TELKOMNIKA Telecommunication Computing Electronics and Control
Vol. 22, No. 1, February 2024, pp. 86~94
ISSN: 1693-6930, DOI: 10.12928/TELKOMNIKA.v22i1.25264

Two new classes of conjugate gradient method based on logistic mapping

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Article Info

Article history:

Received Apr 16, 2023
Revised Jun 25, 2023
Accepted Aug 14, 2023

Keywords:

Conjugate gradient method
Descent condition
Global convergence condition
Logistic mapping
Unconstrained optimization

ABSTRACT

Following the standard methods proposed by Polak-Ribiere-Polyak (P-R), in this work we introduce two new non-linear conjugate gradient methods for solving unconstraint optimization problem, our new methods based on P-R. Standard method (P-R) have performance well in numerical result but does not satisfy global convergency condition. In this paper we modified double attractive and powerful parameters that have better performance and good numerical result than P-R method, also each of our robust method can satisfies the descent condition and global convergency condition by using wolf condition. More over the second method modified by logistic mapping form, the main novelty is their numerical results and demonstrate performance well with compare to a standard method.

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1. INTRODUCTION

Optimization is a process for finding the most curtail value for given functions within a specific domain, this process is mostly studied and used in mathematics, computer and physics science, the principal part of the unconstrained optimization is minimizing an objective function that rely on real variable, with no restrictions on the values. Mathematically, let $x \in R^n$ be real vector with $n > 1$ component and let $f : R^n \to R$ be smooth function, we consider below unconstrained optimization problem:

$$min\, f(x)\ \forall x \in R^n \tag{1}$$

Where the function $f: R^n \to R$ and its gradient g_i, also conjugate gradient methods (C-G) have played special roles in solving large dimension nonlinear problems. C-G method is an iterative scheme that usually generated a sequence $\{x_i\}$ of an approximated solution for (1), [1]–[4] usually we use the repetition:

$$x_{i+1} = x_i + \varphi_i p_i, i = 0,1,2, \dots \tag{2}$$

Where $\varphi_i > 0$ is a step size evaluated by line searches (Goldstein condition, wolfe condition, sufficient decent condition, and curvature condition) and the search direction p_i is designed by:

$$p_{i+1} = -g_{i+1} + \beta_i\, p_i \tag{3}$$

Where $g_i = \nabla f\ (x_i)$ and β_i is most influential parameter in C-G method. The different choices for the parameter β_i correspond to different C-G method, and most famous β_i check by the following scientists, Hestenes and Stiefel (H-S) [5], Polak-Ribiere-Polyak (P-R) [6], Fletcher and Reeves (F-R) [7], Alaa and Banaz [8], two formulas for β_i as shown in (4) and (5):

$$p_{(i+1)}^{P-R} = -g_{i+1} + (\frac{g_{i+1}^T y_i}{g_i^T g_i})\, p_i \tag{4}$$

$$p_{(i+1)}^{new} = -g_{i+1} + (\frac{g_{i+1}^T g_{i+1}}{p_i^T y_i} - \mu \left(\frac{g_{i+1}^T g_{i+1}}{g_i^T g_i}\right)^2)\, p_i \tag{5}$$

Where $y_i = g_{i+1} - g_i$ and $s_i = \varphi_i p_i = x_{i+1} - x_i$. Furthermore, the design of CG-techniques had been studied by many researchers; for more details see [9]–[14].

In the convergent condition and implementation of C-G method [15]–[17], one often requires the exact and inexact line search such as the wolfe conditions or the strong wolfe conditions. The wolfe line search is to find φ_i such that:

$$f(x_i + \varphi_i p_i) \le f(x_i) + \theta \varphi_i g_i^T p_i \tag{6}$$

$$p_i^T g(x_i + \varphi_i p_i) \ge \sigma g_i^T p_i \tag{7}$$

With $0 < \theta < \sigma$. The strong Wolfe line search is to find φ_i such that:

$$f(x_i + \varphi_i p_i) \le f(x_i) + \theta \varphi_i g_i^T p_i \tag{8}$$

$$|p_i^T g(x_i + \varphi_i p_i)| \le \sigma g_i^T p_i \tag{9}$$

Where $0 < \theta < \sigma < 1$ are constants [18]–[21].

In this paper, will present our new C-G training algorithms and their derivations in section 2. Global convergence condition and descent condition of our modified methods are proved in section 3. Numerical out comes for two new proposed algorithms are discussed in section 4. Finally, we conclude our methos and gives in section 5.

2. DERIVATIVE OF DOUBLE MODIFICATION ON P-R METHOD

2.1. Derivative of first proposed C-G method ($\boldsymbol{\beta_i^{NP\_R}}$)

Consider we have these new approaches:

$$y_i^* = y_i + \frac{\rho}{s_i^T y_i} y_i \text{ where } \rho = \frac{||g_i||^2}{||p_i||^2}$$

And our search direction with new parameter will be of the form:

$$p_{i+1} = -(\mu + 1) g_{i+1} + \beta_i^{NP\_R} p_i \text{ where } \mu \in (0,1) \tag{10}$$

By changing y_i in standard parameter (P-R) that in (4) by y_i^* we have:

$$\beta_i^{NP\_R} = \frac{g_{i+1}^T y_i + g_{i+1}^T y_i (\frac{||g_i||^2}{||p_i||^2\, s_i^T y_i})}{g_i^T g_i} \tag{11}$$

So, we get this new form:

$$\beta_i^{NP\_R} = \beta_i^{P\_R} + \frac{g_{i+1}^T y_i (\frac{||g_i||^2}{||p_i||^2\, s_i^T y_i})}{g_i^T g_i} \tag{12}$$

Now, we will show the steps of solving unconstrained optimization problems by the algorithms (2.2) and (2.3).

2.2. Algorithm of the first new method ($\beta_i^{NP\_R}$)

In this subsection, the outlines of the first new method for solving unconstrained optimization problems is stated.

Step 1 : begin with initial point $x_0 \in R^n$.
Step 2 : set initial search direction $p_0 = -g_0$, $i = 0$.
Step 3 : if $\|g_i\| = 0$ then stop, otherwise go to step 4.
Step 4 : find the step size φ_i by using minimize function $f(x_i + \varphi_i p_i)$.
Step 5 : set $x_{i+1} = x_i + \varphi_i p_i$.
Step 6 : determine g_{i+1}, if $\|g_{i+1}\| \leq 10^{-5}$ stop, else go to step 7.
Step 7 : compute new search direction p_{i+1} by (10).
Step 8 : if $\|g_{i+1}\|^2 \leq \frac{|g_i^T g_{i+1}|}{0.2}$ is satisfied go to step 3,
else $i = i + 1$ and go to step 4.

2.3. Derivative of second proposed C-G method (β_i^{LMP-R})

Consider our new search direction is of the form:

$$p_{i+1}^{NEW} = -(1+\theta)g_{i+1} + \beta_i^{LMP-R}\, p_i \tag{13}$$

The logistic mapping form [1] and standard parameter in (4) gives:

$$\beta_n^{LMP-R} = \mu\, \beta_i^{P-R}(1 - \omega\beta_i^{P-R})$$

Where:

$$\omega = \frac{g_{i+1}\, s_i}{p_i^T y_i}\ , \theta \in (0,1)\ , \mu\epsilon(0,1] \tag{14}$$

This implies that,

$$\beta_i^{LMP-R} = \mu\ \frac{g_{n+1}^T y_n}{g_n^T g_n}(1 - \frac{g_{i+1}\, s_i}{p_i^T y_i}\ \frac{g_{n+1}^T y_n}{g_n^T g_n}) \tag{15}$$

2.4. Algorithm of the second new method ($\beta_i^{LMP\_R}$)

In this subsection, the outlines of the second new method for solving unconstrained optimization problems is stated. The steps of this algorithm are the same steps that in the first new outline (2.2). The only changeable step is step 7, we will compute the new parameter that in (15).

3. DESCENT PROPERTY AND GLOBAL CONVERGENCE CONDITION FOR TWO NEW METHOD

Assumption (A) [22]–[24].

3.1. The level set $\delta = \{x\ |f(x) \leq f(x_i)\}$ at x_i is bounded, there exists a constant $a > 0$ such that:

$$\|x\| \leq a, \forall x \in \delta \tag{16}$$

3.2. If the function f is continuously differentiable and some neighborhood N of δ, and its $g_i(x)$ is Lipschitz continuous with Lipschitz constant $L > 0$, i.e.,

$$\|g(r) - g(t)\| \leq L\|r - t\|\ \forall\ r, t \in \delta \tag{17}$$

3.3. From two assumptions that in (16) and (17), that there exists a positive constant c such that:

$$\|g(c)\| \leq c\ \forall x \in \delta \tag{18}$$

If function f is strongly convex, then there exists a constant $\mu > 0$ such that:

$$\mu\ \|x - y\|^2 \leq (\nabla f\ (x) - \nabla f\ (y))^T\ (x - y), \text{for all } x, y \in \delta \tag{19}$$

3.4. Consider assumption (16) is hold. Suggest that (1) and (2), where p_i is a descent direction and φ_i satisfies the standard wolfe line search. If:

$$\sum_{i\geq 1}\frac{1}{\|p_i\|^2}=\infty \tag{20}$$

Then, $\lim_{i\to\infty} inf\ \|g_i\|=0$

Theorem (3.1): proof that the search direction p_{i+1} with modified parameter β_i^{NP-R} of C-G method is descent direction $g_{i+1}^T p_{i+1}\leq 0$. consider the sequence $\{\ x_i\}$ is in the (2) where the step length α_i defined in two cases, exact line search and inexact line search. Then:

$$p_{i+1}=-(\mu+1)\ g_{i+1}+\beta_i^{NP-R}\ p_i$$

Multiple both sides of equation by g_{i+1}^T and from (12), we get:

$$g_{i+1}^T p_{i+1}=-(\mu+1)\ g_{i+1}g_{i+1}^T+\beta_i^{P\text{-}R}g_{i+1}^T\ p_i+\frac{g_{i+1}^T y_i(\frac{||g_i||^2}{||p_i||^2\ s_i^T y_i})}{g_i^T g_i}g_{i+1}^T\ p_i \tag{21}$$

Then we have two cases for selecting step size [11], so the first one by (ELS) that require of $g_{i+1}^T\ p_i=0$, and the maintain term $||g_{i+1}||^2$ is positive, so:

$$g_{i+1}^T p_{i+1}=-(\mu+1)\ ||g_{i+1}||^2 \tag{22}$$

Thus (22) satisfies descent condition.

But the second case of g_i by ILS, it consists of $g_{i+1}^T\ p_i\neq 0$, in the above form we have $\beta_i^{P\text{-}R}$ so it satisfies the descent condition. By using strong wolfe condition in (21) $\sigma_2 g_i^T\ p_i\leq g_{i+1}^T\ p_i\leq -\sigma_2 g_i^T\ p_i$ where $\sigma_2\in(0,1)$ we get:

$$g_{i+1}^T p_{i+1}\leq -(\mu+1)||g_{i+1}||^2-\frac{g_{i+1}^T y_i(\frac{||g_i||^2}{||p_i||^2\ s_i^T y_i})}{g_i^T g_i}\sigma_2 g_i^T\ p_i$$

In this case if the wolfe condition satisfies, we can say that $g_{i+1}^T\ p_i\leq p_i^T\ y_i$:

$$g_{i+1}^T p_{i+1}\leq -(\mu+1)||g_{i+1}||^2-\frac{g_{i+1}^T y_i(\frac{||g_i||^2}{||p_i||^2\ s_i^T y_i})}{g_i^T g_i}p_i^T\ y_i$$

We know that $s_i=\varphi_i p_i$ and $y_i=g_{i+1}-g_i$, so the equation will be of:

$$g_{i+1}^T p_{i+1}\leq -(\mu+1)||g_{i+1}||^2-\frac{g_{i+1}^T(g_{i+1}-g_i)(\frac{\sigma_2||g_i||^2}{\varphi_i||p_i||^2})}{||g_i||^2}$$

Then,

$$g_{i+1}^T p_{i+1}\leq -(\mu+1)||g_{i+1}||^2-\frac{||g_{i+1}||^2+g_{i+1}^T g_i(\frac{\sigma_2||g_i||^2}{\varphi_i||p_i||^2})}{||g_i||^2}$$

By powell condition -$|g_{i+1}^T g_i|$<-0.2 $||g_{i+1}||^2$:

$$g_{i+1}^T p_{i+1}\leq -(\mu+1)||g_{i+1}||^2-\frac{||g_{i+1}||^2-0.2\ ||g_{i+1}||^2(\frac{\sigma_2||g_i||^2}{\varphi_i||p_i||^2})}{||g_i||^2}$$

$$g_{i+1}^T p_{i+1} \leq -||g_{i+1}||^2[(\mu+1)+\frac{1+0.2\left(\frac{\sigma_2||g_i||^2}{\varphi_i||p_i||^2}\right)}{||g_i||^2}].\text{ Let } \tau=(\mu+1)+\frac{1+0.2\left(\frac{\sigma_2||g_i||^2}{\varphi_i||p_i||^2}\right)}{||g_i||^2}$$

Where $\mu\epsilon(0,1)$ and $\varphi_i, \sigma_2, ||g_i||^2$, and $||p_i||^2$ are positive terms, so $\tau > 0$, hence we get descent condition. Theorem (3.2): consider the assumption (A) is hold. Then the new β_i^{NP-R} of the form (10) and (12), where φ_i is checked by wolfe line search (7), then it satisfies global convergency condition $\lim_{i\to\infty} inf\ \|g_i\| = 0$.

Proof: $p_{(i+1)}^{New} = -\ g_{i+1} + (\frac{g_{i+1}^T y_i}{g_i^T g_i} + \frac{g_{i+1}^T y_i\left(\frac{||g_i||^2}{||p_i||^2\ s_i^T y_i}\right)}{g_i^T g_i})\ p_i$

$$\|p_{i+1}^{NEW}\| \leq (\mu+1)\|g_{i+1}\| + \left\| \frac{g_{n+1}^T y_n}{g_n^T g_n}\right\| \|\ p_i\| + \left\|\frac{g_{i+1}^T y_i\left(\frac{||g_i||^2}{||p_i||^2\ s_i^T y_i}\right)}{g_i^T g_i}\right\| \|\ p_i\| \tag{23}$$

Since $\|g_{n+1}^T y_n\| \leq ||g_{i+1}||||y_n||$. And by (LC) we have $||y_n|| \leq L||\ s_i||$:

$$\|p_{i+1}^{NEW}\| \leq (\mu+1)\|g_{i+1}\| + \frac{L\ ||g_{i+1}||\ ||s_i||}{||g_i||^2}\|\ p_i\| + \frac{L\ ||g_{i+1}||\ ||s_i||\left(\frac{||g_i||^2}{||p_i||^2\ \varphi_i\ y_i^T d_i}\right)}{||g_i||^2}\|\ p_i\| \tag{24}$$

Hence, we know that $p_i^T g_{i+1} \leq p_i^T y_i$, by using (7) and the form ($p_i = -g_i$).

We get this form $||g_i||^2 \geq \frac{-1}{\sigma}\ p_i^T y_i$:

$$\|p_{i+1}^{NEW}\| \leq (\mu+1)c + \frac{\sigma_1 L\ c\ ||s_i||}{p_i^T y_i}\|\ p_i\| + \frac{\sigma_1 L\ c\ ||s_i||\left(\frac{p_i^T y_i}{||p_i||^2\ \varphi_i\ y_i^T d_i}\right)}{p_i^T y_i}\|\ \mathrm{p}_i\| \tag{25}$$

Since $p_i^T y_i \geq\ \theta\frac{\|s_i\|^2}{\varphi_i}$, use this form in (25):

$$\|p_{i+1}^{NEW}\| \leq (\mu+1)c + \frac{\varphi_i\sigma_1 L\ c\ ||s_i||}{\vartheta\|s_i\|^2}\|\ p_i\| + \frac{\varphi_i\sigma_1 L\ c\ ||s_i||\left(\frac{p_i^T y_i}{||p_i||^2\ \varphi_i\ y_i^T d_i}\right)}{\vartheta\|s_i\|^2}\|\ p_i\| \tag{26}$$

After some algebraic operation in (26) we get this simple form:

$$\|p_{i+1}^{NEW}\| \leq (\mu+1)c + \frac{\varphi_i\sigma_1 L\ c}{\vartheta\|\ s_i\|}\|\ p_i\| + \frac{\varphi_i\sigma_1 L\ c}{\vartheta\|\ s_i\|^2}\|\ p_i\|$$

Let $||s_i|| = ||x_{i+1} - x_i||$, $\gamma = \max\{||x_{i+1} - x_i||\}, \forall\ x_{i+1}, x_i \in R$

$\|p_{i+1}^{NEW}\| \leq c((\mu+1) + \frac{\sigma_1 L}{\vartheta} + \frac{\sigma_1 L}{\vartheta\ \gamma}) = \tau^*$

$\therefore \sum_{i\geq 1}\frac{1}{\|p_{i+1}^{NEW}\|^2} \geq \sum_{i\geq 1}\frac{1}{\tau^*} = \infty$

$\Rightarrow \sum_{i\geq 1}\frac{1}{\|d_{i+1}^{NEW}\|^2} = \infty.$

Finally, by using Lemma we get $\lim_{x\to\infty}\inf\|g_{i+1}\| = 0$.

Theorem (3.3): assume that the sequence $\{x_i\}$ is generated by (2), then the search direction in (13) and new parameter β_i^{LMP-R} satisfy the descent property, i.e.,

$$g_{i+1}^T p_{i+1} \leq 0 \tag{27}$$

Proof: multiply both sides of equation by g_{i+1}, to obtain:

$$g_{i+1}^T p_{i+1}^{NEW} = -g_{i+1}^T g_{i+1} + \mu \frac{g_{n+1}^T y_n}{g_n^T g_n}\left(1 - \frac{g_{i+1}\, s_i}{p_i^T y_i}\frac{g_{n+1}^T y_n}{g_n^T g_n}\right) g_{i+1}^T\, p_i \tag{28}$$

If the search direction is chosen by (ELS), then $g_{i+1}^T\, p_i = 0$, so it satisfies (27), however, if the search direction (15) is inexact (i.e.,) $g_{i+1}^T d_i \neq 0$.We concludes that the first two terms of (26) are satisfies the descent condition i.e., $g_{i+1}^T p_{i+1}^{NEW} = -\|g_{i+1}\|^2 + \mu \frac{g_{n+1}^T y_n}{g_n^T g_n} g_{i+1}^T\, p_i - \frac{g_{i+1}\, s_i}{p_i^T y_i} (\frac{g_{n+1}^T y_n}{g_n^T g_n})^2\, g_{i+1}^T\, p_i$ because the PRP method satisfies the (27) condition, so we have:

$$g_{i+1}^T p_{i+1}^{NEW} = -\|g_{i+1}\|^2 - \frac{g_{i+1}\, s_i}{p_i^T y_i} (\frac{g_{n+1}^T y_n}{g_n^T g_n})^2\, g_{i+1}^T\, p_i$$

Therefore, by using the form $s_i = \varphi_i p_i$, we get:

$$g_{i+1}^T p_{i+1}^{NEW} = -\|g_{i+1}\|^2 - \frac{\delta_i (g_{i+1}\, p_i)^2}{p_i^T y_i} (\frac{g_{n+1}^T y_n}{g_n^T g_n})^2$$

Clearly, δ_i, $(g_{i+1}\, p_i)^2, d_i^T y_i$ and$(\frac{g_{n+1}^T y_n}{g_n^T g_n})^2$ are positive term. Because of that it satisfies (27).

Theorem (3.4): consider the assumption (A) is hold. Then the new β_i^{LMP-R} of the form (13) and (15), where φ_i is checked by wolfe condition, then it satisfies $\lim_{i\to\infty} inf\ \|g_i\| = 0$.

Proof: $p_{i+1}^{NEW} = -g_{i+1} + \mu \frac{g_{n+1}^T y_n}{g_n^T g_n}(1 - \frac{g_{i+1}\, s_i}{p_i^T y_i}\frac{g_{n+1}^T y_n}{g_n^T g_n})\, \mathrm{p}_i$

$$\left\|p_{i+1}^{NEW}\right\| \leq \|g_{i+1}\| + \mu \left\|\frac{g_{n+1}^T y_n}{g_n^T g_n}\right\| \|\, p_i\| + \left\|\mu \frac{g_{i+1}\, s_i}{p_i^T y_i}\left(\frac{g_{n+1}^T y_n}{g_n^T g_n}\right)^2\right\| \|\, p_i\| \tag{29}$$

Since $|g_{n+1}^T y_n| \leq \left||g_{i+1}|\right| ||y_n||$. And by (LC) we have$\left||y_n|\right| \leq \mathrm{L}||\, s_i||$

$$\left\|p_{i+1}^{NEW}\right\| \leq \|g_{i+1}\| + \mu \frac{L\|\, g_{i+1}\|\|s_i\|}{\|g_i\|^2} \|\, p_i\| + \mu \frac{\|\, g_{i+1}\|\|s_i\|}{\|p_i^T y_i\|}\left(\frac{L\|\, g_{i+1}\|\|s_i\|}{\|g_i\|^2}\right)^2 \|\, p_i\|$$

$$\left\|p_{i+1}^{NEW}\right\| \leq \mathrm{c} + \mu \frac{\mathrm{c}\,\mathrm{L}\|s_i\|}{\|g_i\|^2} \|\, p_i\| + \mu \frac{c\,\|s_i\|}{\|p_i^T y_i\|}\left(\frac{\mathrm{c}\,\mathrm{L}\|s_i\|}{\|g_i\|^2}\right)^2 \|\, p_i\| \tag{30}$$

Hence, we know that $p_i^T\, g_{i+1} \leq p_i^T\, y_i$, by using (7) and form $p_i = -g_i$. We get this form $||g_i||^2 \geq \frac{-1}{\sigma}\, p_i^T\, y_i$:

$$\left\|p_{i+1}^{NEW}\right\| \leq \mathrm{c} + \mu \frac{c\, L\sigma\|s_i\|}{p_i^T y_i} \|\, p_i\| + \mu \frac{c\,\|s_i\|}{p_i^T y_i}\left(\frac{L\sigma \mathrm{c}\|s_i\|}{p_i^T y_i}\right)^2 \|\, p_i\| \tag{31}$$

Since $p_i^T y_i \geq\ \theta \frac{\|s_i\|}{\varphi_i}$

$$\left\|p_{i+1}^{NEW}\right\| \leq \mathrm{c} + \mu \frac{c\, L\sigma\|s_i\|}{\theta\|s_i\|} \|\, p_i\| + \mu \frac{c\,\|s_i\|}{\theta\|s_i\|}\left(\frac{L\sigma \mathrm{c}\|s_i\|}{\theta\|s_i\|}\right)^2 \|\, p_i\|$$

$$\left\|p_{i+1}^{NEW}\right\| \leq \mathrm{c}\, \mu L(\frac{\sigma}{\theta} + \frac{L\sigma\gamma}{\theta^3}) = \tau^* \tag{32}$$

$$\therefore \sum_{i\geq 1} \frac{1}{\|p_{i+1}^{NEW}\|^2} \geq \sum_{i\geq 1} \frac{1}{\tau^*} = \infty$$

$$\Rightarrow \sum_{i\geq 1} \frac{1}{\|p_{i+1}^{NEW}\|^2} = \infty$$

By using Lemma (4.1), we get:

$$\lim_{i\to\infty} inf\ \|\, g_{i+1}\| = 0.$$

4. NUMERICAL RESULTS AND COMPARISONS

Here, we show some numerical experiments of the new formulas in conjugate gradient method. Each problem has been tested with different values of n ranged from 4 to 5,000, where n is a number of variables of each test function [25]. We did our implementations in the FORTRAN 95 language with given initial points. Table 1 shows comparison tests consist of well-known non-linear functions with 8 different functions for each new method. For restart we use the powell condition. On the other hand, the comparative results illustrate in Table 2 which contain number of iteration (NI) and number of function (NF), the experimental results verify that the new algorithms are superior to standard (P-R), in general the rate of improvement of the first new method β_i^{NP-R} is 3.238% in (NI) and 11.7118% in (NF), also for the second method β_i^{LMP-R} that applied in logistic mapping is 40.556% in (NI) and 25.6373% in (NF), in general the magical improvement between (NI) and (NF) are 66.1933%.

Table 1. Shows the comparison between standard P-R method and two modified methos β_i^{NP-R} and β_i^{LMP-R}

Test function	Recip			
n	4	100	1000	5000
NI (P-R)	9	9	9	9
NI (NP-R)	7	8	8	8
NF (P-R)	28	28	28	28
NF (NP-R)	19	23	23	23
Test function	OSP			
n	4	100	1000	5000
NI (P-R)	9	52	179	340
NI (NP-R)	9	48	129	275
NF (P-R)	52	173	570	114
NF (NP-R)	51	161	391	824
Test function	Wood			
n	4	100	1000	5000
NI (P-R)	38	39	39	44
NI (NP-R)	30	30	32	32
NF (P-R)	83	85	85	95
NF (NP-R)	67	68	72	72
Test function	Rozen			
n	4	100	1000	5000
NI (P-R)	18	19	21	21
NI (NP-R)	18	18	21	21
NF (P-R)	46	49	55	55
NF (NP-R)	46	46	53	53
Test function	cubic			
n	4	100	1000	5000
NI (P-R)	29	31	31	31
NI (NP-R)	25	25	25	26
NF (P-R)	79	84	84	84
NF (NP-R)	73	73	73	75
Test function	Powell 3			
n	4	100	1000	5000
NI (P-R)	27	48	63	137
NI (NP-R)	18	44	48	103
NF (P-R)	55	97	127	289
NF (NP-R)	37	89	96	230
Test function	Miele			
n	4	100	1000	5000
NI (P-R)	75	115	145	181
NI (NP-R)	54	56	64	78
NF (P-R)	185	295	382	480
NF (NP-R)	128	144	165	205
Test function	Powell			
n	4	100	1000	5000
NI (P-R)	98	124	138	138
NI (NP-R)	17	17	22	31
NF (P-R)	128	283	319	319
NF (NP-R)	38	38	64	97
Total of NI (P-R)			1328	
Total of NI (NP-R)			1285	
Total of NF (P-R)			4440	
Total of NF (NP-R)			3920	

Test function	Box			
n	4	100	1000	5000
NI (P-R)	22	22	23	30
NI (LMP_R)	17	19	22	22
NF (P-R)	159	159	171	270
NF (LMP_R)	103	130	171	174
Test function	Wolf			
n	4	100	1000	5000
NI (P-R)	11	49	64	99
NI (LMP_R)	11	43	48	99
NF (P-R)	24	99	129	214
NF (LMP_R)	24	87	97	214
Test function	Miele			
n	4	100	1000	5000
NI (P-R)	37	44	50	50
NI (LMP_R)	41	57	58	59
NF (P-R)	116	148	180	180
NF (LMP_R)	104	142	144	146
Test function	Wood			
n	4	100	1000	5000
NI (P-R)	29	30	30	30
NI (LMP_R)	28	28	28	29
NF (P-R)	67	69	69	69
NF (LMP_R)	63	63	63	64
Test function	Sum			
N	4	100	1000	5000
NI (P-R)	3	14	23	31
NI (LMP_R)	3	14	20	28
NF (P-R)	11	80	127	145
NF (LMP_R)	11	71	102	109
Test function	Cubic			
n	4	100	1000	5000
NI (P-R)	20	21	21	21
NI (LMP_R)	19	19	19	19
NF (P-R)	55	59	59	59
NF (LMP_R)	54	54	54	54
Test function	Rosen			
n	4	100	1000	5000
NI (P-R)	15	15	15	15
NI (LMP_R)	12	12	12	12
NF(P-R)	41	41	41	41
NF (LMP_R)	32	32	32	32
Test function	OSP			
n	4	100	1000	5000
NI (P-R)	8	49	161	276
NI (LMP_R)	8	47	156	276
NF (P-R)	45	176	493	844
NF (LMP_R)	45	146	475	828
Total of NI (P-R)			2266	
Total of NI (LMP-R)			1347	
Total of NF (P-R)			4864	
Total of NF (LMPR)			3617	

Table 2. The percentage of improvement between the new methods β_i^{NP-R} and β_i^{LMP-R} with PRP method

Tools	Standard P-R (%)	New β_i^{NP-R}	New β_i^{LMP-R}
NI	100	96.762	59.4439
NF	100	88.2882	74.3626

5. CONCLUSION

We have shown two new powerful and superior non-linear C-G methods for solving unconstrained problems. Consider the standard approach proposed by P-R, we modified P-R two times, so on the one hand the first modified method delightful have double properties that performance well in numerical outcomes with compare to standard P-R and satisfies the descent and global convergent condition. On the other hand, the second modified method we applied in the form of logistic mapping and a new search direction, because of that it satisfies the global convergent and descent condition, farther more in numerical outcomes illustrate their magical performance with measured to standard P-R method.

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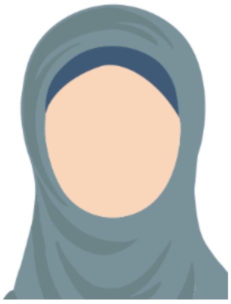

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