The comparison of underwater source localization between Riemannian MFP and blind channel equalizer

Tran Cao Quyen[1] Tran Linh Huong Giang[2]
[1]Department of Wireless, Faculty of Electronics and Telecommunications, University of Engineering and Technology, Hanoi, Vietnam
[2]Department of Culture, Faculty of Chinese Language and Culture, University of Language and International Studies, Hanoi, Vietnam

Article Info

Article history:

Received Apr 6, 2025
Revised Sep 5, 2025
Accepted Sep 10, 2025

Keywords:

Blind equalizer
Multipath
Riemannian distance
Riemannian matched field processing
Statistical signal-to-noise ratio

ABSTRACT

Blind channel equalization (BCE) has been widely used in underwater communications due to its strong robustness against multipath propagation and its suitability for rapidly varying environments. However, there has been little research on the application of BCE for underwater source localization. On the other hand, conventional matched field processing (MFP), and particularly Riemannian MFP (RMFP), have been regarded as highly effective for this problem. In this paper, based on the statistical characterization of the signal-to-noise ratio (SNR) in underwater acoustic channels, we propose a method for estimating the channel transfer function, which is then used to construct a blind channel equalizer. A source localization approach using the proposed BCE is also presented. The localization performance using BCE is comparable to that of RMFP, achieving a depth error of 10 meters and a range error of 100 meters, while requiring significantly lower computational complexity.

Corresponding Author:

Tran Cao Quyen
Department of Wireless, Faculty of Electronics and Telecommunications
University of Engineering and Technology
144 Xuan Thuy, Hanoi, Vietnam
Email: quyentc@vnu.edu.vn

1. INTRODUCTION

The problem of underwater source localization using the matched field gives very accurate results because it takes into account the influence of the environment, which is the ocean waveguide model, sound velocity profile, source Doppler, and seabed properties [1] There are two sources of data used in matched field processing (MFP). The first source is the measured data from a vertical hydrophone array with N elements. The second source is the modeled data of the ocean waveguide. According to researchers [2], [3], the ocean waveguide is formed by the interface between a sea surface and a seabed, which can be considered as an absolutely rigid bottom or an elastic bottom. With different bottoms, the sound reflection coefficient will be different, affecting the sound field at the receiver.

The normal mode method [4]-[6] considers the sound oscillations along the depth to create modes that satisfy the Helmholtz wave equation, the sound pressure at the receiving point will be the sum of those modes. The normal mode method will be independent of the distance and therefore has certain limitations and the Parabolic approximation method is born as a distance-dependent method. Indeed, the parabolic approximation method [7]-[9] calculates the pressure grid according to the distance, the pressure at the later point is the result of the pressure at the previous point. Another advantage of this method is that it is very suitable for the practical layered ocean environment. When the sound propagation model is determined, the source localization is the inverse problem of the sound propagation problem MFP [1].

The development of MFP method from traditional MFP [1], mode-based MFP [10], maximum entropy MFP [11], MFP based on Karhunen-Loeve expansion [12], adaptive MFP [13] and recently Riemannian MFP (RMFP) [14]-[16]. The later methods are born, the more capable they are of the robustness to mismatch due to errors in environmental assessment (sound velocity, seabed), or errors in determining the properties of the transmitting and receiving hydrophones (not properly calibrated).

On the other hand, blind channel equalizers (BCE) have been widely used in underwater communications. For instance, as in study [17] the BCE that employs phase-locked loop (PLL) and exploits the cyclostationary property of modulated signals has been implemented. BCEs are commonly used in underwater acoustic communication systems to mitigate inter-symbol interference (ISI) caused by multipath propagation [18]. The BCE based on blind deconvolution techniques was proposed in [19] where the system's performance has been evaluated using both simulated and experimental data. The BCE utilizing higher-order statistics to reduce transmission time slots and energy consumption of underwater modems was proposed in [20]. Xiao and Yin [21], the BCE employing recursive least squares (RLS) with an adaptive forgetting factor was introduced to improve convergence speed and reduce steady-state error. Recently, Silva and Fernandes [22] one uses the adaptive equalizer which is called blind linear equalizer using genetic algorithm (BLE-GA) a method combining stochastic linear programming with genetic algorithms for blind adaptive equalization. Demonstrated strong noise resilience, rapid convergence, and scalability across high-order quadrature amplitude modulation (QAM) systems (e.g., 64-QAM); Hao *et al.* [23] one uses the adaptive equalizer which based on a floating decision feedback equalizer (DFE), tailored for SerDes receivers and focuses on high-speed serial communication (32 Gbps).

The question is: can BCE be used for underwater source localization? theoretically, if the received data is processed by a BCE that effectively compensates for signal distortion caused by the channel, then the received signal will closely resemble the original source signal. In this paper, we propose a BCE with its channel impulse response consisting of three paths: a direct path, a surface-reflected path, and a bottom-reflected path [24]-[27]. The received signal is used to estimate the channel impulse response, and then it is equalized based on this estimation. After that, underwater source localization is performed using BCE and compared with that of using RMFP, achieving a depth error of 10 meters and a range error of 100 meters, while requiring significantly lower computational complexity. The simulations using SACLANT acoustic dataset [28] have verified this result. The rest of the paper is organized as follows. Section 2 introduces the RMFP. The next section presents the underwater localization using proposed BCE. The underwater source localization using RMFP and proposed BCE are presented in section 4. Finally, we conclude the paper.

2. RIEMANNIAN MATCHED FIELD PROCESSING

MFP in general and RMFP in particular have taken into account the nature of the environment in their signal processing, thus increasing the accuracy of the problem of underwater source localization. Assuming there is a sound source located at coordinate $p_s = (r_s, z_s)$, we use a vertical hydrophone array with N sensors at coordinate $p_a = (r_a, z_a), a = \overline{1, N}$, the sound pressure field obtained is [14]-[16].

$$A_{p_s}(p_s, p_a) = C.B(p_s, p_a) + N(p_a) \tag{1}$$

where C is the spectral density of the sound source, B is the green function derived from the normal mode model (including the characteristics of the layered ocean waveguide environment, sea surface and seabed properties and sound velocity profile), N is the ocean background noise uncorrelated with C.

From there, the measured cross-spectral density matrix is:

$$\overline{R}_{p_s} = \sum_{m=1}^{M} \left[A_{p_s}\right]_m \left[A_{p_s}\right]^H{}_m \tag{2}$$

The normalized measurement cross-spectral matrix is given by:

$$R_{p_s} = \frac{\overline{R}_{p_S}}{\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{M}\left|(\overline{R}_{p_S})_{mn}\right|^2}} \tag{3}$$

The Frobenius norm define that $\|X\|_F{}^2 = \sum_{ij} x_{ij} = tr(XX^H)$ where $x_{ij}$is element of matrix X and H is the transpose conjugate [12].

Predicted sound sources at the positions of greatest correlation between the replica field cross-spectral matrix and the actual measured field from the vertical hydrophone array. The matched field processor based on Riemannian geometry is minimization of specific Riemannian distance. The minimization process is done over a set of all modeled field replicas position $\hat{p} = (\hat{r}, \hat{z})$ in this case it is the cross-spetral density however in general case it is a manifold. The shortest path between two points in a manifold should be a geodesic path.

The advantage of the RMFP compared to other MFP is that it exploits the curvature of acoustic rays by using the Riemannian distance measure instead of the Euclidean measure. This allows dealing with multiple field replicas and reduces the mismatch cases. The methodology of the sound source localization on the basis of Riemnnian MFP [16], i.e.,

$$(\hat{r}_s, \hat{z}_s)_{d_{SIM}} = \underset{\hat{p}}{argmin}\sqrt{tr[(sinR_{p_s})^2] + tr[(sinR_{\hat{p}})^2] - 2tr[sin(R_{p_s})sin(R_{\hat{p}})]} \quad (4)$$

3. THE UNDERWATER LOCALIZATION USING BLIND CHANNEL EQUALIZATION

3.1. Sea ambient noise

Turbulence, distant shipping, breaking waves and thermal noise produced ambient noise. The power spectral densities (p.s.d.) of the those noise components are specified by empirical formula (5) and are expressed in dB re µPa/Hz as functions of frequency in kHz, as in [29]. We may conclude that the ambient noise is approximated as Gaussian but it cannot be regarded as white.

$$\begin{aligned}
&10\,log\,N_t\,(f) = 17 - 30\,log\,f \\
&10\,log\,N_s\,(f) = 40 + 20(s - 0.5) + 26\,log\,f - 60\,log(f + 0.03) \\
&10\,log\,N_w\,(f) = 50 + 7.5w^{1/2} + 20\,log\,f - 40\,log(f + 0.4) \\
&10\,log\,N_{th}\,(f) = -15 + 20\,log\,f
\end{aligned} \quad (5)$$

The ambient noise varying to the wind speed w and the distance shipping activity s as in Figure 1. The wind speed can be 0 m/s (calm) or 10 m/s (medium) whereas shipping activity can be numbered 0, 0.5 or 1 according to sparse or dense activity. From the Figure 1, one could say that the noise p.s.d. decays at a rate of approximately 18 dB/decade (the straight dashed line).

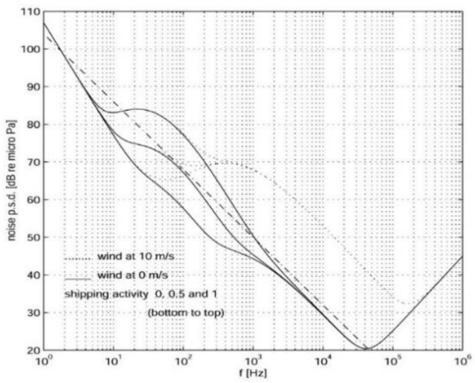

Figure 1. Power spectral density of the sea ambient noise as in [29]

3.2. Sound absorbtion

The total path loss is given by (6) [29]:

$$A(l, f) = (l/l_{ref})^k a(f)^{l-l_{ref}} \quad (6)$$

where *f* is the signal frequency, *l* and l_{ref} are the transmission and reference distance, respectively. The path loss exponent k are from 1 for cylindrical to 2 for spherical spreading or in between.

The absorption coefficient can be written empirically as the function of frequency *f* in kHz and, a*(f)* in dB/km as (7).

$$10\,log\,a\,(f) = 0.11\frac{f^2}{1+f^2} + 44\frac{f^2}{4100+f^2} + 2.75.10^{-4}f^2 + 0.003 \quad (7)$$

The absorption coefficient is depicted in Figure 2.

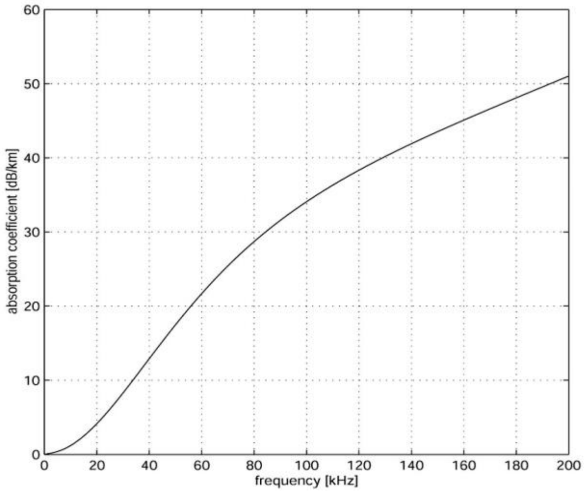

Figure 2. Absorption coefficient**,** 10 log *a*(*f*) **i**n dB/km as in [29]

3.3. Power spectral density of ambient noise with absorption

The fact that the attenuation grows with frequency and the p.s.d of noise reduces with frequency. Consequently, the signalto-noise ratio (SNR) varies over the signal bandwidth. The SNR can be expressed as a function of a narrow band of frequencies Δf and p.s.d of the transmitted signal $S_l(f)$.

$$SNR(l,f) = \frac{S_l(f)\Delta f}{A(l,f)N(f)\Delta f} \tag{8}$$

For any given distance, the narrowband SNR is thus a function of frequency, as shown in Figure 3.

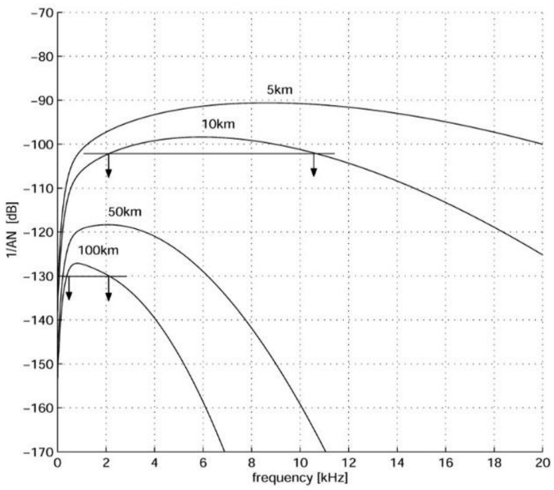

Figure 3. Signal-to-noise ratio in an acoustic channel depends on the frequency and distance through the factor $1/A(l;f)N(f)$ as in [29]

3.4. Channel model with three paths

The formation of multipaths in shallow seawater channels is mainly governed by the phenomenon of reflection and refraction of sound rays. Reflection can occur at the sea surface, the seabed or any objects. The channel model with 3 paths: straight rays, sea surface reflection rays and sea bottom reflection rays is described in Figure 4.

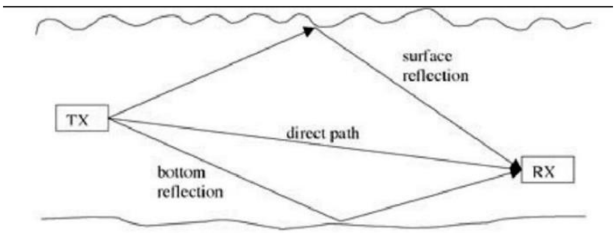

Figure 4. The channel model with 3 paths: straight rays, sea surface reflection rays and sea bottom reflection rays

Each path acts as a low-pass filter $H_p(f,t)$. The reference path transfer function $H_0(f)$, distance of 1 km, spreading coefficient k=1.5 is as shown in Figure 5 as:

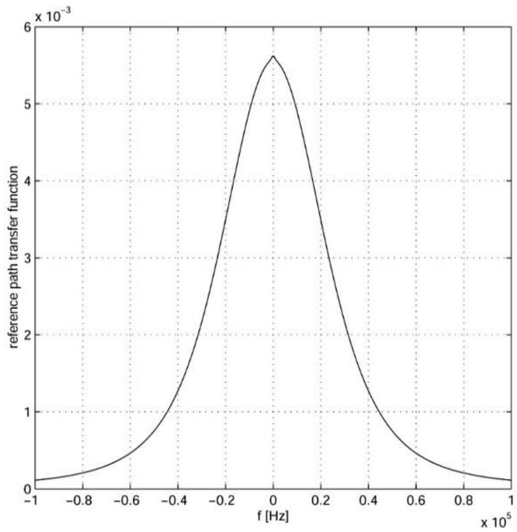

Figure 5. Reference path transfer function $H_0(f)$ with path length l_0=1 km and spreading factor k=1.5 as in [29]

The transfer function [29] of the pth path is described as (9).

$$H_p(f,t) = \frac{\Gamma_p}{\sqrt{A(l_p,f)}} \tag{9}$$

where $\Gamma_p$is the cumulative reflection coefficient along the path of the sound ray, $A(l_p, f)$ is the path loss of the pth ray.

At the receiver, the filtering effect is essentially similar across all propagation paths, since the absorption coefficient varies only slightly. The dominant variations arise from the reflection coefficient and the transmission distance. Accordingly, the transfer function of the p-th path can be expressed as (10).

$$\begin{aligned} H_p(f,t) &= h_p H_0(f) \\ h_p &= \frac{\Gamma_p}{\sqrt{(l_p/l_0)^k a^{l_p - l_0}}} \end{aligned} \tag{10}$$

The maximum sound transmission distance in SACLANT's experiment is 5.8 km. Therefore, the transmission time of direct path is 3,86 s ($t = \frac{d}{c} = \frac{5800\ m}{1500\ m/s} = 3.86\ s$) so the maximum spread delay in this channel is several ms.

In North Elba experiment, the direct path is 5.8 km, the maximum depth is 112 m, the signal bandwidth at low frequency range (appriximetely 3 kHz) so the number of multipaths are as (11).

$$L \leq B.T = 3KHz \times 1ms = 3(paths) \tag{11}$$

Reflection coefficient at sea surface, $\Gamma_s = -1$. The properties of the seabed in the North Elba experiment [28] are given by Figure 6 as:

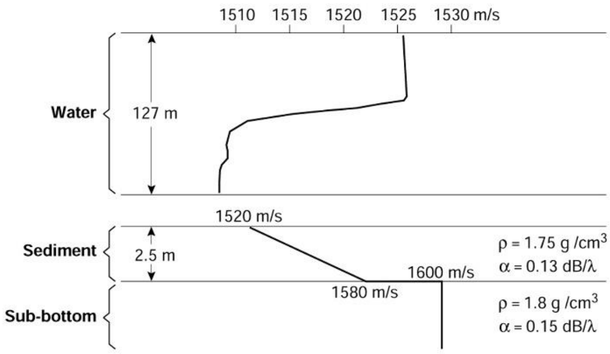

Figure 6. Seafloor parameters in the North Elba experiment

with this seabed property, we can deduce the bottom reflection coefficient $\Gamma_b = 0.2 \sim 0.5$. The transfer function of the 3-ray multipath model is:

$$H(f) = H_{dicrect}(f) + H_{surface}(f)e^{-j2\pi f\tau_1} + H_{bottom}(f)e^{-j2\pi f\tau_2}$$
$$H_{dicrect}(f) = \frac{1}{\sqrt{A(l_p, f)}}$$
$$H_{surface}(f) = \frac{-1}{\sqrt{A(l_p, f)}}$$
$$H_{bottom}(f) = \frac{0.2}{\sqrt{A(l_p, f)}} \tag{12}$$

where τ_1, τ_2 are the delay times of the reflected rays on the sea surface and seabed compared to the direct ray.

From the results of section 3.3, we can deduce the normalized power spectral density of sea ambient noise with absorption with a distance up to 10 km and maximum frequency of 20 KHz as shown in Figure 7. From the above discussion, and on the basis of (12) we can deduce the normalized channel tranfer function with three paths as in Figure 8. It is the goal of our methodology.

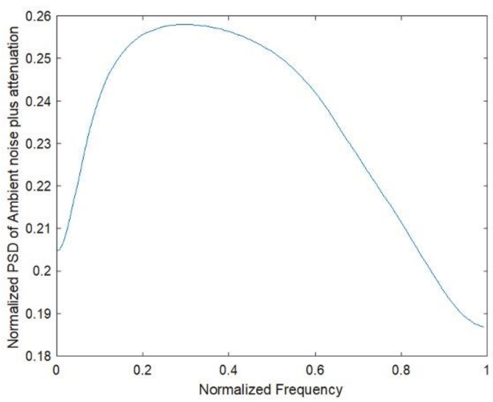

Figure 7. Normalized power spectral density of sea ambient noise with absorption with a distance up to 10 km and maximum frequency of 20 KHz

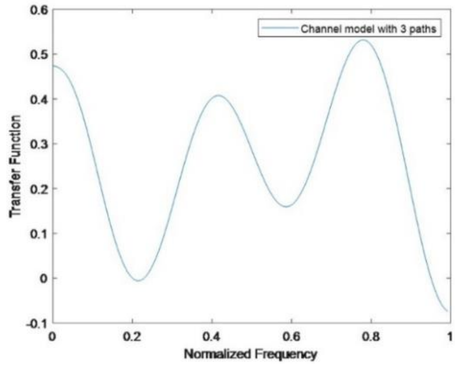

Figure 8. The normalized transfer function of the channel model with three paths

3.5. The underwater localization using blind channel equalizer

The configuration of a receiver with blind channel equalizer is depicted as in Figure 9. First, the estimated multipath hydroacoustic channel is performed as presented in section 3.4. Next, the blind channel equalizer operates on the basis of inversion of the channel transfer function or channel impulse response.

In general, the power spectral density of the ocean embient noise with absorption will determine the shape of the reference transfer function Figure 5. The number of multipaths and their strength determine the blurrednes of a received signal. When the product of delay and bandwidth determines the number of multipaths, the intensity of the sound ray depends mainly on the reflection coefficients. All of these issues have been presented in detail in the channel model in section 3.4.

The methodology of the source localization using the proposed BCE is as shown in Figure 10. First, the data received from the vertical hydrophone array is processed through the proposed BCE. Second, this data is correlated with the signal after channel equalization. The output of the correlator shows peaks corresponding to the estimated source locations.

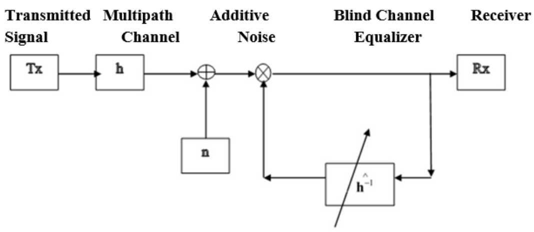

Figure 9. Configuration of a receiver with proposed BCE

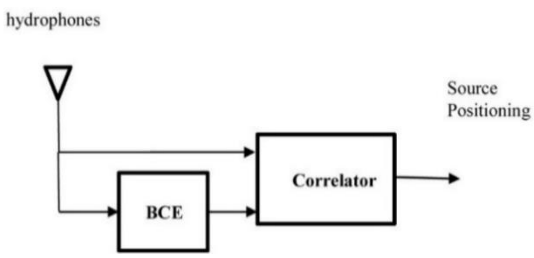

Figure 10. The underwater localization using blind channel equalizer

4. UNDERWATER SOURCE LOCALIZATION USING RIEMANNIAN MFP AND BLIND CHANNEL EQUALIZER

Passive array sonar data from the SACLANTC 1993 North Elba experiment, accessible on the internet, was utilized for the analysis [28] before June 2025. Now the center changed to CMRE (NATO STO CMRE). The SONAR data is now available by request directly to CMRE. The original time series was transformed into a collection of MATLAB `.mat` files, each comprising a data matrix "dat" of dimensions 48 sensors by 64,000 samples. Each file corresponds to roughly one minute of observation. The vertical array consisted of 48 hydrophones with an element spacing of 2 m, providing a total aperture of 94 m and extending from 18.7 m to 112.7 m in depth. The source radiated a pseudo-random noise (PRN) signal with a center frequency of 350 Hz.

In view of RMFP, we apply the (4) for finding the source localization. In view of BCE, we apply the (12) for construction the normalized transfer fucntion of the channel. The proposed BCE operates on the basis of inversion of the channel transfer function. The data received from the vertical hydrophone array is processed through the proposed BCE. Then, this data is correlated with the signal after channel equalization. The output of the correlator shows peaks corresponding to the estimated source location.

It is clear that in proposed BCE, the only measured data is used wheareas in RMFP not only measured data but modeled data are used. The complexity of RMFP depends on how many field replicas are used while in proposed BCE depends on convergence speed and the inversion of transfer function. However, the time delay of computation in the proposed BCE is acceptable for the underwater channel in this case. This is because, the maximum delay spread in the underwater channel is several milli seconds compared to micro seconds of the time delay of computation.

The other benefit of the underwater source localization using the proposed BCE is that it could be apply for spy systems. In those systems, it is not neccesity to transmite a training sequence for channel sounding. The results of locating the sound source using the RMFP are depicted through the normalized power in Figure 11 and the ambiguity surface in Figure 12. When using the proposed BCE, the normalized powers and ambiguity surfaces are shown in Figures 13 and 14 respectively.

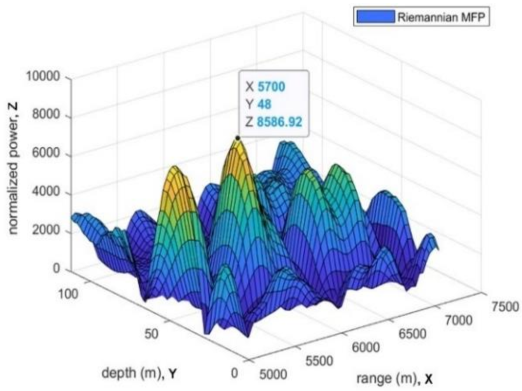

Figure 11. The normalized power of the underwater source using RMFP

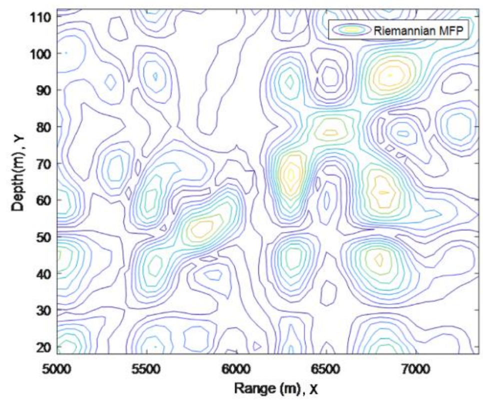

Figure 12. The ambiguity surface of the underwater source using RMFP

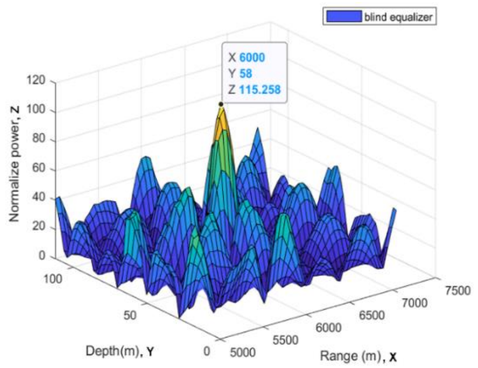

Figure 13. The normalized power of the underwater source using the proposed BCE

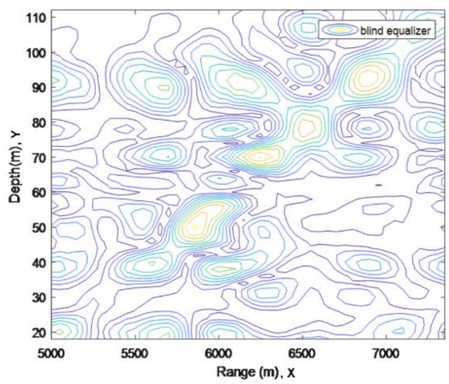

Figure 14. The ambiguity surface of the underwater source using the proposed BCE

The sources located at depth of 48 m and at range of 5700 m for RMFP; at depth of 58 m and at range of 6000 m for the proposed BCE. In summary, the performance of the source localization using RMFP and BCE is depeicted as in Table 1.

Table 1. The result of source localization using RMFP and proposed BCE

Metric	True source	RMFP	Proposed BCE
Range	5700 m	5700 m	5800 m
Depth	48 m	48 m	58 m
Complexity		High	Medium
Spy system		Yes	Yes

Surprisingly, using the proposed blind channel equalizer the underwater source could be located. There is an error of 10 m in depth and 300 m in distance between the two methods. However, the possibility of using the proposed BCE is completely proven and the result of source localization could be compared to RMFP. The reason is that in shallow sea water, curved sound rays do not prevail over straight ones.

We may conclude that in shallow underwater environment, we can use both RMFP or BCE for source localization. Indeed, the main differences between our proposed BCE and the adaptive equalizer approaches in [22], [23] are as follows: firstly, our method is applied to the problem of underwater source localization instead of communication; secondly, our method achieves faster convergence since it does not require pilot signals as in [22], [23]. This is particularly significant in channels with limited and scarce bandwidth. However, this comes at the cost of reduced accuracy if the statistical characteristics of the channel SNR are deviated from the real ones.

5. CONCLUSION

A blind channel equalizer comprising three rays has been proposed. Its transfer function is estimated on the basis of the statistical analysis of the SNR in the underwater acoustic channel. A source localization method based on correlation and this blind equalizer is also introduced. The localization performance is comparable to the FMFP approach, while offering lower computational complexity. The processing delay is acceptable, as underwater acoustic channels typically tolerate delay spreads on the order of several milliseconds. Both BCE and RMFP show potential for use in spy systems. The limitations of the proposed method is the reducement of its accuracy when the statistical SNR of the underwater acoustic channel deviates from its actual value. However, the proposed method should be implemented in passive SONAR systems of VietNam navy as well as of the Republic of Indonesia navy. In the future, we will analyze the maximum allowable deviation between the nominal SNR distribution and the actual SNR one for the problem of underwater source localization.

ACKNOWLEDGMENTS

We would like to thank SACLANTC for providing access of SONAR array data and this work has been supported by University of Engineering and Technology (VNUH).

FUNDING INFORMATION

The University has science and technology funds to support lecturers in international publication

AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Tran Cao Quyen	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓		✓
Tran Linh Huong Giang	✓									✓	✓		✓	

C : **C**onceptualization	I : **I**nvestigation	Vi : **Vi**sualization
M : **M**ethodology	R : **R**esources	Su : **Su**pervision
So : **So**ftware	D : **D**ata Curation	P : **P**roject administration
Va : **Va**lidation	O : Writing - **O**riginal Draft	Fu : **Fu**nding acquisition
Fo : **Fo**rmal analysis	E : Writing - Review & **E**diting	

CONFLICT OF INTEREST STATEMENT
Author state no conflict of interest.

DATA AVAILABILITY
The data that support the findings of this study will be available in http://spib.rice.edu/spib/saclant.html

REFERENCES

[1] A. Tolstoy, "Review of matched field processing for environmental inverse problems," *International Journal of Modern Physics C*, vol. 03, no. 04, pp. 691–708, Aug. 1992, doi: 10.1142/s0129183192000439.
[2] C. L. Pekeris, "Theory of propagation of explosive sound in shallow water," in *Memoir of the Geological Society of America*, 1948, pp. 1–116. doi: 10.1130/MEM27-2-p1.
[3] F. B. Jensen, W. A. Kuperman, M. B. Porter, H. Schmidt, and J. F. Bartram, " Computational ocean acoustics ," *The Journal of the Acoustical Society of America*, vol. 97, no. 5, pp. 3213–3213, 1995, doi: 10.1121/1.411832.
[4] R. H. Ferris, "Comparison of measured and calculated normal-mode amplitude functions for acoustic waves in shallow water," *The Journal of the Acoustical Society of America*, vol. 52, no. 3B, pp. 981–989, Sept. 1972, doi: 10.1121/1.1913204.
[5] M. B. Porter, "The KRAKEN normal mode program," *Naval Research Laboratory*, pp. 1–198, 1992.
[6] S. J. Levinson, R. A. Koch, E. K. Westwood, S. K. Mitchell, and C. V. Sheppard, "An efficient and robust method for underwater acoustic normal mode computations," *The Journal of the Acoustical Society of America*, vol. 95, no. 5_Supplement, pp. 2909–2909, 1994, doi: 10.1121/1.409303.
[7] F. D. Tappert, "The parabolic approximation method," in *Wave Propagation and Underwater Acoustics*, Berlin, Heidelberg: Springer Berlin Heidelberg, 2008, pp. 224–287. doi: 10.1007/3-540-08527-0_5.
[8] D. Lee, "The state-of-the-art parabolic equation approximation as applied to underwater acoustic propagation with discussions on intensive computations," *The Journal of the Acoustical Society of America*, vol. 76, no. S1, pp. S9–S9, Oct. 1984, doi: 10.1121/1.2022120.
[9] D. A. Outing, W. L. Siegmann, and M. D. Collins, "Generalization of the rotated parabolic equation to variable slopes," *The Journal of the Acoustical Society of America*, vol. 114, no. 4_Supplement, pp. 2428–2429, 2003, doi: 10.1121/1.4778936.
[10] E. C. Shang, "Source depth estimation in waveguides," *The Journal of the Acoustical Society of America*, vol. 77, no. 4, pp. 1413–1418, 1985, doi: 10.1121/1.392034.
[11] E. A. Robinson, "Fundamentals of Seismic Exploration," in *Maximum-Entropy and Bayesian Methods in Inverse Problems*, Dordrecht: Springer Netherlands, 1985, pp. 171–210. doi: 10.1007/978-94-017-2221-6_7.
[12] Q. Wang and Q. Jiang, "Simulation of matched field processing localization based on empirical mode decomposition and karhunen-loève expansion in underwater waveguide environment," *EURASIP Journal on Advances in Signal Processing*, vol. 2010, no. 1, p. 483524, Dec. 2010, doi: 10.1155/2010/483524.
[13] K. L. Gemba, W. S. Hodgkiss, and P. Gerstoft, "Adaptive and compressive matched field processing," *The Journal of the Acoustical Society of America*, vol. 141, no. 1, pp. 92–103, 2017, doi: 10.1121/1.4973528.
[14] S. Finette and P. C. Mignerey, "Stochastic matched-field localization of an acoustic source based on principles of Riemannian geometry," *The Journal of the Acoustical Society of America*, vol. 143, no. 6, pp. 3628–3638, Jun. 2018, doi: 10.1121/1.5040492.
[15] R. Cao, K. Yang, Y. Ma, Q. Yang, and Y. Shi, "Passive broadband source localization based on a Riemannian distance with a short vertical array in the deep ocean," *The Journal of the Acoustical Society of America*, vol. 145, no. 6, pp. EL567-EL573, Jun. 2019, doi: 10.1121/1.5111971.
[16] T. C. Quyen, "Underwater source localization using cylindrical hydrophone array and riemannian matched field processing," *Akustika*, vol. 42, pp. 7–13, 2022, doi: 10.36336/akustika20224214.
[17] H. Sen Hung and C. C. Chang, "Blind adaptive equalizer for underwater communications," *Ocean '04 - MTS/IEEE Techno-Ocean '04: Bridges across the Oceans - Conference Proceedings*, vol. 1, pp. 34–39, 2004, doi: 10.1109/oceans.2004.1402891.
[18] J. Wang, H. Huang, C. Zhang, and J. Guan, "A study of the blind equalization in the underwater communication," *Proceedings of the 2009 WRI Global Congress on Intelligent Systems, GCIS 2009*, vol. 3, pp. 122–125, 2009, doi: 10.1109/GCIS.2009.61.
[19] S. I. Siddiqui and H. Dong, "Blind deconvolution based equalizer for underwater acoustic communications," *Applied Acoustics*, vol. 149, pp. 114–122, Jun. 2019, doi: 10.1016/j.apacoust.2019.01.022.
[20] K. C. H. Blom, H. S. Dol, A. B. J. Kokkeler, and G. J. M. Smit, "Blind equalization of underwater acoustic channels using implicit higher-order statistics," in *2016 IEEE Third Underwater Communications and Networking Conference (UComms)*, IEEE, Aug. 2016, pp. 1–5. doi: 10.1109/UComms.2016.7583462.
[21] Y. Xiao and F. Yin, "Blind equalization based on RLS algorithm using adaptive forgetting factor for underwater acoustic channel," *China Ocean Engineering*, vol. 28, no. 3, pp. 401–408, Jun. 2014, doi: 10.1007/s13344-014-0032-5.
[22] C. A. D. Silva and M. A. C. Fernandes, "New approach of blind adaptive equalizer based on genetic algorithms," *Telecom*, vol. 6, no. 1, p. 6, Jan. 2025, doi: 10.3390/telecom6010006.
[23] G. Hao, W. Yi, L. Bin, C. J. Jun, S. Fan, and F. Pan, "Adaptive equalization for 32 Gbps SerDes receivers using a floating decision feedback equalizer algorithm," in *ACM International Conference Proceeding Series*, New York, NY, USA: ACM, Dec. 2025, pp. 165–171. doi: 10.1145/3718391.3718421.
[24] J. G. Proakis, *Digital Communications*, 3rd ed. 1995.
[25] S. Sampei, "*Applications of digital wireless technologies to global wireless communications*," Prentice Hall PTR, p. 558, 1997.
[26] H. Monson H., *Statistical digital signal processing and modeling*, John Wiley & Sons, INC., 1996.
[27] A. B. Carlson, *Communication systems: An introduction to signal and noise in electrical engineering*. New York: McGraw-Hill, 1986.
[28] SACLANT Sonar Data — Signal processing information base (SPIB). [Online]. Available: https://www.cmre.nato.int (Accessed: April. 01, 2025)
[29] M. Stojanovic, "Underwater acoustic communications: Design considerations on the physical layer," in *2008 Fifth Annual Conference on Wireless on Demand Network Systems and Services*, IEEE, Jan. 2008, pp. 1–10. doi: 10.1109/WONS.2008.4459349.

BIOGRAPHIES OF AUTHORS

Tran Cao Quyen is the member of Falculty of Electronics and Telecommunications, University of Engineering and Technology, Vietnam National University (VNUH), Vietnam. His major subject consists of Antenna and propagation, RADAR, SONAR. He is the author over 30 technical papers and supervises 2 Ph.D. students, 3 master students and dozens of Bachelors in Electronics and Telecomunications. Over his professional career, he successfully combined academic teaching with practical application of the results of his research. For instance, his product, namely, OFDM underwater acoustic modem using IC technology has been received much attention from Vietnam National University as weel as Vietnam Navy. For the problem of underwater source localization, he interested in Riemannian Matched Field Processing and suggested to Czech Republic Navy as well as Vietnam Navy using a passive SONAR system which is embedded the proposed algorithms. He can be contacted at email: quyentc@vnu.edu.vn.

Tran Linh Huong Giang is the member of Vietnam National University, Hanoi-University of Languages and International Studies. She obtained her Ph.D. degree in Jilin University in 2015. Her major subjects are Chinese Language and Culture. Over her professional career, she obtained a lot of Awards from the Center for Language Education and Cooperation in Nationwide as well as in Confucius Institute. She can be contacted at email: giangtlh@vnu.edu.vn.