Enhanced fuzzy logic control for overcoming intrinsic resistance in inverted pendulum systems

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ABSTRACT

The paper delves into an in-depth analysis of the intrinsic resistance of the inverted pendulum system which causes the modeling of the system to differ from the actual system. Our primary objective revolves around the implementation and subsequent optimization of fuzzy logic controllers (FLC), drawing inspiration from human perceptual assessments. The processing comprises comprehensive mathematical system modeling, intrinsic resistance examination, and improved fuzzy logic control with detailed membership function and rule design. In addition, we conduct a comparative analysis with the widely recognized linear quadratic regulator (LQR) algorithm, which is considered the conventional control algorithm. The result demonstrates that the improved FLC outperforms the conventional LQR algorithm overshoot mitigation, thereby underscoring its superior efficacy and optimality.

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1. INTRODUCTION

Almost all dynamics systems with widespread applications like aircraft, robotics arms, humanoid robots, and satellites have multi-dimensional structures [1]. However, it is very complex to analyze and solve these dynamic equations. So, the basic single-dimensional system which has a simple differential dynamic equation like an inverted pendulum is the fundamental concept in control theory [2], [3]. The inverted pendulum is a prevalent control model for learning control theory because of its unstable characteristic. Besides that, it occurs in several scientific and industrial applications like humanoid robots and self-balancing vehicles. In Figure 1, the operation of the self-balancing vehicle is based on balancing the inverted pendulum in three coordinates. Therefore, this nonlinear system is an important and realistic system to learn about.

In recent research, several problems of inverted pendulum systems are discussed. The directional inverted pendulum normally known as the cart and pole system is a typical model for applying a new nonlinear control technique [4]–[7]. Double inverted pendulum which is harder to control is widely used as a model for testing adaptive control and intelligent control algorithms [8]–[10]. The rotary inverted pendulum (RIP) is also considered an excellent model for learning energy-based control methods [11]–[16]. The combination of rotary and double inverted pendulum is also developed [17]– [20]. Other researchers mention swing-up methods for types of inverted pendulums [21]–[23]. Nevertheless, their models are implemented without considering the intrinsic resistance that affects the system when controlling the pendulum at high speed. The sliding friction and rotational friction of the pendulum itself are considered intrinsic resistance of an inverted pendulum. It is necessary to consider this intrinsic resistance when establishing the inverted pendulum system model for system accuracy. Nguyen *et. Al.* [24], Sambo *et. Al.* [25] analyze the stability of the pendulum system without considering the effect of the resistance. In references [26], [27] focus on designing an effective control method to maintain the pendulum angle, yet the intrinsic resistance between the cart and the platform is not adequately taken into account.

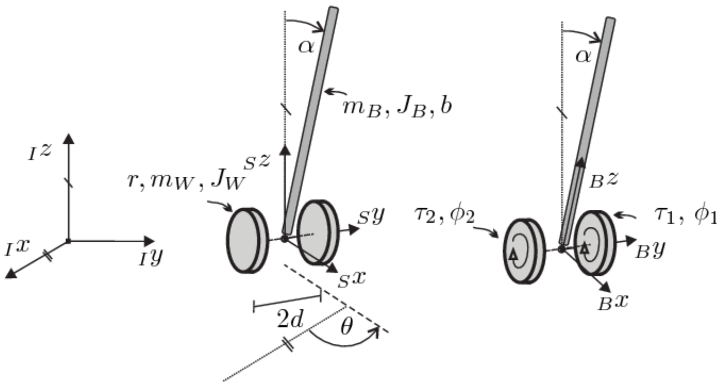

Figure 1. The inverted pendulum theory applied in self-balancing vehicles [28]

When we analyzed the dynamics equation of the system in detail, both the sliding friction between the cart and the rail and the rotational friction from the pendulum itself are considered in a mathematical model. Considering rotational friction makes the system more complex and difficult to balance. In addition to stabilizing the inverted pendulum, the controller must be flexible and appropriate for solving nonlinear system problems. Fuzzy logic control is suitable for nonlinear systems like the inverted pendulum due to its inherent capacity to manage the system without a need for an accurate mathematical system model. Fuzzy logic control demonstrates its prominence when compared to popular controllers in industrial applications such as the linear quadratic regulator (LQR) algorithm. Fuzzy logic control requires the designer experience in the system [29], [30]. The rule base is the most important condition to stabilize the system. Other parts, such as membership functions, defuzzification method, and feedback gain improve the performance [31], [32]. Hence, designing complicated fuzzy is deliberated.

The study focus on improving fuzzy logic control for stabilizing the inverted pendulum system with intrinsic resistance. Specifically, the study aims to understand how the fuzzy logic control method performs in comparison to the conventional LQR algorithm when applied to an inverted pendulum system affected by rotational friction and sliding friction, which are considered intrinsic resistance. The focus is on analyzing the system's stability, overshoot, and performance under different intrinsic resistance values. The result shows that fuzzy logic control keeps the stability of the system when applying a series of different intrinsic resistances. In addition, fuzzy logic control surpasses the LQR algorithm in terms of overshoot percentage, oscillation, and stability. The feasibility of the proposed control scheme was assessed through simulation results conducted using MATLAB Simulink.

2. INTRINSIC RESISTANCE ANALYSIS OF THE INVERTED PENDULUM MODEL

The inverted pendulum consists of a moving cart and a rotating pendulum. The resistance of the system includes the sliding friction force between the carriage and the slide and the rotational friction force of the rotating shaft. The other resistance is not considered. The model of the inverted pendulum is shown in Figure 2. In Figure 2, M is the mass of the cart, m is the mass of the pendulum, α is the coefficient of sliding friction, β is the coefficient of rotational friction, $g$is the acceleration of gravity. U is the force acting on the cart. The cart's position is x. The cart velocity is $\dot{x}$. The falling angle is θ and the angular velocity is $\dot{\theta}$.
From Figure 2, the position of the pendulum is derived:

$$\begin{aligned} x_p &= x + l\sin\theta \\ y_p &= l\cos\theta \end{aligned} \tag{1}$$

Derivative (1), we get the velocity of the pendulum as (2):

$$\dot{x}_p = \dot{x} + l\dot{\theta}\cos\theta$$
$$\dot{y}_p = -l\dot{\theta}\sin\theta \tag{2}$$

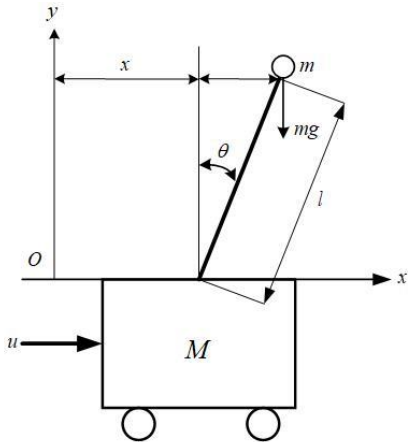

Figure 2. Model of inverted pendulum

Since the velocity of the cart and the pendulum, the kinetic energy of the system can be expressed as (3):

$$T = T_C + T_P = \frac{1}{2}(M+m)\dot{x} + \frac{1}{2}ml^2\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\theta \tag{3}$$

Besides, the potential energy of the system is calculated as (4):

$$U = U_C + U_P = mgl\cos\theta \tag{4}$$

Combining both kinetic energy and potential energy, we get the Lagrangian of the inverted pendulum:

$$L = T - U = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\theta - mgl\cos\theta \tag{5}$$

The effect of the sliding friction force between the carriage and the slide is considered together with the effect of the rotational friction force of the rotating shaft. Hence, the total resistance energy applied to the inverted pendulum is derived as (6):

$$P = P_1 + P_2 = \frac{1}{2}\beta\dot{\theta}^2 + \frac{1}{2}\alpha\dot{x}^2 \tag{6}$$

The Euler-Lagrange equations are used to derive the equations of motion for a physical system, and these equations are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \frac{\partial P}{\partial \dot{x}} = u$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} + \frac{\partial P}{\partial \dot{\theta}} = 0 \tag{7}$$

Combining (1)–(6) and analyzing (7), we get the differential equations for the inverted pendulum system:

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta = u + ml\theta^2\sin\theta - \alpha\dot{x}$$
$$ml^2\ddot{\theta} + ml\ddot{x}\cos\theta = mgl\sin\theta - \beta\dot{\theta} \tag{8}$$

Solving (8), we get the dynamic equation of the inverted pendulum system:

$$\ddot{x} = \frac{(u + ml\dot{\theta}\sin\theta - \alpha\dot{x})l - (mgl\sin\theta - \beta\dot{\theta})\cos\theta}{(M+m)l - ml\cos^2\theta} \tag{9}$$

$$\ddot{\theta} = \frac{(mgl\sin\theta - \beta\dot{\theta})(M+m) - (u + ml\dot{\theta}^2 \sin\theta - \alpha\dot{x})ml\cos\theta}{(M+m)ml^2 - m^2l^2\cos^2\theta} \tag{10}$$

Assuming the state variables $x_1 = x, x_2 = \dot{x}, x_3 = \theta, x_3 = \dot{\theta}$. Then, the system state-space is derived as (11):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{(u + mlx_4{}^2 \sin x_3 - \alpha x_2)l - (mgl\sin x_3 - \beta x_4)\cos x_3}{(M+m)l - ml\cos^2 x_3} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{(mgl\sin x_3 - \beta x_4)(M+m) - (u + mlx_4{}^2 \sin x_3 - \alpha x_2)ml\cos x_3}{(M+m)ml^2 - m^2l^2\cos^2 x_3} \end{aligned} \tag{11}$$

The α and β values represent the intrinsic resistances of the inverted pendulum system, contributing to increased model complexity and the necessity for accurate parameters in conventional control schemes. As the system becomes highly nonlinear, conventional linear control theory is inapplicable. Therefore, the exploration of nonlinear control methods, tailored to address the complexities inherent in this nonlinear system, becomes imperative.

3. CONVENTIONAL AND PROPOSED FUZZY LOGIC CONTROL METHODS

The adoption of enhanced fuzzy logic control is proposed for controlling the inverted pendulum with intrinsic resistances. One notable feature of fuzzy logic control is that it is a model-free approach, which is especially useful for handling the system's intrinsic complexity and nonlinearity. Fuzzy logic control, in contrast to conventional control techniques, is excellent at capturing and managing uncertainties, which makes it a good fit for situations where it could be difficult to build an accurate mathematical model. Effective control techniques for the inverted pendulum system with inherent resistances are made possible by this flexible and adaptable fuzzy control approach, potentially leading to increased stability and performance.

A fuzzy logic controller (FLC)'s architecture and designing processing are shown in Figure 3. It consists of essential elements: a control rule base, a fuzzification procedure that transforms system feedback signal into fuzzy values to support rule-based reasoning, and a defuzzification step that converts the inferred fuzzy output into a distinct and accurate control signal for the system.

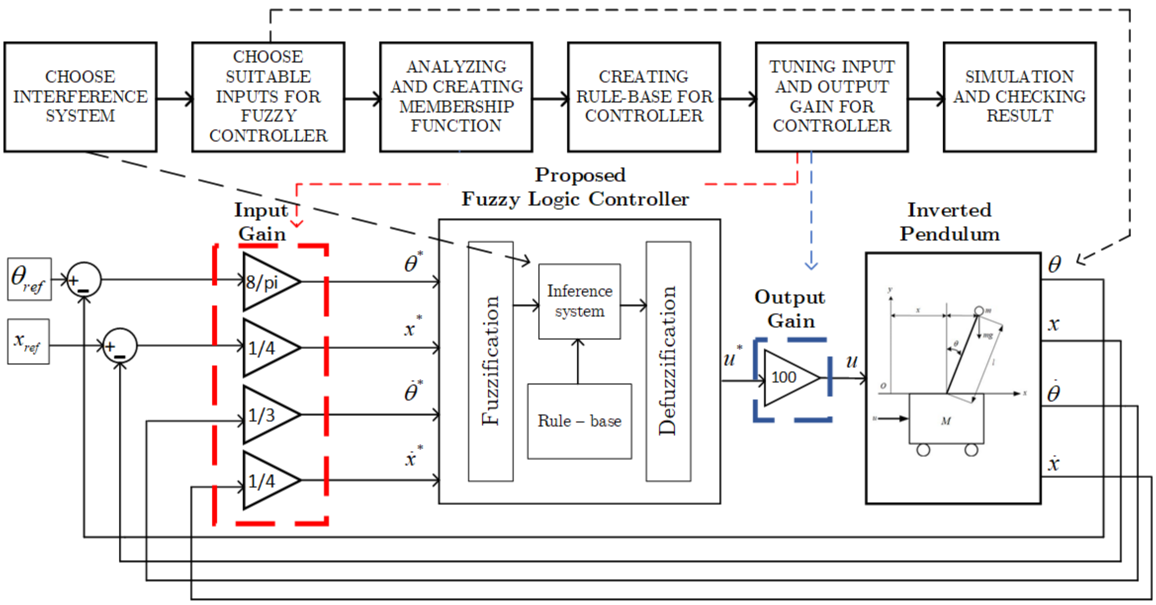

Figure 3. Fuzzy logic control's architecture and designing processing

3.1. Conventional fuzzy logic control

In the controller designing process, the choice of an interference system is important because it will affect the efficiency and computational complexity of the controller. Many inverted pendulum studies are using Mamdani-type interference systems to design the controller. For the mamdani fuzzy interference system (FIS), the output of each rule in the rule-base is the fuzzy set and the output of the model is calculated as a combination of these fuzzy sets. So, it requires a defuzzification method to convert fuzzy set values into numerical values for control in the final stages. In Mamdani interference, there are two popular defuzzification methods: the mean of maxima method (MOM) and the centroid of gravity (COG) method [33]. The MOM method can be represented through mathematical formulas as (12):

$$y^* = mean\{y: \mu(y) = 1\} \tag{12}$$

Where y^* is the crisp output value, $\mu(y)$ is the membership function of output. The MOM defuzzification methods may introduce nonlinear behavior leading to system instability, rendering them unsuitable for control problems [34]. In the COG defuzzification method, the output of the centroid-based method is computed by (13):

$$y^* = \frac{\int_{-\infty}^{\infty} y\mu(y)dy}{\int_{-\infty}^{\infty} \mu(y)dy} \tag{13}$$

Because the computing process has an integral operator, the COG methods require a high computation in discrete time. Consequently, (13) has high computational complexity because it involves a complete fractional calculation. That increases implementation costs for the processing unit. Therefore, a simpler calculation and defuzzification method needs to be found to save costs and calculation time.

3.2. Proposed fuzzy logic control with modified sugeno interferences

To optimize the calculation process, the Takagi–Sugeno–Kang (TSK) interferences are adopted thanks to their simplicity and efficiency. Compared to the conventional Mamdani FIS, the Sugeno FIS calculation process relies on crisp values for the control, allowing us to bypass defuzzification steps without compromising the stability and accuracy of the control system. In the proposed method, the consequent clause y_i of $i'th$ rule in rule-base system is a mapping function belonging to the input variable:

$$y_i = f(x_1, x_2, x_3, x_4) \tag{14}$$

In general, the mapping function f in (14) can be a nonlinear, linear function, or only a constant value. The fuzzy output is devided into seven levels and the function f is defined as a constant value for each level for simplicity in this study. The overall output value is calculated as (15):

$$y^* = \sum_i \beta_i y_i, \tag{15}$$

Where β_i is the degree of truth for the i^{th} rule. The computing processing in TSK (15) is more simple than mamdani FIS output in (13) because there is no integral function in (15). Consequently, the adoption of the TSK interference system is advocated for its ability to expedite the calculation process in the fuzzy controller design.

The TSK FIS has minimized the computational complexity of the controller, enabling us to select additional inputs and establish more control rules. To conduct a more comprehensive system evaluation, the controller's inputs comprise the angle, position angular velocity, and velocity of the cart ($x_1 = \theta$; $x_2 = \dot{\theta}; x_3 = x; x_4 = \dot{x}$). Using these four parameters makes the controller more effective and accurate because the interference system gets more assessments about the system. Reminding the state-space equation denoted as (11), the state variables featured in this equation are also the fuzzy controller's input variables. The objective of the controller is to maintain equilibrium by balancing the pendulum at a deflection angle of zero ($\theta = 0$).

Initiating the design process, we embark on crafting a control rule base by enumerating conceivable combinations of state variables within the state space equation. Fuzzy values such as negative (NE), zero (ZE), and positive (PO) are set to denote the direction of the state variable movement. For the output variable of the controller, we also set the fuzzy values as negative big (NB), negative medium (NM), negative small (NS), zero (ZE), positive small (PS), positive medium (PM), positive big (PB) to feature for consequent values y_i in (13). Figures 4(a) and (b) show membership function of the cart's position (x) and membership function of the cart's velocity ($\dot{x}$). Figures 4(c) and (d) show membership function of the falling angle (θ) and membership function of the angular velocity ($\dot{\theta}$). Similarly, the membership function output variable is shown in Figure 5. Figure 6 shows example combinations of control rules based on state variables and Table 1 shows the composed rule base for the inverted pendulum system.

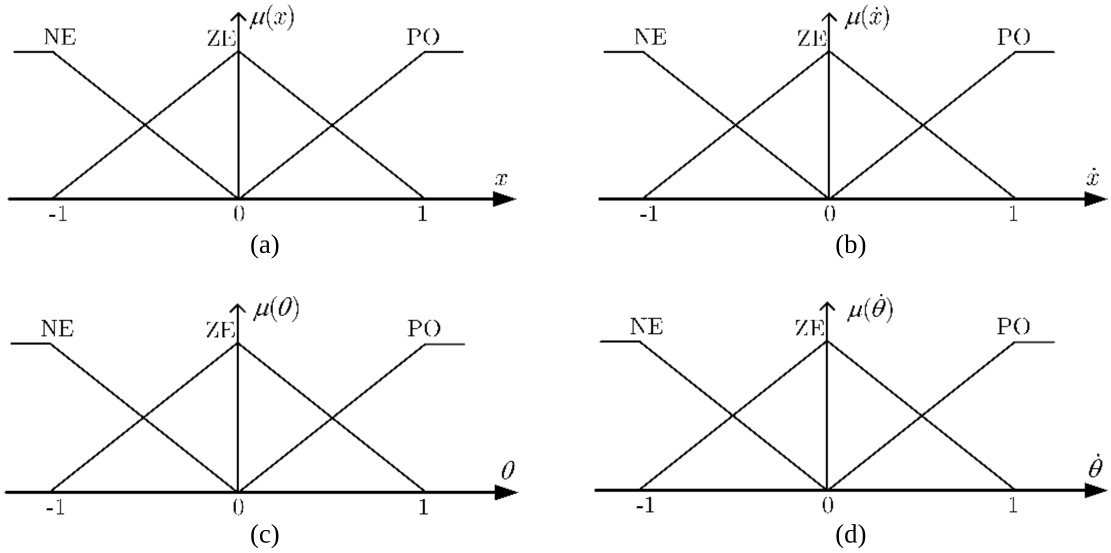

Figure 4. Proposed membership functions for inputs of fuzzy logic control; (a) membership function of the cart's position (x), (b) membership function of the cart's velocity ($\dot{x}$), (c) membership function of the falling angle (θ), and (d) membership function of the angular velocity ($\dot{\theta}$)

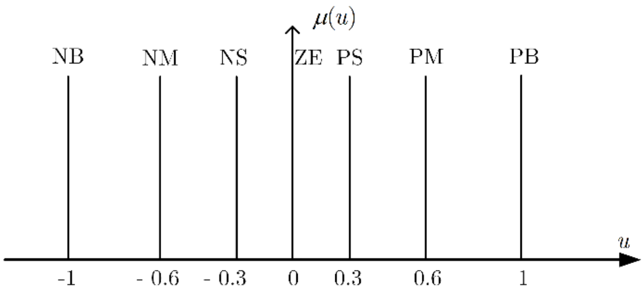

Figure 5. Membership function of the force acting on the cart (u)

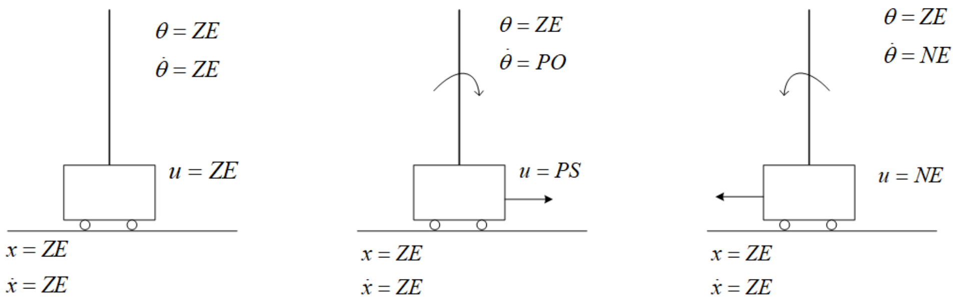

Figure 6. Example combinations of control rule based on state variables

To optimize control results, besides building membership functions and output values clearly, the input gain and output gain coefficients can also be controlled as shown in the block diagram in Figure 3. A similar theory of tuning input and output gains is described in the book "Tunning via Scaling Universes of Discourse" presented by Passino *et al.* in [34]. Based on this control theory, it is possible to tune these gains to improve the control performance of the inverted pendulum. The proposed gains consist of the output gain (K_u), and the input gain for each system variable ($K_\theta, K_{\dot{\theta}}, K_x, K_x$) are shown in Table 2.

Table 1. Rule base for pendulum using TSK type fuzzy inference system

u		$\theta:\dot{\theta}$ NE: NE	NE: ZE	NE: PO	ZE: NE	ZE: ZE	ZE: PO	PO: NE	PO: ZE	PO: PO
$x:\dot{x}$	NE: NE	NB	NB	NM	NB	NM	NS	NM	NS	ZE
	NE: ZE	NB	NM	NS	NM	NS	ZE	NS	ZE	PS
	NE: PO	NM	NS	ZE	NS	ZE	PS	ZE	PS	PM
	ZE: NE	NB	NM	NS	NM	NS	ZE	NS	ZE	PS
	ZE: ZE	NM	NS	ZE	NS	ZE	PS	ZE	PS	PM
	ZE: PO	NS	ZE	PS	ZE	PS	PM	PS	PM	PB
	PO: NE	NM	NS	ZE	NS	ZE	PS	ZE	PS	PM
	PO: ZE	NS	ZE	PS	ZE	PS	PM	PS	PM	PM
	PO: PO	ZE	PS	PM	PS	PM	PB	PM	PB	PB

Table 2. Proposed input gain and output gains

Gains	Values
K_u	100
K_θ	2.5
$K_{\dot{\theta}}$	0.25
K_x	0.67
$K_{\dot{x}}$	0.25

4. SIMULATION RESULTS AND DISCUSSION

4.1. Inverted pendulum parameters and simulation platform

In this section, we assess the performance of the proposed FLC by employing MATLAB Simulink to simulate the inverted pendulum system. The simulation aims to evaluate the effectiveness of our controller in maintaining the stability of the pendulum and achieving the setpoint position of the cart. For an objective comparison, we compared our results with the LQR controller discussed in [35] because the LQR controller is the popular model-based controller for nonlinear systems. LQR is a model-based control scheme with optimized root locus placement and minimized cost equation [36]. For this reason, the LQR control scheme obtains a good steady state performance. The simulation parameters are presented in Table 3. In the simulation scenario, the value of α is constant, while different values of β are taken into consideration. To evaluate the quality of each control method, we analyzed the following criteria: overshoot of the inverted pendulum angle (e_θ) and overshoot of the cart position (e_x), and the settling time of inverted pendulum angle (t_θ) and the settling time of the cart position (t_x), which is the time when the system's response converges and stays within 5% of the reference value.

Table 3. Simulation parameters of inverted pendulum

Parameters	Symbol	Value
Mass of the cart	M	1.096 Kg
Mass of the rod	m	0.109 Kg
Sliding friction coefficient	α	0.1
Length of the rod	l	0.5 m

4.2. Performance evaluation (fuzzy logic and conventional linear quadratic regulator controllers)

To evaluate the control performance, the step responses of the cart position are examined with a setpoint of position is 0.1 m. In addition, the intrinsic resistance coefficients (β) are varied to verify the effectiveness of the proposed FLC controller. The series of intrinsic resistance coefficients are selected as follows: $\beta_1 = 0,\ \beta_2 = 0.005,\ \beta_3 = 0.017,$ and $\beta_4 = 0.035$. In terms of inverted pendulum angle performance, Figures 7(a) and (b) show the step response of the inverted pendulum angle when $\beta = 0$ and $\beta = 0.005$. Figures 7(c) and (d) show the step response of the inverted pendulum angle when $\beta = 0.017$ and $\beta = 0.035$.

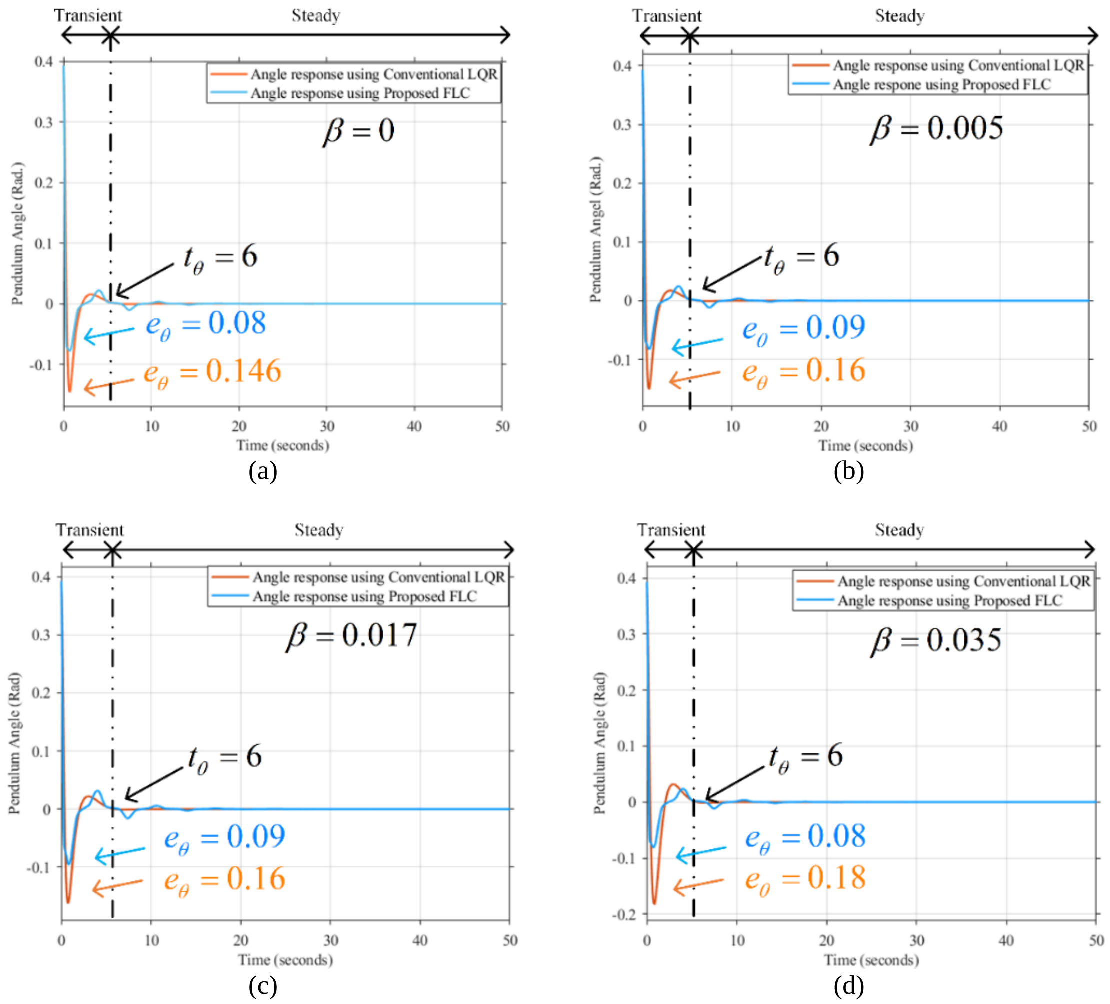

Figure 7. Response of pendulum angle when; (a) $\beta = 0$, (b) $\beta = 0.005$, (c) $\beta = 0.017$, and (d) $\beta = 0.035$

In terms of inverted pendulum cart position performance, Figures 8(a) and (b) show the response of cart position when $\beta = 0$ and $\beta = 0.005$. Figures 8(c) and (d) show the response of cart position when $\beta = 0.017$ and $\beta = 0.035$. To enhance reader comprehension, the results have been summarized in Table 4.

When $\beta = \beta_1 = 0$, the overshoot of the pendulum angle (e_θ) with the proposed FLC control scheme is smaller than that of the conventional LQR control scheme during the transient period. Similarly, the proposed FLC control scheme maintains the small overshoot of the pendulum angle when $\beta = 0.005$, $\beta = 0.017$, and $\beta = 0.035$. These results highlight the advantage of the fuzzy controller in the transient period because of the detailed fuzzy rule and membership function.

In the cart position performance, the settling time of the conventional LQR control scheme is shorter than that of the FLC control scheme thanks to its model-based. Meanwhile, the overshoot of cart position (e_x) with the proposed FLC control scheme is smaller than the conventional LQR (0.56<0.8) when $\beta = \beta_1 = 0$. When increasing the intrinsic resistance coefficient β ($\beta = 0.005$, $\beta = 0.017$, and $\beta = 0.035$), the overshoots using the proposed FLC control scheme are also smaller than the overshoots using conventional LQR.

It is important to note that the LQR mainly depends on the prior knowledge of the state space equation, and the detailed system model of the inverted pendulum is considered. On the other hand, the proposed FLC does not require the system model and parameters. Hence, the LQR control scheme may have a faster cart position response than that of the FLC control scheme. However, the proposed FLC control scheme has a smaller overshoot in cart position and pendulum angle. In the steady-state error is zero in both FLC and LQR control schemes, which ensures the performance in the steady-state.

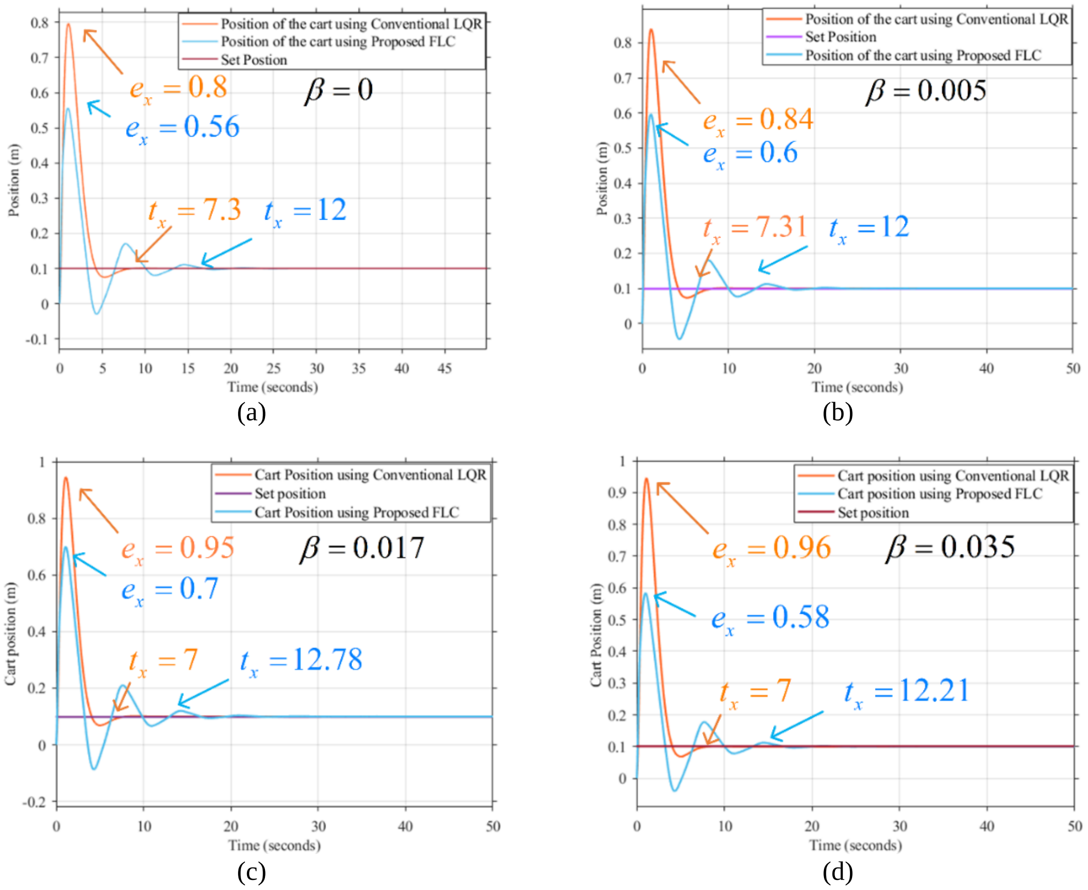

Figure 8. Response of cart position when; (a) $\beta = 0$, (b) $\beta = 0.005$, (c) $\beta = 0.017$, and (d) $\beta = 0.035$

Table 4. Simulation result

β	Controller	Pendulum angle		Cart position	
		e_θ (rad)	t_θ (s)	e_x (m)	t_x (s)
0	FLC	0.08	6.01	0.56	12
	LQR	0.146	6.12	0.8	7.3
0.005	FLC	0.08	6.03	0.6	12
	LQR	0.15	6.14	0.84	7.31
0.017	FLC	0.09	6.03	0.7	12.77
	LQR	0.16	6.2	0.95	7.1
0.035	FLC	0.08	6.01	0.58	12.21
	LQR	0.18	6.14	0.96	7

4.3. Discussion of findings and potential future research

Through this work, we contribute to advancing the understanding and control of inverted pendulum systems, particularly in the presence of intrinsic resistance. For the purpose of comparison, the LQR control algorithm is selected because of its well-established effectiveness in optimizing the performance of dynamic systems, especially the inverted pendulum system. In terms of improved performance, the proposed fuzzy logic control has advantages in terms of overshoot percentage, oscillation, and stability compared to the LQR algorithm. In terms of unimproved performance parameters, the settling time for the pendulum angle is nearly identical for both the proposed fuzzy logic control and the LQR control algorithm. However, the LQR control algorithm exhibits a significant overshoot, yet its oscillation decay appears to be faster compared to that of the proposed FLC.

For potential future research, from the author's perspective, Type-2 fuzzy logic may enhance system performance while maintaining system stability. Type-2 fuzzy logic is more complicated than Type-1, involving the management of not only the uncertainty in the variable but also the uncertainty in the membership function. It extends traditional fuzzy logic concepts to handle even higher levels of uncertainty, offering a more nuanced and flexible approach to modeling complex systems. However, this advancement comes at the cost of increased computational complexity. The choice between Type-1 and Type-2 fuzzy logic hinges on the specific requirements and the level of uncertainty present in the problem at hand.

5. CONCLUSION

An inverted pendulum model with intrinsic resistance is established in this study, along with the design of an enhanced FLC with detailed membership functions and rules. The study employs the Euler-Lagrange method for deriving dynamic equations, and fuzzy logic control is introduced as a model-free approach, adept at addressing the inherent complexity of the inverted pendulum system. MATLAB Simulink has offer a comprehensive evaluation of the proposed FLC in comparison to the LQR, and the result focus on overshoot and settling time under various intrinsic resistance coefficients. The detailed FLC scheme consistently outperforms the conventional LQR, notably minimizing overshoot by 30 percentage in both the inverted pendulum angle and cart position. This advantage is particularly pronounced under conditions of varying intrinsic resistance coefficients. While the LQR showcases faster cart position response due to its model-based cost function, the FLC maintains stability with reduced overshoot, demonstrating its effectiveness in navigating the intrinsic complexity and nonlinearity of the system. By specifically addressing challenges introduced by sliding and rotational friction, our study focuses on enhancing stability and performance, particularly in applications such as self-balancing vehicles. Theoretical analysis and simulation results have validated the efficacy of the proposed control scheme.

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