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Optimal Economic Ordering Policy with Trade Credit and Discount Cash-Flow Approach

Hao Jiaqin*[1], Mo Jiangtao[2], Min Jie[3]
[1]School of Mathematics and Statistics, Suzhou University, Suzhou, Anhui 234000, China
[2]College of Mathematics and Information Science, Guangxi University, Nanning, Guangxi 530004, China
[3]School of Mathematics and Physics, Anhui Jianzhu University, Hefei, Anhui 30601, China
*Corresponding author, e-mail: h-j-q-2-0-@163.com

Abstract

In this paper, an inventory model for deteriorating items under two levels of trade credit will be established. The trade credit policy depends on the retailer's order quantity. When the retailer's order quantity is greater than or equal to a predetermined quantity, both of the supplier and the retailer are taking trade credit policy; otherwise, the delay in payments is not permitted. Since the same cash amount has different values at different points of time, the discount cash-flow (DCF) is used to analysis the inventory model. The purpose of this paper is to find an optimal ordering policy to minimizing the present value of all future cash-flows cost by using DCF approach. The method to determine the optimal ordering policy efficiently is presented. Some numerical examples are provided to demonstrate the model and sensitivity of some important parameters are illustrated the optimal solutions.

Keywords*: two-level trade credit, deteriorating items, order quantity dependent credit, discount cash-flow, EOQ*

1. Introduction

In the traditional economic order quantity models assumed the purchaser must pay for the items as soon as the items received. However, in real markets, to stimulate retailer's ordering qualities the supplier allows a certain fixed permissible delay in payment to settle the amount. Similarly, a retailer may offer his/her customers a permissible delay period to settle the outstanding balance when he/she received a trade credit by the supplier, which is a two-level trade credit. Huang [1] was the first to explore an EOQ model under the two-level trade credit. Kreng and Tan [2] and Ouyang et al [3] proposed to determine the optimal replenishment decisions if the purchasers order quantity is greater than or equal to a predetermined quantity. Teng et al [4] extended the constant demand to a linear non-decreasing demand function of time and incorporate supplier offers a permissible delay linked to order quantity under two levels of trade credit. Teng et al [5] established an EOQ with trade credit financing for a linear non-decreasing demand function of time. Paulus [6] shed light on how search strategy can be used to gain the maximum benefit of information search activities. Feng et al [7] investigated the retailer's optimal cycle time and optimal payment time under the supplier's cash discount and trade credit policy within the EPQ framework. Wang et al [8] established an economic order quantity model for deteriorating items with maximum lifetime and credit period increasing demand and default risk. Liao [9] developed an inventory model by considering two levels of trade credit, limited storage capacity. Wu et al [10] discussed an economic order quantity model under two levels trade credit, and assumed deteriorating items have their expiration dates. Enda et al [11] presented a generic solution to the sensitive issue of PCI Compliance. Teng et al [12] proposed an EPQ model from the seller's prospective to determine his/her optimal trade credit period, and in his paper production cost declined and obeyed a learning curve phenomenon.

However, the above inventory models did not consider the effects of the time value of money. In fact, as the value of money changes with time, it is necessary to take the effect of the time value of money on the inventory policy into consideration. Chang et al [13] investigated the DCF approach to establish an inventory model for deteriorating items with trade credit based on the order quantity. Chung and Liao [14] adopted the DCF approach to discuss the effect of trade credit depending on the ordering quantity. Liao and Huang [15] extended the inventory model to

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consider the factors of two levels of trade credit, deterioration and time discounting.

In this paper, we develop an inventory system for deteriorating items. Firstly, the items start deteriorating from the moment they are put into inventory. Secondly, if the retailer's order quantity is greater than or equal to a predetermined quantity, both of the supplier and the retailer are taking trade credit policy; otherwise, the delay in payments is not permitted. Thirdly, the present value of all future cash-flows cost instead of the average cost. The theorems are developed to efficiently determine the optimal cycle time and the present value of the total cost for the retailer. Finally, numerical examples and sensitive analysis of major parameters are given to illustrate the theoretical result obtain some managerial insight.

2. Notations and Assumption

2.1. Notations

The following notations are used throughout this paper.

A	the ordering cost one order;	c	unit purchasing cost per item;
p	unit selling price per item $p > c$;	h	holding cost per unit time excluding interest charges;
D	demand rate per year;	r	the continuous rate of discount;
W	quantity at which the delay in payments is permitted;	T_d	the time interval that W units are depleted to zero;
T	the cycle time;	Q	the retailer' order quantity per cycle;
$I(t)$	the inventory level at the time of t;	$PV_\infty(T)$	the present value of all future cash-flow cost.

2.2. Assumptions

The assumptions in this paper are as follows:

(1) Time horizon is infinite, and the lead time is negligible; replenishment are instantaneous, and shortage is not allowed;

(2) A constant θ ($0 < \theta < 1$) fraction of the on-hand inventory deteriorates per unit of time and there is no repair or replacement of the deteriorated inventory;

(3) If $Q \ge W$, both the fixed trade credit period M offered by the supplier and the trade credit N offered by the retailer are permitted. Otherwise, the delay in payments is not permitted. The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost until the end of the trade credit period offered by the supplier. That is to say, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit; When $T \ge M$, the account is settled at $T = M$ and the retailer would pay for the interest charges on items in stock with rate I_p over the interval $[M, T]$; when $T \le M$, the account is also settled at $T = M$ and the retailer does not need to pay any interest charge of items in stock during the whole cycle; The fixed credit period offered by the supplier to the retailer is no less to his/her customers, i.e. $0 < N \le M$.

3. Mathematical Model

Based on above assumptions, depletion due to demand and deterioration will occur simultaneously. The inventory level of the system can be described by the following differential equation $I'(t) + \theta I(t) = -D$, $0 \le t \le T$, $I(T) = 0$.

The solution to the above equation is $I(t) = D\left[e^{\theta(T-t)} - 1\right]/\theta$, $0 \le t \le T$.

So the retailer's order size per cycle is $Q = I(0) = D\left(e^{\theta T} - 1\right)/\theta$, $0 \le t \le T$.

If $Q = W$, we get T_d : $T_d = \dfrac{1}{\theta}\ln\left(1 + \dfrac{\theta W}{D}\right)$.

The present value of all future cash-flow cost $PV_\infty(T)$ consists of the following elements:

(1) The present value of order cost: $V_O = A/\left(1 - e^{-rT}\right)$;

(2) The present value of holding cost excluding interest charges:

$$V_H = \frac{hD}{\theta\left(1-e^{-rT}\right)}\left(\frac{e^{\theta T}-e^{-rT}}{\theta+r}+\frac{e^{-rT}-1}{r}\right)$$

(3) The present value of purchasing cost:

when $Q < W$ ($T < T_d$), $V_C = \dfrac{cD\left(e^{\theta T}-1\right)}{\theta\left(1-e^{-rT}\right)}$;

when $Q \ge W$ ($T \ge T_d$), $V_C = \dfrac{cDe^{-rM}}{\theta\left(1-e^{-rT}\right)}\left(e^{\theta T}-1\right)$;

(4) The present values of interest charged and earned are addressed as follows:

when $0 < T < T_d$ $(Q < W)$, there is no interest earned, that is $V_{IE}=0$. The present value of interest charged is

$$V_{IP} = \frac{cI_pD}{\theta\left(1-e^{-rT}\right)}\left(\frac{e^{\theta T}-e^{-rT}}{\theta+r}+\frac{e^{-rT}-1}{r}\right)$$

when $0 < T \le N$ and $T_d \le T$ $(W \le Q)$, there is no interest charged, that is $V_{IP}=0$. The present value of interest earned is

$$V_{IE} = \frac{pI_eDT}{r\left(1-e^{-rT}\right)}\left(e^{-rN}-e^{-rM}\right).$$

when $N < T \le M$ and $T_d \le T$ $(W \le Q)$, there is no interest charged, that is $V_{IP}=0$, and the present value of interest earned is

$$V_{IE} = \frac{pI_eD}{r^2\left(1-e^{-rT}\right)}\left[(1+rN)e^{-rN}-rTe^{-rM}-e^{-rT}\right].$$

when $M < T$ and $T_d \le T$ $(W \le Q)$, the present value of interest charged is given by

$$V_{IP} = \frac{cI_pD}{\theta\left(1-e^{-rT}\right)}\left[\frac{e^{\theta T-(\theta+r)M}-e^{-rT}}{\theta+r}+\frac{e^{-rT}-e^{-rM}}{r}\right];$$

The present value of interest earned is

$$V_{IE} = \frac{pI_eD}{r^2\left(1-e^{-rT}\right)}\left[(1+rN)e^{-rN}-(1+rM)e^{-rM}\right].$$

Therefore, the present value of all future cash-flow cost, $PV_\infty(T)$, can be expressed as $PV_\infty(T) = V_O + V_H + V_C + V_{IP} - V_{IE}$.

Consequently, based on the values of T_d, N, M,three possible cases: (1) $0 < T_d \le N$, (2) $N < T_d \le M$, and (3) $M < T_d$ will be occur.

Case 1 $0 < T_d \le N$

$$PV_\infty(T) = \begin{cases} PV_1(T), & 0 < T \le T_d, \\ PV_2(T), & T_d \le T \le N, \\ PV_3(T), & N \le T \le M, \\ PV_4(T), & M \le T, \end{cases}$$

where

$$PV_1(T) = \frac{1}{1-e^{-rT}}\left\{A+\frac{cD}{\theta}\left(e^{\theta T}-1\right)+\frac{\left(h+cI_p\right)D}{r\theta}\left[\frac{re^{\theta T}+\theta e^{-rT}}{\theta+r}-1\right]\right\};$$

$$PV_2(T) = \frac{1}{1-e^{-rT}}\left\{A+\frac{D}{\theta}ce^{-rM}\left(e^{\theta T}-1\right)+\frac{hD}{r\theta}\left[\frac{re^{\theta T}+\theta e^{-rT}}{\theta+r}-1\right]-\frac{D}{r}pI_eT\left(e^{-rN}-e^{-rM}\right)\right\};$$

$$PV_3(T) = \frac{1}{1-e^{-rT}}\left\{A+\frac{c}{\theta}De^{-rM}\left(e^{\theta T}-1\right)+\frac{hD}{r\theta}\left[\frac{re^{\theta T}+\theta e^{-rT}}{\theta+r}-1\right]-\frac{D}{r^2}pI_e\left[(1+rN)e^{-rN}-rTe^{-rM}-e^{-rT}\right]\right\};$$

$$PV_4(T)=\frac{1}{1-e^{-rT}}\left\{A+\frac{c}{\theta}De^{-rM}\left(e^{\theta T}-1\right)+\frac{hD}{r\theta}\left[\frac{re^{\theta T}+\theta e^{-rT}}{\theta+r}-1\right]+\frac{D}{r\theta}cI_p\left[\frac{re^{\theta T-(\theta+r)M}+\theta e^{-rT}}{\theta+r}-e^{-rM}\right]\right.$$
$$\left.-\frac{pI_eD}{r^2}\left[(1+rN)e^{-rN}-(1+rM)e^{-rM}\right]\right\}.$$

Case 2 $N<T_d\le M$.

$$PV_\infty(T)=\begin{cases}PV_1(T), & 0<T\le T_d,\\ PV_3(T), & T_d\le T\le M,\\ PV_4(T), & M\le T.\end{cases}$$

Case 3 $M<T_d$

$$PV_\infty(T)=\begin{cases}PV_1(T), & 0<T\le T_d,\\ PV_4(T), & T_d\le T.\end{cases}$$

4. Theoretical Results

The objective in this paper is to find the replenishment time T^* to minimize the present value of all future cash-flow cost of the retailer.

To simply the proof process of this model, the following lemma is given.

Lemma 1. Let x^* denotes the minimum point of the function of $F(x)$ on interval $[a,b]$. Suppose $f(x)$ is continuous function and increasing on $[a,b]$, and $F'(x)=f(x)e^{-rx}/(1-e^{-rx})^2$. We have the following results.
(a) if $f(a)\ge 0$, then $x^*=a$; (b) if $f(a)<0<f(b)$, then $x^*=x_0$, where x_0 is the unique solution of $f(x)=0$ on $[a,b]$; (c) if $f(b)\le 0$, then $x^*=b$.

4.1 when $0<T_d\le N$

Case 1 $0<T\le T_d$

Taking derivative of $PV_1(T)$ with respect to T , we obtain $PV_1'(T)=f_1(T)e^{-rT}/(1-e^{-rT})^2$.
where $f_1(T)=-r(1-e^{-rT})PV_1(T)+(e^{rT}-1)D\left\{(h+cI_p)(e^{\theta T}-e^{-rT})/(\theta+r)+ce^{\theta T}\right\}$ and $f_1(0)=-rA$.

Thus, we know that $f_1'(T)=De^{\theta T}\left[h+cI_p+(\theta+r)c\right](e^{rT}-1)\ge 0$.

From the above analysis and lemma 1(1-2), we have the following results.

Lemma 2. Let T_1^* is the minimum point of $PV_1(T)$ on $(0,T_d]$.
If $f_1(T_d)\le 0$, then $T_1^*=T_d$; else, $T_1^*=T_1^0$, where T_1^0 is the unique solution of $f_1(T)=0$ on $(0,T_d]$.

Case 2 $T_d\le T\le N$

Taking derivative of $PV_2(T)$ with respect to T , we obtain $PV_2'(T)=f_2(T)e^{-rT}/(1-e^{-rT})^2$,
where $f_2(T)=-r(1-e^{-rT})PV_2(T)+(e^{rT}-1)D\left\{\frac{h}{\theta+r}(e^{\theta T}-e^{-rT})+ce^{\theta T-rM}-\frac{pI_e}{r}(e^{-rN}-e^{-rM})\right\}$.
Similarly, taking derivative of $f_2(T)$ with respect to T , we know that $f_2'(T)=(e^{rT}-1)Dg(T)$,
where $g(T)=he^{\theta T}+(r+\theta)ce^{\theta T-rM}-pI_e(e^{-rN}-e^{-rM})$.

Lemma 3. Let T_2^* is the minimum point of $PV_2(T)$ on $[T_d,N]$.
(1) when $g(T_d)\ge 0$
(a) if $f_2(T_d)\ge 0$, then $T_2^*=T_d$; (b) if $f_2(T_d)<0\le f_2(N)$, $T_2^*=T_2^0$, where T_2^0 is the unique solution of $f_2(T)=0$ on $[T_d,N]$; (c) if $f_2(N)<0$, then $T_2^*=N$.

(2) when $g(T_d) < 0 < g(N)$, where $T_2^\#$ is unique solution of $g(T) = 0$ on $[T_d, N]$.

(i) If $f_2(T_2^\#) > 0$, then $T_2^* = T_d$.

(ii) when $f_2(T_2^\#) \le 0$, (a) if $f_2(N) < 0$, then $PV_2(T_2^*) = \min\{PV_2(T_d), PV_2(N)\}$ and $T_2^* = \arg\min\{PV_2(T_d), PV_2(N)\}$; (b) if $f_2(N) \ge 0$, when $f_2(T_d) \le 0$, $T_2^* = T_2^0$ where T_2^0 is the unique solution of $f_2(T) = 0$ on $[T_d, N]$; else, $PV_2(T_2^*) = \min\{PV_2(T_d), PV_2(T_2^2)\}$ and $T_2^* = \arg\min\{PV_2(T_d), PV_2(T_2^2)\}$, where T_2^2 is the largest solution of $f_2(T) = 0$ on $[T_d, N]$;

(3) when $g(N) < 0$, then $PV_2(T_2^*) = \min\{PV_2(T_d), PV_2(N)\}$ and $T_2^* = \arg\min\{PV_2(T_d), PV_2(N)\}$.

Proof: Since $g'(T) = \theta e^{\theta T}\left[h + (r+\theta)ce^{-rM}\right] \ge 0$, we know $g(T)$ is increasing on $[T_d, N]$.

(1) when $g(T_d) \ge 0$, then $g(T) \ge 0$, that is $f_2(T) \ge 0$. From lemma 1, the results are proofed.

(2) When $g(T_d) < 0 < g(N)$, the equation of $g(T) = 0$ has a unique root (say $T_2^\#$). In this situation, $g(T) \le 0$ for $\left[T_d, T_2^\#\right)$, $g(T) \ge 0$ for $\left[T_2^\#, N\right]$. Thus, $f_2(T)$ is decreasing on $\left[T_d, T_2^\#\right]$ and increasing on $\left(T_2^\#, N\right]$.

As follows, we will discuss the property of $f_2(T)$ on $\left[T_d, T_2^\#\right]$ and $\left(T_2^\#, N\right]$.

(A). In this case we discuss the property of $f_2(T)$ on $\left[T_d, T_2^\#\right]$.

From the above analysis, we know that $f_2(T)$ is decreasing on $\left[T_d, T_2^\#\right]$, and have the following results.

If $f_2(T_2^\#) \ge 0$, then we have $f_2(T) \ge 0$ for $\left[T_d, T_2^\#\right)$; If $f_2(T_2^\#) < 0 < f_2(T_d)$, then the equation $f_2(T) = 0$ has a unique root (say T_2^1) on $\left(T_d, T_2^\#\right)$, and $f_2(T) \ge 0$ for $T \in \left[T_d, T_2^1\right]$, $f_2(T) < 0$ for $T \in \left(T_2^1, T_2^\#\right)$; If $f_2(T_d) < 0$, then $f_2(T) < 0$ on $\left[T_d, T_2^\#\right]$.

(B) In this case we discuss the property of $f_2(T)$ on $\left(T_2^\#, N\right]$.

From the above analysis, $f_2(T)$ is increasing on $\left(T_2^\#, N\right]$, and have the following results.

If $f_2(T_2^\#) \ge 0$, then we have $f_2(T) \ge 0$ for $\left(T_2^\#, N\right]$; If $f_2(T_2^\#) < 0 < f_2(T_d)$, then the equation $f_2(T) = 0$ has a unique root (say T_2^2) on $\left(T_2^\#, N\right)$, and $f_2(T) \le 0$ for $\left[T_d, T_2^2\right]$, $f_2(T) > 0$ for $\left(T_2^2, N\right]$; If $f_2(N) \le 0$, then $f_2(T) \le 0$ on $\left(T_2^\#, N\right]$.

From (A) and (B), we have the following results

(i) if $f_2(T_2^\#) \ge 0$, then $f_2(T) \ge 0$ on $[T_d, N]$. Thus, $PV_2(T)$ is increasing on $[T_d, N]$. Thus, $T_2^* = T_d$;

(ii) if $f_2(T_2^\#) < 0$, (a) In this case, we suppose $f_2(N) < 0$. If $f_2(T_d) < 0$, then $f_2(T) \le 0$; else, $f_2(T) \ge 0$ for $\left[T_d, T_2^1\right]$ and $f_2(T) \le 0$ for $\left(T_2^1, N\right]$. therefor, we obtain that if $f_2(T_d) < 0$, then $PV_2(T)$ is decreasing on $[T_d, N]$; else, $PV_2(T)$ is increasing on $\left[T_d, T_2^1\right]$ and decreasing on $\left(T_2^1, N\right]$. Hence, when $f_2(N) < 0$, $PV_2(T_2^*) = \min\{PV_2(T_d), PV_2(N)\}$ and $T_2^* = \arg\min\{PV_2(T_d), PV_2(N)\}$. (b) In this situation, we suppose $f_2(N) \ge 0$. If $f_2(T_d) < 0$, then $f_2(T) \le 0$ for $\left[T_d, T_2^2\right]$ and $f_2(T) \ge 0$ for $\left(T_2^2, N\right]$. For the convenience of that problem, we denote T_2^0 is the unique solution of $f_2(T) = 0$ on $[T_d, N]$. In this case, $T_2^0 = T_2^2$. Thus, we obtain that $PV_2(T)$ is decreasing on $\left[T_d, T_2^0\right]$ and increasing on $\left(T_2^0, N\right]$. Thus, $PV_2(T_2^*) = PV_2(T_2^0)$ and $T_2^* = T_2^0$. If $f_2(T_d) \ge 0$, then $f_2(T) \ge 0$ for $\left[T_d, T_2^1\right]$ and $\left(T_2^2, N\right]$, $f_2(T) \le 0$ for $\left(T_2^1, T_2^2\right]$. Thus, $PV_2(T)$ is increasing on $\left[T_d, T_2^1\right]$ and $\left(T_2^2, N\right]$, and decreasing on $\left(T_2^1, T_2^2\right]$. Hence, $PV_2(T_2^*) = \min\{PV_2(T_d), PV_2(T_2^2)\}$ and $T_2^* = \arg\min\{PV_2(T_d), PV_2(T_2^2)\}$, where T_2^2 is the largest solution of $f_2(T) = 0$ on $[T_d, N]$.

(3) when $g(N) < 0$, we have $g(T) < 0$. Thus, $f_2(T)$ is decreasing on $[T_d, N]$.

(a) if $f_2(N)\ge 0$, then $f_2(T)\ge 0$. Therefor, $PV_2(T)$ is increasing on $[T_d,N]$. Thus, $T_2^*=T_d$;
(b) if $f_2(N)<0<f_2(T_d)$, there exists a unique solution T_2^3 on $f_2(T_2^3)=0$, and $f_2(T)\ge 0$ for $[T_d,T_2^3]$, $f_2(T)<0$ for $(T_2^3,N]$. $PV_2(T)$ is increasing on $[T_d,T_2^3]$ and decreasing on $(T_2^3,N]$. Hence, $PV_2(T_2^*)=\min\{PV_2(T_d),PV_2(N)\}$ and $T_2^*=\arg\min\{PV_2(T_d),PV_2(N)\}$;
(c) if $f_2(T_d)\le 0$, then $f_2(T)<0$. $PV_2(T)$ is decreasing on $[T_d,N]$. Hence, $T_2^*=N$. From the above analysis, we know that when $g(N)<0$, $PV_2(T_2^*)=\min\{PV_2(T_d),PV_2(N)\}$ and $T_2^*=\arg\min\{PV_2(T_d),PV_2(N)\}$.

Case 3 $N<T\le M$

Taking derivative of $PV_3(T)$ with respect to T , we obtain $PV_3'(T)=f_3(T)e^{-rT}/(1-e^{-rT})^2$.
where $f_3(T)=-r(1-e^{-rT})PV_3(T)+(e^{rT}-1)D\left\{\frac{h}{\theta+r}(e^{\theta T}-e^{-rT})+ce^{\theta T-rM}-\frac{1}{r}pI_e(e^{-rT}-e^{-rM})\right\}$.

Thus, we know that $f_3'(T)=(e^{rT}-1)D\left[he^{\theta T}+(\theta+r)ce^{\theta T-rM}+pI_e e^{-rM}\right]\ge 0.$

From the above analysis and lemma 1, we obtain the lemma 4.

Lemma 4. Let T_3^* is the minimum point of $PV_3(T)$ on $[N,M]$.
(a) if $f_3(N)\ge 0$, then $T_3^*=N$; (b) if $f_3(N)<0<f_3(M)$,then $T_3^*=T_3^0$, where T_3^0 is the unique solution of $f_3(T)=0$ on $[N,M]$; (c) if $f_3(M)\le 0$, then $T_3^*=M$.

Case 4 $M\le T$

Taking derivative of $PV_4(T)$ with respect to T , we obtain $PV_4'(T)=f_4(T)e^{-rT}/(1-e^{-rT})^2$,
where $f_4(T)=-r(1-e^{-rT})PV_4(T)+(e^{rT}-1)D\left[\frac{h}{\theta+r}(e^{\theta T}-e^{-rT})+ce^{\theta T-rM}+\frac{cI_p}{\theta+r}(e^{\theta T-(\theta+r)M}-e^{-rT})\right]$, and $\lim_{T\to+\infty}f_4(T)=+\infty$.

Therefor, we know that $f_4'(T)=e^{\theta T}D(e^{rT}-1)\left[h+(\theta+r)ce^{-rM}+cI_p e^{-(\theta+r)M}\right]\ge 0.$

From the above analysis and lemma 1(1-2), we obtain the lemma 5.

Lemma 5. Let T_4^* is the minimum point of $PV_4(T)$ on $[M,+\infty)$

if $f_4(M)\ge 0$, then $T_4^*=M$; else, $T_4^*=T_4^0$, where T_4^0 is the unique solution of $f_4(T)=0$ on $[M,+\infty)$.

From lemmas 2-5, the following theorem is obtained.

Theorem 1. The optimal cycle time T^* and the present value of all future cash-flow cost $PV_\infty(T^*)$ will be determined by the following equation

$$PV_\infty(T^*)=\min\{PV_1(T_1^*),PV_2(T_2^*),PV_3(T_3^*),PV_4(T_4^*)\}.$$

4.2 when $N<T_d\le M$

Case 5 $T_d\le T\le M$, we obtain the following lemma.

Lemma 6. Let T_5^* is the minimum point of $PV_3(T)$ on $[T_d,M]$.
(a) if $f_3(T_d)\ge 0$, then $T_5^*=T_d$; (b) if $f_3(T_d)<0<f_3(M)$,then $T_5^*=T_5^0$, where T_5^0 is the unique solution of $f_3(T)=0$ on $[T_d,M]$; (c) if $f_3(M)\le 0$, then $T_5^*=M$.

From the lemmas of 2, 3and 6, we have the follow theorem.

Theorem 2 when $N < T_d \le M$, the optimal cycle time T^* and the present value of all future cash-flow cost $PV_\infty(T^*)$ will be determined by the following

$$PV_\infty\left(T^*\right) = \min\left\{PV_1\left(T_1^*\right), PV_3\left(T_5^*\right), PV_4\left(T_4^*\right)\right\}.$$

4.3 when $M < T_d$

Case 6 $T_d \le T$, we obtain the following lemma.

Lemma 7. Let T_6^* is the minimum point of $PV_4(T)$ on $[T_d, +\infty)$.
If $f_4(T_d) \ge 0$, then $T_6^* = T_d$; else, $T_6^* = T_6^0$, where T_6^0 is the unique solution of $f_6(T) = 0$ on $[T_d, +\infty)$.
From the lemma of 2 and 7, we have the theorem 3.

Theorem 3 when $M < T_d$, the optimal cycle time T^* and the present value of all future cash-flow cost $PV_\infty(T^*)$ will be determined by the following equation $PV_\infty\left(T^*\right) = \min\left\{PV_1\left(T_1^*\right), PV_4\left(T_6^*\right)\right\}$.

5. Numerical examples

To illustrate the results obtained in this paper, we provide the following numerical examples.

Example 1. Let $c = 5$, $p = 7$, $I_p = 0.15$, $I_e = 0.1$, $h = 0.1$, $D = 2500$, $r = 0.2$, $\theta = 0.08$, $M = 0.3$, $N = 0.1$. Results are summarized in Table 1.

Table 1. The impact of change of A and W on $PV_\infty(T^*)$ and T^*

A	W	T_d	$PV_\infty\left(T^*\right)$	T^*
10	150	0.0599	58683	$T_2^* = T_2^0 = 0.0666$
	450	0.1787	59637	$T_5^* = T_d = 0.1787$
	950	0.3743	62488	$T_6^* = T_d = 0.3743$
100	150	0.0599	62196	$T_3^* = T_3^0 = 0.1855$
	450	0.1787	62196	$T_5^* = T_5^0 = 0.1855$
	950	0.3743	63736	$T_6^* = T_d = 0.3743$
350	150	0.0599	67131	$T_4^* = T_4^0 = 0.3421$
	450	0.1787	67131	$T_4^* = T_4^0 = 0.3421$
	950	0.3743	67202	$T_6^* = T_d = 0.3743$

Example 2. Let $A = 100$, $c = 5$, $p = 7$, $I_p = 0.15$, $I_e = 0.1$, $h = 0.1$, $D = 2500$, $r = 0.2$, $\theta = 0.08$, $W = 450$. Results are summarized in Table 2.

The following inferences can be made based on table 1 and table2:
(1) For fixed other parameters, the larger the value of A, the larger the values of $PV_\infty\left(T^*\right)$ and T^* .
(2) For fixed other parameters, when $0 < T_d \le N$, the values of $PV_\infty\left(T^*\right)$ and T^* are not changed whatever the value of W ; when $N < T_d \le M$ and $M < T_d$, if $T^* \ne T_d$, then larger the value of W , the larger the value of $PV_\infty\left(T^*\right)$ but the value of T^* is not changed; if $T^* = T_d$, the larger the value of W , the larger the values of $PV_\infty\left(T^*\right)$ and T^* .
(3) For fixed other parameters, when $Q \ge W$, the larger the value of M , the smaller the values of

$PV_{\infty}(T^{*})$; the larger the value of N, the larger the values of $PV_{\infty}(T^{*})$; when $T^{*} \neq T_{d}$, the lager the values of M and N, the larger the value of T^{*}; when $T^{*} = T_{d}$, the value of T^{*} is keeping a constant when the values of M and N.

Table 2. The impact of change of M and N on $PV_{\infty}(T^{*})$ and T^{*}

	T_d	M	N	$PV_{\infty}(T^{*})$	T^{*}
$0 < T_d \leq N$	0.0200	0.2	0.05	64083	0.1785
			0.1	64264	0.1840
		0.3	0.05	62016	0.1799
			0.1	62196	0.1855
$N < T_d \leq M$	0.1787	0.2	0.05	64083	0.1787
			0.1	64264	0.1840
		0.3	0.05	62016	0.1799
			0.1	62196	0.1855
$M < T_d$	0.3743	0.2	0.05	65785	0.3743
			0.1	65875	0.3743
		0.3	0.05	63646	0.3743
			0.1	63736	0.3743

Example 3. Let $A = 100$, $c = 5$, $p = 7$, $I_p = 0.15$, $I_e = 0.1$, $h = 0.1$, $D = 2500$, $r = 0.2$, $\theta = 0.08$, $M = 0.3$, $N = 0.1$, $W = 450$. Figures 1 and 2 show the change of the values of $PV_{\infty}(T^{*})$ and T^{*} when the parameter of the discount rate r is changed from (0,1).

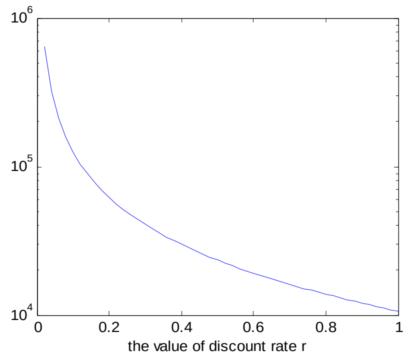

Figure 1. The impact of change of r on $PV_{\infty}(T^{*})$

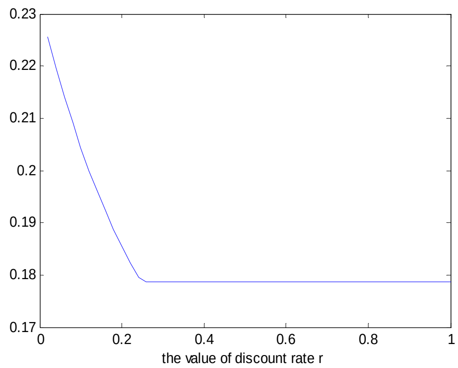

Figure 2. The impact of change of r on T^{*}

The following inferences can be made based on figuer1-2:

(1) For fixed other parameters, the larger the value of r, the smaller the value of $PV_{\infty}(T^{*})$.

(2) For fixed other parameters, when $T^{*} \neq T_{d}$, the larger the value of r, the smaller the value of T^{*}; when $T^{*} = T_{d}$, the value of T^{*} is not impact to the value of r.

6. Conclusions

In this paper, we develop an inventory system for deteriorating items under permissible delay in payments. The primary difference of this paper as compared to previous studies is that we introduce a generalized inventory model by relaxing the traditional EOQ model in the following three ways: (1) the items deteriorate continuously; (2) if the retailer's order quantity is greater than or equal to a predetermined quantity, then both of the supplier and the retailer are taking trade credit police; otherwise, the delay in payments is not permitted; (3) the present value of all future cash-flows cost instead of the average cost. The proposed of the paper is minimizing the present value of all future cash-flow cost of the retailer. In addition, the optimal solutions to the model have been discussed in detail under all possible situations. Three easy-to-use theorems are developed to find the optimal ordering policies for the considered problem, and these theoretical results are illustrated by some numerical examples.

In regards to future research, one could consider incorporating more realistic assumptions into the model, such as the demand dependents the selling price, quantity discounts, supply chain coordination, etc.

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