Bayes estimation of a two-parameter exponential distribution and its implementation

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ABSTRACT

Life test data analysis is a statistical method used to analyze time data until a certain event occurs. If the life test data is produced after the experiment has been running for a set amount of time, the life time data may be type I censored data. When conducting observations for survival analysis, it is anticipated that the data would conform to a specific probability distribution. Meanwhile, to determine the characteristics of a population, parameter estimation is carried out. The purpose of this study is to use the linear exponential loss function method to derive parameter estimators from the exponential distribution of two parameters on type I censored data. The prior distribution used is a non-informative prior with the determination technique using the Jeffrey's method. Based on the research results that have been obtained, application is carried out on real data. This data is data on the length of time employees have worked before they experienced attrition with a censorship limit based on age, namely 58 years, obtained from the Kaggle.com website. Based on the estimation results, the average length of work for employees is 6.29427 years. This shows that employees tend to experience attrition after working for a relatively long period of time.

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1. INTRODUCTION

Employee reduction is an important issue in the world of business and human resource management. Employee attrition is a gradual reduction in the number of workers in a company that occurs for several reasons such as retirement, illness, death, termination of employment, resignation, or other reasons which are then not replaced [1]. This is in line with the sustainable development goals (SDGs) in point 8 (decent work and economic growth). Survival analysis is a statistical analysis that is useful for analyzing data related to the time an event occurs from the beginning until the occurrence of a certain event such as recovery, failure, or death [2]. Data obtained from life test observations can be complete data or censored data. Data may be deemed censored if it is obtained before all of the data is observed throughout its lifetime, after the observation period has ended, or for other reasons [3]. The type of data censorship that is often used is type I censorship. Type I censorship is observations that are stopped when a certain time period is reached [4]. For type I sensors, the research object is included in the study at the same time as the research period has been established [5]. There are three things in determining the timing of survival analysis, namely start point, end point, and measurement scale [6].

Apart from the concept of data censoring, in carrying out survival analysis observations the distribution function is also used. One distribution function that is often used and has an important role in survival analysis is the exponential distribution. The exponential distribution has the role of measuring the level of failure that may occur in an opportunity. Bain and Engelhardt [7] based on the parameters, the exponential distribution is divided into two, namely the one-parameter exponential distribution and the two-parameter exponential distribution.

After knowing the distribution of the data that will be used, the next step is to estimate the parameters. The classical approach and the Bayesian approach are the two approaches used in estimation theory to estimate population parameters given data distribution [8]. The Bayesian approach is an estimation method that combines a prior distribution and a sample distribution [9]. The Bayesian approach has several methods, one of which is linear exponential loss function. Parameter estimation research using the linear exponential method in the Bayes approach is considered good and most commonly used [10]. The expected value of the linear exponential loss function is minimized when using the linear exponential method for parameter estimation.

In the Bayesian method, the parameter is treated as a random variable that has a probability distribution [11]. The probability distribution of unknown parameters selected based on two approaches, namely subjective and based on previous research is called the prior distribution. Prior distributions consist of two types, namely informative prior distributions and non-informative prior distributions. A non-informative prior distribution is a prior distribution for which no information about the parameter θ is known, one of which is the Jeffrey's prior [12]. The non-informative prior distribution has a less significant influence on the posterior distribution, but will still have an influence on the analysis results. The analysis results from non-informative prior distributions are more influenced by the data, so they will provide more neutral estimates.

Research on estimating continuous distribution parameters using a Bayesian approach to survival analysis was carried out by [13] which explains the use of the Bayesian linear exponential loss function method in estimating exponential distribution parameters on type I censored data using Jeffrey's prior. Furthermore, research conducted by [14] explains the use of the Bayesian linear exponential loss function method in estimating exponential distribution parameters using Gamma priors. The author wishes to conduct research on parameter estimation from a two-parameter exponential distribution on type I censored data using the Bayesian linear exponential loss function method. Jeffrey's non-informative prior is the prior distribution that is used. This is based on the background that was previously explained.

2. METHOD

2.1. Linear exponential loss function

Linear exponential loss function is one of the loss functions used in statistical analysis for estimation and forecasting problems that show exponential growth on one side of zero and almost linear on the other side of zero [15]. The linear exponential function shows a high degree of flexibility in measuring estimation, because linear exponential is able to adjust varying weights. This method has been widely used by several researchers, including [16], [17]. The formula for the linear exponential loss function for parameter θ is:

$$loss(\hat{\theta},\theta) = \exp\left(a(\hat{\theta}-\theta)\right) - a(\hat{\theta}-\theta) - 1, a \neq 0 \quad (1)$$

Using the linear exponential loss function, the value of the posterior expectation for parameter θ, represented by E_θ, can be expressed as (2):

$$E_\theta\left(loss(\hat{\theta},\theta)\right) = \exp(a\hat{\theta})\, E_\theta\left(\exp(-a\hat{\theta})\right) - a\left(\hat{\theta} - E_\theta(\theta)\right) - 1 \quad (2)$$

where a is a control parameter that sets the magnitude of the discrepancy between the estimated and actual values.

2.2. Prior Jeffrey's

Non-informative prior distribution is a concept in Bayesian statistics that refers to a prior distribution used to describe prior knowledge about a particular parameter. A non-informative priors approach is important when not having sufficient knowledge or keeping the analysis results free from bias. One form of non-informative prior that is often used is the Jeffrey's method [12]. As per the Jeffrey's method approach [13], it is recommended that the previous distribution have a square root of the Fisher information. Fisher's parameter θ for information value is defined as (3):

$$I(\theta) = -E\left[\frac{\partial^2 \ln(f(t))}{\partial\theta^2}\right] \quad (3)$$

Then, a prior distribution can be formulated from Fisher information.

$$P(\theta) \propto \sqrt{I(\theta)} \tag{4}$$

where, f(t) is the probability density function (PDF) of a two-parameter exponential distribution.

3. RESULTS AND DISCUSSION

3.1. Analysis of bayes estimation

$T_i \sim Exp(\theta, \mu)$ can be used to represent the surviving observation data, which are supposed to be T_1, T_2, … , T_n from a random variable T of size n with an exponential distribution with parameters θ and μ. As per reference Lee and Wang [18], the PDF serves as a tool to gauge the Likelihood that an individual would encounter a specific event during the time frame t to (t+Δt). So, the PDF is [19].

$$f(t;\theta;\mu) = \begin{cases} \theta e^{-\theta(t-\mu)};\ t \geq \mu, \theta > 0 \\ 0\ ;\ t < 0 \end{cases} \tag{5}$$

In forming the Likelihood function, a survival function formula is needed. According to Tolba [20], the survival function is a statistical analysis used to understand phenomena related to time. In (6) is the survival function of the two-parameter exponential distribution.

$$S(t) = e^{-\theta(t-\mu)} \tag{6}$$

The random variable from type I censored data has the following Likelihood function after deriving the survival function formula [21].

$$L(\vartheta) = \prod_{i=1}^{n} f(t_i)^{\delta_i}\, S(t_i)^{1-\delta_i} \tag{7}$$

$$L(\vartheta) = \prod_{i=1}^{n} \left(\theta e^{-\theta(t_i-\mu)}\right)^{\delta_i} \left(e^{-\theta(t_i-\mu)}\right)^{1-\delta_i} \tag{8}$$

$$L(\vartheta) = \theta^{\sum_{i=1}^{n}\delta_i} e^{-\theta\sum_{i=1}^{n}(t_i-\mu)} \tag{9}$$

To generate the Likelihood function, use (7). It is therefore possible to determine the posterior distribution, which is denoted by $P(\vartheta|X_i)$.

3.1.1. Analysis of bayes estimation with assuming both parameters are variable

The formula for the Likelihood function has been obtained, then the natural logarithm of the Likelihood function will be calculated.

$$ln\left(L(\vartheta)\right) = ln\left(\theta e^{-\theta(t-\mu)}\right) \tag{10}$$

$$ln\left(L(\vartheta)\right) = ln(\theta) - \theta(t-\mu) \tag{11}$$

The results of $ln\left(L(\vartheta)\right)$ will then be used to determine the Fisher's information value obtained from the first and second derivatives of $ln\left(L(\vartheta)\right)$. So, Fisher's information value will be a determinant [22].

$$I(\vartheta) = -E\begin{bmatrix} \frac{\partial^2 ln(L(\vartheta))}{\partial\theta^2} & \frac{\partial^2 ln(L(\vartheta))}{\partial\theta\partial\mu} \\ \frac{\partial^2 ln(L(\vartheta))}{\partial\mu\partial\theta} & \frac{\partial^2 ln(L(\vartheta))}{\partial\mu^2} \end{bmatrix} \tag{12}$$

$$I(\vartheta) = -E\begin{bmatrix} -\frac{1}{\theta^2} & 1 \\ 1 & 0 \end{bmatrix} \tag{13}$$

Based on the Fisher's information matrix according to (4) with reference to (12), the form of the prior distribution can be described as (14) and (15):

$$P(\vartheta) \propto \sqrt{det\ I(\vartheta)} \tag{14}$$

$$P(\vartheta) \propto 1 \tag{15}$$

The posterior distribution in the Bayes theorem is produced by combining the Jeffrey's prior distribution which was previously computed with the Likelihood function [23]. A Bayesian approach foundation for statistical inference will be the posterior distribution [24]. According to Zhang and Gao [25], the posterior distribution denoted by $P(\vartheta|T_i)$ has (16) to (18):

$$P(\vartheta|T_i) = \frac{L(\vartheta).P(\vartheta)}{\int_0^\infty \int_0^\infty L(\vartheta).P(\vartheta)\, d\mu\, d\theta} \quad (16)$$

$$P(\vartheta|T_i) = \frac{\theta^{\sum_{i=1}^{n}\delta i}\, e^{-\theta \sum_{i=1}^{n}(t_i-\mu)}.(1)}{\int_0^\infty \int_0^\infty \theta^{\sum_{i=1}^{n}\delta i}\, e^{-\theta \sum_{i=1}^{n}(t-\mu)}.(1)\, d\mu\, d\theta} \quad (17)$$

$$P(\vartheta|T_i) = \frac{\theta^{\sum_{i=1}^{n}\delta i}\, e^{-\theta \sum_{i=1}^{n}(t_i-\mu)}}{\int_0^\infty \theta^{\sum_{i=1}^{n}\delta i} e^{-\theta \sum_{i=1}^{n} t_i}\left(\int_0^\infty e^{\theta\mu}\, d\mu\right) d\theta} = 0 \quad (18)$$

The result of the integral over the parameter μ is infinity, so that if the integral over θ is multiplied by infinity it will still produce an infinite value. This results in the posterior distribution equation being zero. So, parameter estimation using the linear exponential loss function method using the Jeffrey's $P(\vartheta)$ prior cannot be applied.

3.1.2. Analysis of bayes estimation with assuming parameter θ is constant and parameter μ is variable

The first and second derivatives of $ln(L(\vartheta))$ are as (19):

$$\frac{\partial^2\, ln(L(\vartheta))}{\partial \mu^2} = 0 \quad (19)$$

Based on the results of the second derivative of the function $ln(L(\vartheta))$ on the parameter μ, the Fisher information value for the parameter μ can be described as (20) and (21):

$$I(\mu) = -E\left(\frac{\partial^2\, ln(L(\vartheta))}{\partial \mu^2}\right) \quad (20)$$

$$I(\mu) = 0 \quad (21)$$

The calculation results in (22) show that the Fisher information value is zero, so the parameter estimation cannot be applied assuming the parameter θ is constant and the parameter μ is variable.

3.1.3. Analysis of bayes estimation with assuming parameter θ is variable and parameter μ is costant

The first and second derivatives of $ln(L(\vartheta))$ are as (22):

$$\frac{\partial^2\, ln(L(\vartheta))}{\partial \theta^2} = -\frac{1}{\theta^2} \quad (22)$$

Based on the results of the second derivative of the function $ln(L(\vartheta))$ on the parameter θ, the Fisher information value for the parameter μ can be described as (23) and (24):

$$I(\theta) = -E\left(\frac{\partial^2\, ln(L(\vartheta))}{\partial \theta^2}\right) \quad (23)$$

$$I(\theta) = \frac{1}{\theta^2} \quad (24)$$

In accordance with the value of the Fisher information matrix obtained in (25), the Jeffrey's prior distribution $P(\theta)$ can be determined as (25):

$$P(\theta) \propto \frac{1}{\theta} \quad (25)$$

Next, determine the formula for the posterior distribution.

$$P(\theta|T_i) = \frac{L(\vartheta).P(\theta)}{\int_0^\infty L(\vartheta).P(\theta)\, d\theta} \quad (26)$$

$$P(\theta|T_i) = \frac{\theta^{\sum_{i=1}^{n}\delta i}\, e^{-\theta \sum_{i=1}^{n}(t_i-\mu)}.\left(\frac{1}{\theta}\right)}{\int_0^\infty \theta^{\sum_{i=1}^{n}\delta i}\, e^{-\theta \sum_{i=1}^{n}(t_i-\mu)}.\left(\frac{1}{\theta}\right) d\theta} \quad (27)$$

$$P(\theta|T_i) = \frac{\theta^{(\sum_{i=1}^{n}\delta i)-1}\, e^{-\theta \sum_{i=1}^{n}(t_i-\mu)}}{\int_0^\infty \theta^{(\sum_{i=1}^{n}\delta i)-1}\, e^{-\theta \sum_{i=1}^{n}(t_i-\mu)}\, d\theta} \quad (28)$$

Estimating the previously unknown parameter θ comes next after obtaining the posterior distribution. In this research, the Bayesian linear exponential loss function method will be used to estimate the parameter θ. The loss function $loss(\hat{\theta};\theta)$ is described as (29):

$$loss(\hat{\theta};\theta) = exp\left(a(\hat{\theta}-\theta)\right) - a(\hat{\theta}-\theta) - 1, a \neq 0 \tag{29}$$

In a two-parameter exponential distribution, the posterior expectation value (E_θ) of the loss function in (30) is minimized to produce the Bayesian linear exponential estimation of the parameter θ. This process is explained as (30) and (31):

$$E_\theta\left(loss(\hat{\theta},\theta)\right) = E_\theta\left[e^{a(\hat{\theta}-\theta)} - a(\hat{\theta}-\theta) - 1\right] \tag{30}$$

$$E_\theta\left(loss(\hat{\theta},\theta)\right) = e^{a\hat{\theta}}.E_\theta(e^{-a\theta}) - a\left(\hat{\theta} - E_\theta(\theta)\right) - 1 \tag{31}$$

The next step is to minimize (32) by finding the first derivative of $\hat{\theta}$ then setting it to zero. So, we will get a Bayesian linear exponential estimator of the parameter θ as (32)-(34):

$$\frac{\partial\left(E_\theta\left(loss(\hat{\theta};\theta)\right)\right)}{\partial\hat{\theta}} = 0 \tag{32}$$

$$\frac{\partial\left(e^{a\hat{\theta}}.E_\theta(e^{-a\theta}) - a\left(\hat{\theta}-E_\theta(\theta)\right) - 1\right)}{\partial\hat{\theta}} = 0 \tag{33}$$

$$\hat{\theta} = -\frac{1}{a}\left(ln\left(E_\theta(e^{-a\theta})\right)\right) \tag{34}$$

From this estimator, the value of $E_\theta(e^{-a\theta})$ is then calculated which is the posterior expected value of $e^{-a\theta}$ so that it can be described as (35) and (36):

$$E_\theta(e^{-a\theta}) = \int_0^\infty e^{-a\theta} \,.\, P(\theta \mid Ti)\, d\theta \tag{35}$$

$$E_\theta(e^{-a\theta}) = \int_0^\infty e^{-a\theta} \left(\frac{\theta^{(\sum_{i=1}^n \delta i)-1}\, e^{-\theta\sum_{i=1}^n (t_i-\mu)}}{\int_0^\infty \theta^{(\sum_{i=1}^n \delta i)-1}\, e^{-\theta\sum_{i=1}^n (t_i-\mu)}\, d\theta}\right) d\theta \tag{36}$$

Thus, the parameter estimator θ from the two-parameter exponential distribution using the Bayesian linear exponential method is as (37):

$$\hat{\theta} = -\frac{1}{a}\left(ln\left(\int_0^\infty e^{-a\theta} \,.\, \left(\frac{\theta^{(\sum_{i=1}^n \delta i)-1}\, e^{-\theta\sum_{i=1}^n (t_i-\mu)}}{\int_0^\infty \theta^{(\sum_{i=1}^n \delta i)-1}\, e^{-\theta\sum_{i=1}^n (t_i-\mu)}\, d\theta}\right) d\theta\right)\right) \tag{37}$$

3.2. Implementation of bayes estimation analysis

After obtaining parameter estimates from the two-parameter exponential distribution using the Bayesian linear exponential loss function approach based on Jeffrey's prior and the assumptions that are met, namely the assumption parameter θ variable and the parameter μ considered constant. Next, the results of these parameter estimates will be applied to real data. The data that will be used is secondary data in the form of data on how long employees have worked before they experienced attrition in 2021. This data was obtained from the Kaggle.com website entitled "IBM HR Analytics Employee Attrition & Performance". In this study, a sample of 50 employees was taken with a censorship limit based on age, namely 58 years. After determining the censorship limit, out of 50 employees who could be observed, there were 48 employees. Testing the data distribution is the first action that can be done before estimating parameters using actual data. This study employed a two-parameter exponential data distribution with the following hypothesis:
H_0 = the data follow a two-parameter exponential distribution.
H_1 = the data does not follow a two-parameter exponential distribution.

By using the testing criteria $a = 0.05$, the decision H_0 will be rejected if the $p - value < a$. The results of the distribution suitability test obtained a $p - value$ of 0.43377. This shows the $p - value > 0.05$, so the decision is to accept H_0. So, it can be concluded that the data on employee length of work in 2021 follows a two-parameter exponential distribution.

After knowing the shape of the data distribution, the next step is to determine the estimated value of the μ parameter in the data on employee length of work in 2021. Based on the assumption that the μ parameter is constant and the θ parameter is variable, the estimated value of the μ parameter will be taken from $\min(T_i)$ or T_1. So, it can be said that the estimated value of the parameter μ is the same as the smallest T value from the data. The smallest T value from employee length of work data in 2021 is 1 year. Thus, it can be said that the μ parameter's estimated value is 1.

The data distribution has been met and the estimated value of the parameter μ has been determined. Next, the value of a is determined based on the estimation results that have been obtained. This is done by taking the value of a that is closest to the average value of all $\hat{\theta}$ calculation results. The value of a has the condition that $a \neq 0$. The value of a that will be used is a=-0.25; -0.50; -0.75; -1 and a=0.25; 0.50; 0.75; 1. The average of all $\hat{\theta}$ values is 0.126316. So, the value of $\hat{\theta}$ that is closest to the average value is $\hat{\theta}$ using the value a=0.25. Aside from that, it is evident that $\hat{\theta}$ increases with decreasing a value. On the other hand, $\hat{\theta}$ will have a smaller value if an is higher. Since there will be less of a discrepancy between the estimated and actual values, an increased value of a will yields more accurate estimation results.

Once the value of a is known, the estimated value of parameter θ can be calculated. Based on the calculation results, the estimated parameter θ with a value of a=0.25 and $\hat{\mu}$=1 in the employee length of work data in 2021 is 0.188605. The average length of time employees have worked is 6.29427 years. This shows that employees tend to experience attrition after working for a relatively long period of time. However, it also depends on the reasons that cause the attrition itself, such as retirement, illness, death, termination of employment, resignation, or other reasons.

The probability that an employee will work before they experience attrition within a certain period of time can be determined by conducting a survival analysis. The chance that an employee can work for more than 40 years before they experience attrition is 0.0073 or 0.73%. This shows that the longer an employee works at a company, the less likely they are to stay at that company. This means that the possibility of employees remaining is very small after a certain period of time. As explained, this is also influenced by several reasons that cause attrition itself, such as retirement, illness, death, termination of employment, resignation, or other reasons that can increase the probability of attrition as the length of service increases.

4. CONCLUSION

The estimation results were applied to secondary data, namely data regarding the length of time employees worked before they experienced attrition in 2021. The research results show that the estimated value of the parameter θ is 0.188605. With an average value of 6.29427 years and an estimated probability value that an employee can work for more than 40 years before they experience attrition is 0.0073 or 0.73%. These findings suggest that after working for a considerable amount of time, employees typically experience attrition. The longer an employee works at a company, the less likely they are to stay at the company. So, the advice that can be given is that further research can be carried out regarding data on employee length of work in detail based on the factors that cause attrition in each employee, so that we can find out which factors cause attrition that dominate the most in the company.

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