228	5.6620	6.2146	0.0156	0.0312	0.0112
0.1	7.5228	5.2210	6.3697	0.0067	0.0100	0.0075
1.0	7.5520	5.6076	6.2595	0.0083	0.0089	0.0103

7. CONCLUSION

We have presented the problem of measurements contaminated by outliers in nonlinear systems using moving horizon estimation approach. The estimation consider two cases, measurement affected by outlier, and the other case where we suppose no outlier in the batch. The stability of the proposed moving horizon estimator is discussed. The efficiency of such filter is discussed compared with the EKF and KF. Such KF/EKF turns out to be sensitive to the choice of the threshold, while the proposed filter approach is easier to be tuned via the selection of μ and more robust to outliers. The performance of the proposed method is illustrated by the mean of simulation and RMSE result. One the challenges in such approach is the number of outlier presence in the same window size as well as the challenge of computational complexity; which are considered for further investigation in the future.

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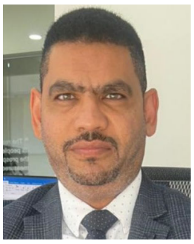
BIOGRAPHIES OF AUTHORS

Moath Awawdeh (Senior Member IEEE, CSS) received the B.S. in Electrical Engineering from Al Balqa Applied University for Engineering Technology, Jordan, in 2010, and Ph.D. in Control Engineering from the University of Genoa, Italy, 2015. His main research interests include outlier detection, data mining, machine learning, signal processing, and control engineering applications. He is currently working on the problem of outlier detection and correction in LTI systems with a specific approach of state estimation and measurement validation. He is the editor-in-chief of JAETS and Omdena JAII journal and serves as co-chair of ASET multi-conferences. He is currently with the Higher Colleges of Technology, United Arab Emirates He can be contacted at email: mawawdeh@hct.ac.ae.



Tarig Faisal received the master's degree in Mechatronics Engineering from IIUM University, in 2006, and the Ph.D. degree in Signal Processing from the University of Malaya, Malaysia, in 2011. He has been the Dean of academic operations at the Higher Colleges of Technology, since 2018. He has more than 20 years of academic and industry experience of which he worked as an engineer, an assistant professor, the programs chair, the head of department, the division chair, and the campus Director. His research interests include biomedical signal processing, intelligent systems, robotics, control, embedded system design, the IoTs, machine learning, and outcome-based education. He has been a reviewer for multiple journals including IEEE, Elsevier, Taylor & Francis, and Springer Nature. He is also a Charted Engineering as well a Senior Fellow HEA. He can be contacted at email: tfaisal@hct.ac.ae.

Anees Bashir received bachelor in Electronics and Communication Engineering from Anna University, India in 2012, and master degree in Applied Electronics Engineering from M.S. University India, in 2014. Her main research interests include embedded system design, signal processing, outlier detection and data mining. She is currently working on her research based on Moving-horizon strategy for anomaly detection in discrete LTI systems. She can be contacted at email: abashir1@hct.ac.ae.

Abdel Ilah Nour Alshbatat was born in Tafila, Jordan, in 1968. He received the B.S. degree in Electrical Engineering / Communications from Mutah Universty / Jordan in 1991, and the M.S. degree in Computer Engineering / Embedded System from Yarmouk University / Jordan in 2004, and the Ph.D. in Electrical and Computer Engineering / Networks from Western Michigan University / USA in 2010. In 1991, he joined Royal Jordanian Air Force (RJAF) and served in the directorate of communication and electronics ground communication branch, during which, he was responsible for the wireless communication and digital microwave sections. In 2010, he joined the Faculty of Tafila Technical University and is currently associate professor in the Department of Communications, Electronics and Computer Engineering. His current research interests include computer networks, wireless networks, embedded systems, UAV (unmanned aerial vehicle) and communication systems. He can be contacted at email: a.alshabatat@ttu.edu.jo.

Rana T. H. Momani an associate professor in measurement and evaluation. She graduated from Yarmouk University. She worked for several universities in Jordan and in the Arab World. her research interests in SEM, IRT, and cognitive abilities. She published several papers in world journals and currently serving as academic consultant at Ai-Enabled Consulting and Capacity Building, responsible for developing data mining curriculum and personalized training. She is currently with Zarqa University and can be contacted at email: rmomani@zu.edu.jo.