

# Application of A Self-adaption Dual Population Genetic Algorithm in Multi-objective Optimization Problems

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## Abstract

*Multi-objective evolutionary algorithm is a powerful tool in resolving multi-objective optimization problems. This algorithm inherits the advantages of parallel random search, strong global searching capability and the ability to solve highly-complicated non-linear problems of evolutionary algorithm and it is usually used in the optimization problems with multiple mutual conflicts. However, such algorithms are slow in convergence and easy to be trapped in local optimal solution. This paper proposes a multi-objective dual population genetic algorithm (MODPGA) and explores the improvement strategies of multi-objective genetic algorithm. The adoption of self-adaption and dual population strategy can guarantee that the algorithm of this paper can converge to Pareto solution set in a reliable and quick manner and it can perform more extensive search on the objective function space and conduct more samples on multi-objective functions so as to be closer to the approximate optimal solution set of global optimal solutions. This solution set also includes more optimal feasible points and provides reliable basis for the decision making.*

**Keywords:** multi-objective optimization, genetic algorithm, adaptive dual population

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## 1. Introduction

Multi-objective optimization is an NP difficult problem which widely exists in reality and the optimization of the overall objective is realized by weighing the optimization of multiple objectives under one overall objective, therefore, it becomes very difficult to take the weights of every sub-objective into consideration and require high latitude and large scale [1]. Worse still, the traditional optimization means are even more stringent on the form of the objective function. Generally, the problems usually have more than one solution and the results of every solution are incomparable. So far, the multi-objective optimization theory not only makes important achievements including the formation and improvement of a set of optimization theory which parallels to the single-objective optimization, but it has also been applied more and more extensively [2].

Every objective in the multi-objective optimization problem is called a sub-objective. Due to the mutual influence and interaction among each sub-objective, the multi-objective optimization meets the optimal conditions of every sub-objective as well as the constraints of the interactive relationship among the sub-objectives [3]. Multi-objective optimization problem was first raised by V.Pareto, a French economist when researching the economic balance. At that time, he had summarized numerous objectives which are hard to compare as multi-objective optimization problem from the perspective of political economy. After that, Von.Neumann and J.Morgenstern came up with the multi-objective decision-making problem which has multiple conflicting decision makers in game theory. T.C. Koopmans was the first to bring forth Pareto optimal solution after putting forward multi-objective optimization problems from the analysis of production and distribution. Z. Johnsen systematically proposed the research report on multi-objective decision-making model, which is a turning point when the multi-objective optimization has been developed greatly. The numerous evolutionary algorithms such as genetic algorithm, particle swarm optimization algorithm and fish swarm algorithm have come into being in recent years to solve multi-objective optimization problems, nevertheless, these algorithms are slow in convergence and easy to be trapped in local optimal solutions and they need further improvements [5].

This paper presents a multi-objective genetic algorithm by combining self-adaption, dual population strategy and genetic algorithm to settle multi-objective optimization problems. It firstly conducts mathematical description of the multi-objective optimization problems. Then it elaborates the basic principles of genetic algorithm, based on which, it designs the multi-objective genetic algorithm in accordance with self-adaption and dual population strategy. Finally, it is about the performance test and analysis of this algorithm.

## 2. The Mathematical Description of Multi-objective Optimization Problem

At present, multi-objective optimization problem has been applied more and more widely, involving many fields. In daily life and projects, optimization is not only required in one index but it demands several indexes to achieve the optimization at the same time. A great number of problems can be reduced as the multi-objective optimization problem to make multiple objectives to realize optimization under certain constraint conditions [6]. The mathematical description of multi-objective optimization problem is made of decision variable, objective function and constraint condition. Because of the differences of the application fields of multi-objective optimization problem, the corresponding mathematical description is different, including general multi-objective optimization, dynamic multi-objective optimization, certain multi-objective optimization and uncertain multi-objective optimization. The multi-objective optimization problem can be described as follows mathematically:

$$\begin{aligned}
 \text{Min(&Max)} \quad y = f(x) = [f_1(x), f_2(x), \dots, f_n(x)] (n = 1, 2, \dots, N) \\
 \text{S. t.} \quad & g(x) = [g_1(x), g_2(x), \dots, g_k(x)] \leq 0 \\
 & h(x) = [h_1(x), h_2(x), \dots, h_m(x)] = 0 \\
 & x = [x_1, x_2, \dots, x_d, \dots, x_D] \\
 & x_{d\_min} \leq x_d \leq x_{d\_max} (d = 1, 2, \dots, D)
 \end{aligned} \tag{1}$$

Here:  $x$  is a  $D$ -dimensional decision variable,  $y$  is the objective function,  $N$  is the number of total optimization objectives,  $f_n(x)$  is the  $n$ th sub-objective function,  $g(x)$  is  $K$  inequality constraints,  $h(x)$  is  $M$  equality constraints, the constraints constitute the feasible region and  $x_{d\_min}$  and  $x_{d\_max}$  are the upper and lower bounds of vector search. The above equation is the multi-objective optimization problem which including the minimization problem (min), the maximization problem (max) and certain multi-objective optimization problem [7].

The concept of the optimal solution or Pareto optimal solution to multi-objective optimization problem is as follows:

Definition 1: If any  $d$  which meets  $d \in [1, D]$  and  $x_d^* \leq x_d$  and which has  $d_0 \in [1, D]$  and  $x_{d_0}^* < x_{d_0}$ , the vector  $x^* = [x_1^*, x_2^*, \dots, x_D^*]$  dominates the vector  $x = [x_1, x_2, x_d, \dots, x_D]$ .

If  $f(x^*)$  dominates  $f(x)$ , it must meet two conditions:  $\forall n, f_n(x^*) \leq f_n(x) \quad n = 1, 2, \dots, N$ ,  $\exists n_0, f_{n_0}(x^*) < f_{n_0}(x) \quad 1 \leq n_0 \leq N$ .

The dominance relation of  $f(x)$  is consistent with that of  $x$ .

Definition 2: Pareto optimal solution is the solution which can't be dominated by any solution in the feasible solution set. If  $x^*$  is a point in the search space,  $x^*$  is the Pareto optimal solution when and only when no  $x$  (in the feasible region of the search space) can make  $f_n(x) \leq f_{n_0}(x^*)$  when  $n = 1, 2, \dots, N$ .

Definition 3: For a multi-objective optimization problem  $f(x)$ ,  $f(x^*)$  is the global optimal solution when and only when any  $x$  (in the search space) can make  $f(x^*) \leq f(x)$ .

Definition 4: The set consisting of all Pareto optimal solutions is the Pareto optimal set of the multi-objective optimization problem and it can also be the acceptable or effective solution set.

The optimization process of multi-objective problem is indicated as Figure 1.

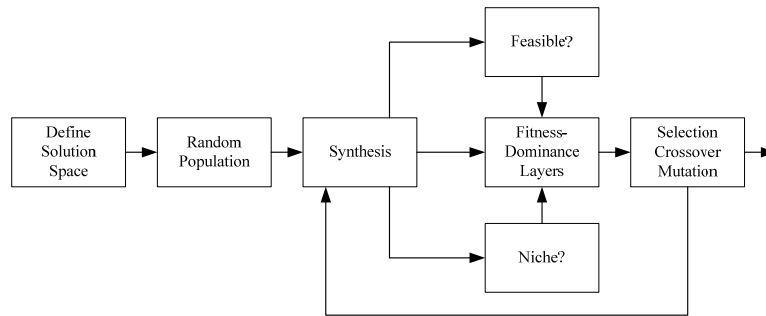


Figure 1. Optimization process

### 3. Genetic Algorithm

The main characteristic of genetic algorithm is that the strategy of population search and the information exchange between the individuals in the population are not based on gradient information. It can be used to handle the complicated and non-linear problems which are difficult to be solved by traditional search methods in particular and it can also be widely applied in such fields as combinatorial optimization, machine learning, self-adaptive control, planning and design as well as artificial life. As a global optimization search algorithm, genetic algorithm is one of the core intelligent computation technologies in the 21st century for it is easy and universal to apply, it has strong robustness, it can be used in parallel processing and it has a wide application scope [8].

#### 3.1. The Principles and Methods of Genetic Algorithm

##### (1) Chromosome Encoding

Encoding refers to the transformation method to transfer the feasible solutions to one problem to the search space which genetic algorithm can handle.

De Jong once proposed two practical coding principles: one is to use the encoding plan which is related to the problem to be solved and which has lower-order, short-defined length pattern and the other is to utilize the encoding plan which can give natural presentation or description to the problem or which has the minimum coded character set [9].

Encoding methods include the following: binary encoding method, gray-code encoding method, floating-point number encoding method, parameter cascade encoding method and multi-parameter cross-over encoding method.

##### (2) Fitness Computation

Basically, there are three methods to transform the objective function value  $f(x)$  of a certain point in the solution space to the fitness function value  $Fit(f(x))$  of the corresponding individual in the search space:

(a) Directly transform the objective function value  $f(x)$  to be solved to the fitness function value  $Fit(f(x))$  and make  $Fit(f(x)) = \begin{cases} f(x) & \text{The objective function is maximized.} \\ -f(x) & \text{The objective function is minimize} \end{cases}$

(b) As for the minimization problem, perform the following transformation  $Fit(f(x)) = \begin{cases} C_{\max} - f(x) & f(x) < C_{\max} \\ 0 & \text{others} \end{cases}$ . Here,  $C_{\max}$  is the maximum input value of  $f(x)$ .

(c) If the objective function is the minimization problem,

$$Fit(f(x)) = \frac{1}{1 + c + f(x)}, \quad c \geq 0, c + f(x) \geq 0 \quad (2)$$

If the objective function is the maximization problem,

$$Fit(f(x)) = \frac{1}{1 + c - f(x)}, \quad c \geq 0, c - f(x) \geq 0 \quad (3)$$

### 3.2. The Process of Genetic Algorithm

The genetic operations of genetic algorithm in the entire evolution process are random, but the characteristic it presents is full search. It can effectively use the previous information to predict the optimization point set with improved expected performance in the next generation. After the continuous evolution from generation to generation, it is finally converged to the individual which can adapt to the environment at most and the optimal solution to the problem can be obtained. Genetic algorithm involves five elements: parameter coding, setting of initial population, design of fitness function, design of genetic operation and setting of control parameters [10]. The operations of genetic algorithm are as follows:

#### (1) Selection

Selection operation combines elite selection and roulette wheel selection. At first, it directly copies several elite individuals to the population in the next generation and select the rest individuals with roulette wheel method. In this way, it can not only preserve the excellent individuals in the population, but also protect the diversity of the individuals in the population.

#### (2) Crossover and Mutation

In the genetic operations, perform crossover and mutation operations at the crossover probability  $P_c$  and mutation probability  $P_m$ . After crossover and mutation operations, conduct validation test on the newly-generated individuals to check whether the solutions of the new individuals meet the sequence constraints. If so, it proves that these new individuals are effective; if not, they are invalid and adjustments needs to be made on them. Redistribute some operations to make them effective genes [11, 12].

The basic flowchart of genetic algorithm is indicated as Figure 2.

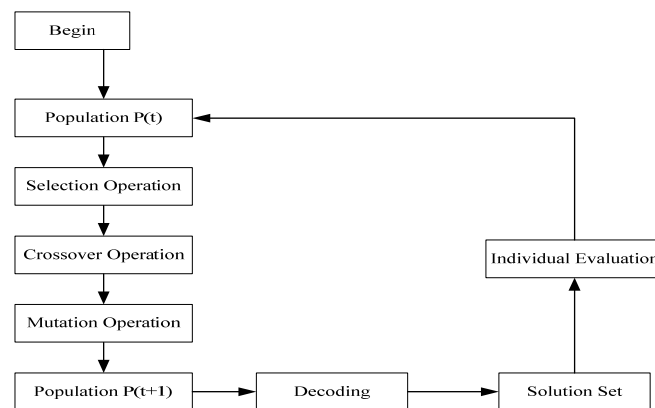


Figure 2. Basic flowchart of genetic algorithm

### 4. Design of Multi-objective Genetic Algorithm Based on Self-adaption and Dual Population Strategy

In the genetic evolution, the differences among the fitness of the individuals in the population vary from the differences of the evolution. At the early evolution, the difference is big, but it becomes small in the late evolution. In order to guarantee that the individuals can be selected in early evolution to preserve the diversity of the individuals in the population and highlight the excellent individuals in the late evolution to improve the competitiveness of the individuals, this paper has come up with a Multi-objective Dual Population Genetic Algorithm (MODPGA). The steps of MODPGA are as follows [13, 14]:

(1) When the variable part  $X_u^i$  of the individual  $X_l^{(i-1)n_l+j}$  ( $j=1, \dots, n_l$ ) in the population  $P_{ui}$  ( $i=1, \dots, n_u$ ) mutates, randomly select individuals to generate the variable part of the mutation vector from the variable population  $P_u$  (the size is  $n_u$ ).

(2) When the variable part  $X_l^{(i-1)n_l+j}$  of the individual  $X_l^{(i-1)n_l+j}$  ( $j=1, \dots, n_l$ ) in the population  $P_{ui}$  ( $i=1, \dots, n_u$ ) mutates, randomly select individuals to generate the variable part of

the mutation vector from the variable population II  $P_i$  (the size is  $n_i$ ) without being limited to the population I  $P_{ui}$ .

(3) In order to protect the evolution structure of population I, MODPGA updates the population I  $P_{ui}$  in a dynamic manner so as to realize the dynamic update of the entire population I.

(4) Preserve the non-dominant elite individuals with the level of  $ND_u = 1$  and  $ND_l = 1$  by using external archival strategy.

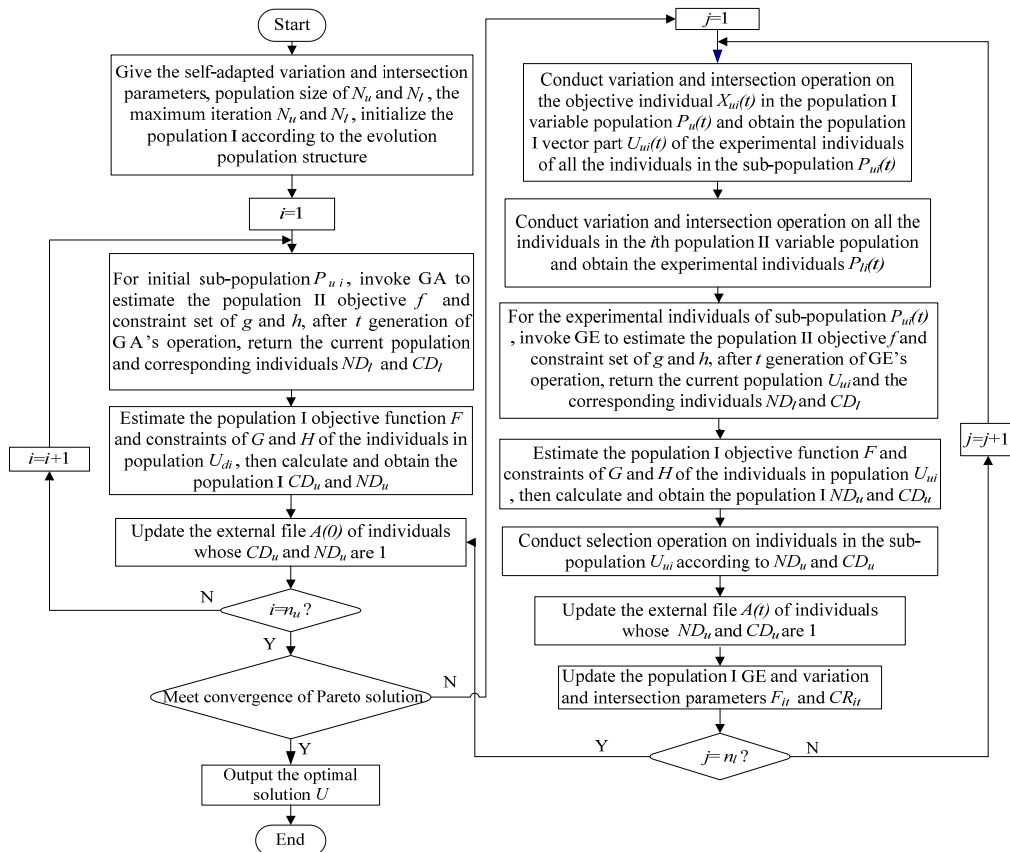


Figure 3. Flowchart of MODPGA

The process for MODPGA to conduct mutation and crossover operations to generate new experimental individuals includes two stages.

Stage I: Randomly select three different individuals from the existing objective individual in  $n_u$  different variable populations  $P_u$  to perform variable mutation and crossover operations of population I and generate the variable part of the experimental individuals  $U_i^{(i-1)n_l+j}$  ( $j = 1, \dots, n_l$ ).

Stage II: As for the variable part  $X_i^{(i-1)n_l+j}$  of all the individuals in population  $P_{ui}$ , randomly select individuals to perform mutation and crossover operations from the entire population  $P$  (the size is  $n_u$ ) and generate the variable part of the experimental individual  $U^{(i-1)n_l+j}$ . The flowchart of MODPGA is indicated as Figure 3.

**5. Performance Test and Analysis of the Algorithm in This Paper**

Based on the the theoretical research and experimental verification, this paper proposes a new improved Multi-objective Dual Population Genetic Algorithm (MODPGA). In

order to test the performance of this algorithm, this paper introduces two groups of random test functions to seek the maximum and minimum. These functions select two objectives, two decision variables and one constraint and when these two objectives take their own extremums, the positions of two particles contradict with each other so as to better observe the approximate Pareto optimal solution. The following is the comparison of MODPGA and basic genetic algorithm.

(1) Function 1: Seek the maximums of two objective functions.

$$MAX(f_1 = -x_1^2 + 2x_2; MAX(f_2 = x_1 / 3 + x_2 + 1));$$

Constraints:  $-25 \leq x_1, x_2 \leq 25$ .

Figure 4 is the comparison of Pareto front-ends of Function 1 by basic genetic algorithm and MODPGA respectively.

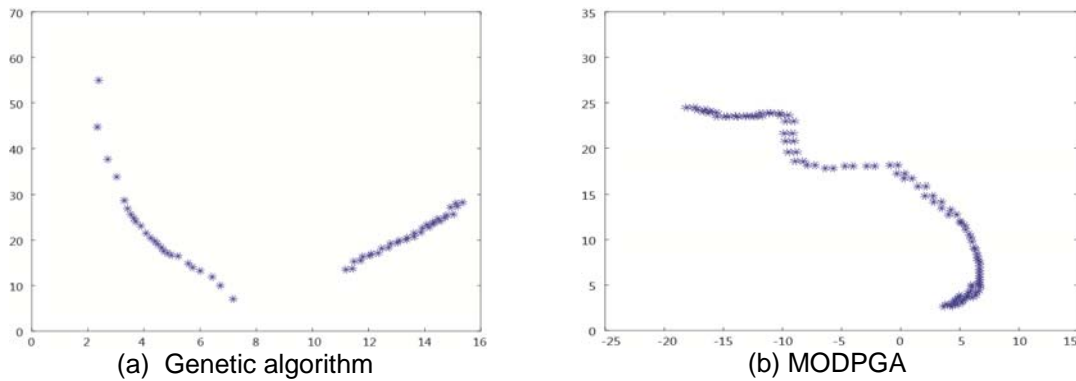


Figure 4. Comparison of pareto front-ends by two algorithms (function 1)

(2) Function 2: Seek the minimums of two objective functions.

$$MIN(f_1 = (x_1 + 3)^2 + 2x_2); MIN(f_2 = 3x_1 + \frac{1}{2x_2});$$

Constraints:  $0 \leq x_1, x_2 \leq 20$ .

In order to preserve the universality, this paper also chooses the fitness function to seek the minimum function and the Pareto front-ends of two algorithms are indicated in Figure 5.

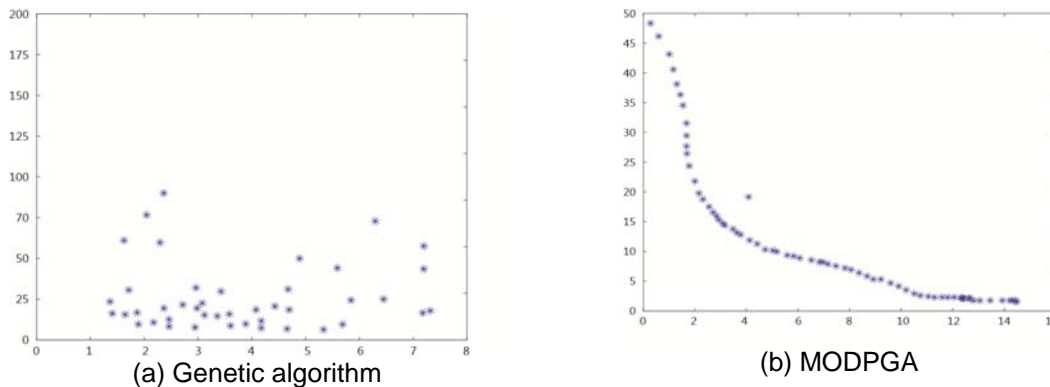


Figure 5. Comparison of pareto front-ends by two algorithms (function 2)

It can be seen clearly from Figure 4 and Figure 5 that after combining self-adaption and dual population strategy, MODPGA can quickly push the population to converge to the real Pareto front-end and to be uniformly distributed along Pareto front-end and it can have

approximate Pareto solution with better diversity. In one word, the optimization performance of MODPGA has been greatly improved compared with the traditional algorithms.

## 6. Conclusion

This paper explores the improvement strategy of multi-objective genetic algorithm and designs a highly-effective adaptive dual population genetic algorithm which can quickly converge to the real Pareto front-end shape. The MODPGA method proposed in this paper has lower time complexity, makes the selection strategy have higher selection pressure and guarantee that the population can converge to the real Pareto front-end part.

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## References

- [1] Karoon Suksonghong, Kittipong Boonlong, Kim-Leng Goh. Multi-objective Genetic Algorithms for Solving Portfolio Optimization Problems in the Electricity Market. *International Journal of Electrical Power & Energy Systems*. 2014; 58(24): 150-159.
- [2] Ioannis K Karathanassis, Elias Papanicolaou, Vassilios Belessiotis, Georgios C Bergeles. Multi-objective Design Optimization of a Micro Heat Sink for Concentrating Photovoltaic/Thermal (CPVT) Systems using a Genetic Algorithm. *Applied Thermal Engineering*. 2013; 59(1): 733-744.
- [3] Rahul Dhabale, Vijay Kumar S Jatti, TP Singh. *Multi-objective Optimization of Turning Process During Machining of AlMg1SiCu using Non-dominated Sorted Genetic Algorithm*. *Procedia Materials Science*. 2014; 6(1): 961-966.
- [4] Chyi-Tsong Chen, Hung-I Chen. Multi-objective Optimization Design of Plate-Fin Heat Sinks using a Direction-based Genetic Algorithm. *Journal of the Taiwan Institute of Chemical Engineers*. 2013; 44(2): 257-265.
- [5] Laxminarayan Sahoo, Asoke Kumar Bhunia, Parmad Kumar Kapur. Genetic Algorithm Based Multi-Objective Reliability Optimization in Interval Environment. *Computers & Industrial Engineering*. 2012; 62(1): 152-160.
- [6] S Pourzeynali, S Salimi, H Eimani Kalesar. Robust Multi-objective Optimization Design of TMD Control Device to Reduce Tall Building Responses Against Earthquake Excitations using Genetic Algorithms. *Scientia Iranica*. 2013; 20(2): 207-221.
- [7] D Gossard, B Lartigue, F Thellier. Multi-objective Optimization of a Building Envelope for Thermal Performance using Genetic Algorithms and Artificial Neural Network. *Energy and Buildings*. 2013; 67(12): 253-260.
- [8] Amani Bensghaier, Lotfi Romdhane, Fethi Benouezdou. Multi-objective Optimization to Predict Muscle Tensions in a Pinch Function using Genetic Algorithm. *Comptes Rendus Mécanique*. 2012; 340(3): 139-155.
- [9] Reza Soleimani, Navid Alavi Shoushtari, Behrooz Mirza, Abdolhamid Salahi. Experimental Investigation, Modeling and Optimization of Membrane Separation using Artificial Neural Network and Multi-objective Optimization using Genetic Algorithm. *Chemical Engineering Research and Design*. 2013; 91(5): 883-903.
- [10] Sachindra K Rout, Balaji K Choudhury, Ranjit K Sahoo, Sunil K Sarangi. Multi-objective Parametric Optimization of Inertance Type Pulse Tube Refrigerator using Response Surface Methodology and Non-dominated Sorting Genetic Algorithm. *Cryogenics*. 2014; 62(7): 71-83.
- [11] Rung-Chuan Lin, Mustafa Y Sir, Kalyan S Pasupathy. Multi-objective Simulation Optimization using Data Envelopment Analysis and Genetic Algorithm: Specific Application to Determining Optimal Resource Levels in Surgical Services. *Omega*. 2013; 41(5): 881-892.
- [12] ászló Daróczy, Gábor Janiga, Dominique Thévenin. Systematic Analysis of the Heat Exchanger Arrangement Problem using Multi-objective Genetic Optimization. *Energy*. 2014; 65(1): 364-373.
- [13] Jafar Ramadhan Mohammed. Comparative Performance Investigations of Stochastic and Genetic Algorithms under Fast Dynamically Changing Environment in Smart Antennas. *International Journal of Electrical and Computer Engineering*. 2012; 2(1): 98-105.
- [14] Amit Kumar Yadav, OP Rahi, Hasmal Malik, Abdul Azeem. Design Optimization of High Frequency Power Transformer by Genetic Algorithm and Simulated Annealing. *International Journal of Electrical and Computer Engineering*. 2011; 1(2): 102-109.