■ 829

System Identification and LMI Based Robust PID Control of a Two-Link Flexible Manipulator

M. Khairudin[1], Z. Mohamed[2], A.R. Husain[2]
[1] Department of Electrical Engineering, Universitas Negeri Yogyakarta, Indonesia
No 1. Jln Colombo Yogyakarta, telp. +62-274-548161
[2] Faculty of Electrical Engineering, Universiti Teknologi Malaysia, Malaysia
e-mail: moh_khairudin@yahoo.com

Abstract

This paper presents investigations into the development of a linear matrix inequalities (LMI) based robust PID control of a nonlinear Two-Link Flexible Manipulator (TLFM) incorporating payload. A set of linear models of a TLFM is obtained by using system identification method in which the linear model represents the operating ranges of the dynamic system. Thus, the LMI constraints permit to robustly guarantee a certain perturbation rejection level and a region of pole location. To study the effectiveness of the controller, initially a PID control is developed for TLFM with varying payloads. The performances of the controllers are assessed in terms of the input tracking controller capability of the system as compared to the response with PID control. Moreover, the robustness of the LMI based robust PID control schemes is discussed. Finally, a comparative assessment of the control strategies is presented.

Keywords: LMI, PID, system identification, two-link flexible manipulator

1. Introduction

Flexible manipulator robots are used in a wide spectrum of applications starting from simple pick and place operations of an industrial robot to micro-surgery, maintenance of nuclear plants and space robotics [1]. Moreover, the dynamic behaviour of the manipulator is significantly affected by payload variations. If the advantages associated with lightness are not to be sacrificed, accurate models and efficient controllers for a TLFM have to be developed.

The main goal of modelling of a TLFM is to achieve an accurate model representing the actual system behaviour. A good agreement between modelling and experiments has been achieved [2]. Zhou *et.al* [3] presents the neural network online modelling technology to approximate the system uncertain model a space manipulator. Dogan and Istefanopulos [4] have developed the finite element models to describe the deflection of a planar two-link flexible robot manipulator. De Luca and Siciliano [5] have utilised the AMM to derive a dynamic model of multilink flexible robot arms limiting to the case of planar manipulators with no torsional effects. Subudhi and Morris [6] have also presented a systematic approach for deriving the dynamic equations for n-link manipulator where two-homogenous transformation matrices are used to describe the rigid and flexible motions respectively.

Newly emerging technique for optimising the controller parameters is the use at linear matrix inequalities (LMI). The works in formulating set of LMIs to overcome the effect on mismatched uncertainties in dynamic system has also surfaced in the literatures [7]. Since LMIs can be solved efficiently by standard numerical algorithms, this has prompted a great number of researchers to describe different control problems in terms of LMIs [8]. Bevrani and Hiyama [9] presented an LMI based robust control to maintain the robust performance and minimize the effect of disturbance and specified uncertainties of power system stabilizers.

On the other hand, the important feature of LMI based robust PID design approach is that the derivative term at the controller appears in such a form that enables to consider the model uncertainties, to be considered in the design. Assuming the structured feedback matrix, this approach is appropriate for decentralized PID control design. The guaranteed cost control presented with a new quadratic cost function including the derivative term for state vector as a tool to influence the overshoot and response rate [10]. Using LMI approach to design a robust PID controller presented [11],[12],[13]. On other hand, Liang *et.al* [14] implemented a fuzzy adaptive PID controller whose duty is to make sure the uncertainty and nonlinearities of

hydraulic erecting mechanism. Using the LQR to solve flexible link robustness and input tracking capability of hub angular position [15].

However, most of the published work on robust PID design via LMI was based on simulation exercises with limited possibility for experimental validation due to complicated controller structure. Moreover, not much work on LMI robust controller of a TLFM with payload been reported. This is a challenging task for a MIMO system and the system behaviour is affected by several factors. This paper presents the design and development of a robust PID control based on LMI for a non-linear two-link flexible manipulator. It is found that the LMI approach has not been explored for control of a two-link flexible manipulator where the system dynamics have uncertainties due to the variation of payloads. Using the robust controller, identified PID gains can be used for all payloads with satisfactory responses. This is an advantage as compared to Ziegler-Nichols (ZN) tuned PID control which needs to be re-tuned for different payloads. Subsequently, the dynamic model is represented into convex formulation which leads to the formulation of system requirement into LMIs representation that can accommodate the convex model. A set of robust PID gains is then obtained by solving the LMIs with desired specifications. For performance assessment, ZN-PID and LMI-PID controllers are compared to control of the manipulator in terms of input tracking, deflection reduction level and robustness to payload variations of both links. Experimental results show that better robustness and system performance are achieved with LMI-PID controller despite using a single set of PID gains.

2. Research Method

The physical parameters of the TLFM system considered in this study are shown in Table 1. M_{h2} is the mass considered at the second motor which is located in between both links, J_{hi} is the inertia of the ith motor and hub. The input torque, $\tau_i(t)$ is applied at each motor and G_i is the gear ratio for the ith motor. Both links and motors are considered to have the same dimensions.

Table 1. Parameters of a TLFM

Symbol	Parameter	Link-1	Link-2	Unit
M_{L1}, M_{L2}	Mass of link	0.08	0.05	kg
ρ	Mass density	2666.67	2684.56	kgm^{-1}
EI	Flexural rigidity	1768.80	597.87	Nm^2
J_h	Motor and hub inertia	1.46 x10^{-3}	0.60 x10^{-3}	kgm^2
M_p	Payload mass max	-	0.1	kg
J_p	Payload inertia max	-	0.05 x 10^{-3}	kgm^2
l	Length of link	0.5	0.5	m
	Width of link	0.03	0.025	m
	Thickness of link	2 x10^{-3}	1.49 x10^{-3}	m
J_o	Moment of inertia	5 x10^{-3}	3.125 x10^{-3}	kgm^2
M_{h2}	Mass of the centre rotor	-	0.155	kg

A nonlinear TLFM is a distributed-parameter system that can be described by an infinite-dimensional mathematical model. In practice, the reduced-order model is used to conform to computational limitations [16]. This system identification to obtain a set of linear models of the TLFM in which the linear model represents the operating ranges of the dynamic system.

The method constructs the system identification for a nonlinear TLFM selectively. In this section, it is indicated that the identification system is consisted of the program based on Matlab. The interface of the identification system application uses identification tools in Matlab. The recognition system described in this paper is the least square offline parametric identification system. The multisine signal produces sinusoids of different amplitudes and frequencies, which are summed to constitute a persistently exciting signal for the identification process.

After the identification results are obtained, it also needs to verify whether this model is applicable. Model validation and simulation consist of comparing the predicted output with the measured output, checking the transient response using a step response plot for the estimated

model, and checking poles and zeros. The result shows that the waveforms of the forecast output and the actual output are basically the same, and the matching degree is about 94%. The parameters of the transfer function model are obtained from least-squares estimation. A sixth-order identified model G(s) that has a good match with the first two modes is obtained:

$$G_{11}(s) = \frac{-5153\ s^5 - 919\ s^4 - 4.518e005\ s^3 + 3.126e006\ s^2 - 8.72e007\ s + 0.700}{s^6 + 21.72\ s^5 + 1492\ s^4 + 2.29e004\ s^3 + 2.989e004\ s^2 - 7.246e007\ s + 2.028e009}$$

For the same processes, a model of link-2 without load, a model of system with load 0.05 kg and 0.1 kg will be obtained.

Simulation results of LMI based robust PID control of the TLFM are presented in the time and frequency domains. The steps that are necessary for the design is as follow: **Step 1**: system identification of a nonlinear TLFM to obtain a sets of linear model of a TLFM (eq.2). **Step 2**: construct the linear model in statespace form. **Step 3**: polytopic models of TLFM without payload, load 0.05 and 0.1 kg (eq.6). **Step 4**: set the upper bound specification (eq. 13). **Step 5**: find *X* such that the inequalities are satisfied (eq.11). **Step 6**: construct LMIs region (ρ,θ,α) using eq.(17), eq.(18) and eq. (19). **Step 7**: using the result of *X* form the previous step, calculate gain *K* (eq.14). **Step 8**: applied the gain *K* to the LMI based robust PID controller for a nonlinear TLFM. **Step 9**: check the output and repeat from step6 to produce the desired output.

In this work, Linear Quadratic Regulator (LQR) approach is considered as a basis for tuning the controller gain since this approach can give nice robustness and it can be formulated in term of performance based optimization problem which can be solved using numerical technique [12].

Proposition 1: Schur Compliments to determine matrix inequality [19]:

$$\begin{bmatrix} V(x) & S(x) \\ S^T(X) & Z(x) \end{bmatrix} > 0 \tag{1}$$

This also known as Schur complement in which this property is very useful to cost the imposed constraint in to LMIs sets.

3. Robust PID Controller Design
Consider an uncertain sixth order of the system for link-1

$$G_1(s) = \frac{n_1 S^5 + n_2 S^4 + n_3 S^3 + n_4 S^2 + n_5 S + n_6}{S^6 + d_1 S^5 + d_2 S^4 + d_3 S^3 + d_4 S^2 + d_5 S + d_6} \tag{2}$$

and similarly the same order of the system for link-2, where the parameter vary in intervals:

$$d_1 \in [\underline{d_1},\overline{d_1}],...d_6 \in [\underline{d_6},\overline{d_6}], n_1 \in [\underline{n_1},\overline{n_1}]...n_6 \in [\underline{n_6},\overline{n_6}] \tag{3}$$

where $\underline{d_i},\overline{d_i}$ and $\underline{n_i},\overline{n_i}$ are lower and upper bounds for the uncertain parameters denumerator and numerator of the system respectively. According to Astrom and Hagglund [20], a PID controller with the structure:

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{4}$$

is adequate for such a system. For a TLFM both two inputs, the resultant negative feedback system.

The objective of PID controller design is to determine PID settings to meet various design specifications. In this paper, the PID controller is designed in the state space settings for the ease of using LMI approach. The feedback system can be expressed in the state space description:

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{5}$$

where y are the system output and the reference input respectively, $x=[x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$ the state with variables. $A \in R^{6x6}, B \in R^{6x1}, C \in R^{1x6}$.

The transfer function models (equation 3) convert to a state-space model. In the state-space model, the PID controller design becomes a static state feedback controller, and the static feedback gain $K=[K_{p1}\ K_{d1}\ K_{i1}\ K_{p2}\ K_{d2}\ K_{i2}]$ simply contains all the PID controller parameters. Note also that there are several uncertain parameters in (4) and the polytopic uncertain set reduces to

$$\Omega = Cov\{[A_{11}, B_{11}], [A_{12}, B_{12}], [A_{21}, B_{21}], [A_{22}, B_{22}], [A_{31}, B_{31}], [A_{32}, B_{32}]\} \tag{6}$$

where the vertex matrices $[A_{ij}, B_{ij}]$ are determined based on the system identification results, i is varying load and j is link.

3.1. LMI Based robust PID Controller

This section presents the concept of LMI and presents the constraints used in the controller synthesis problem which will be used for the robust PID controller design. In several control problems, it is well motivated to base the design on the LQR control theory for its nice robustness [17]. The standard LQR problem is to determine the signal control u which minimizes the quadratic cost:

$$J(u) = \int_0^\infty (x^T Qx + u^T Ru)dt \tag{7}$$

for an initial state x(0), where Q and R are symmetric positive semi-definite matrix and symmetric positive definite matrix, respectively, i.e. $Q^T \geq 0$ and $R=R^T > 0$. Assume that (A,B) are controllable and $(Q^{1/2}, A)$ are observable. It turns out that the solution u^* to this optimal control problem can be expressed [17] in the state feedback form:

$$u^* = -Kx = -R^{-1}B^T Xx$$

where X is the symmetric positive definite solution of the *algebraic Riccati equation* (ARE):

$$A^T X + XA - XBR^{-1}B^T X + Q = 0 \tag{8}$$

and the minimum quadratic cost [12] is given by

$$J_{min} = x^T(0)Xx(0) \tag{9}$$

Thus, the solution to the LQR problem relies on solving the ARE (8). An efficient alternative for this problem is the LMI technique that has emerged recently as a powerful design utility for a variety of control problems due to its convexity [18]. By the LMI technique, the LQR problem can be rephrased as an optimization problem over X and Y:

$$\min_{X,Y} x^T(0)X^{-1}x(0) \tag{10}$$

subject to

$$\begin{bmatrix} AX + XA^T + BY + Y^T B^T & X & Y^T \\ X & -Q^{-1} & 0 \\ Y & 0 & -R^{-1} \end{bmatrix} \leq 0, \; X > 0 \tag{11}$$

where $Y = -KX$. In several practical situations, the objective (10) is represented as:

$$x^T(0)X^{-1}x(0) \leq \gamma \tag{12}$$

where γ is the specified upper bound. The above inequality can also be expressed as LMI:

$$\begin{bmatrix} \gamma & x^T(0) \\ x(0) & X \end{bmatrix} \geq 0 \tag{13}$$

Consequently, the optimization problem in (10) and (11) in converted to seeking a solution (X^*, Y^*) that satisfies a set of LMIs in (11) and (13) and the state feedback gain is given by

$$K = -Y^*(X^*)^{-1} \tag{14}$$

The system matrix $[A, B]$ is usually not precisely known in practice. Assume that $[A_{ij}, B_{ij}]$, is uncertain but belongs to a polytopic set:

$$\Omega = Cov\{[A_{11}, B_{11}], [A_{12}, B_{12}], [A_{21}, B_{21}], [A_{22}, B_{22}], [A_{31}, B_{31}], [A_{32}, B_{32}]\} \tag{15}$$

where *cov* refers to a convex hull, or $[A, B] \in \Omega$ if

$$[A, B] = \sum w(x, u)[A_{ij}, B_{ij}]$$

where *i* and *j* is varying load and link respectively, and w the weighting function constrained between 0 and 1.

3.2. Pole Placement LMIs
Chilali and Gahinet [19] presented a region of the complex plane *S(α,ρ,θ)* where *α,ρ,* and *θ* are minimum decay rate, the disk of radius and inner angle, in the form *x ± jy* satisfy

$$x < -\alpha < 0, \quad |x \pm jy| < \rho, \quad y < cot(\theta)x \tag{16}$$

where *α*, *ρ* are the minimum decay rate and the disk of radius respectively, *θ* is the sector of the centered at the origin and inner angle.

Proposition 2: The closed-loop poles of the system with a state-feedback *u=Kx* are inside the region *S(α,ρ,θ)* if there exists a symmetric definite positive matrix *X* and a matrix *Y* such that

$$AX + XA^T + BY + Y^T B^T + 2\alpha X < 0 \tag{17}$$

$$\begin{bmatrix} -\rho X & XA^T + Y^T B^T \\ AX + BY & -\rho X \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \cos\theta(AX + XA^T + BY + Y^T B^T) & \sin\theta(AX - XA^T + BY - Y^T B^T) \\ \sin\theta(-AX + XA^T - BY + Y^T B^T) & \cos\theta(AX + XA^T + BY + Y^T B^T) \end{bmatrix} < 0 \tag{19}$$

and $K=YX^{-1}$ is the state feedback gain.

4. Results and Discussion

A step signal with amplitude of ±0.5 rad is used as an input position in radian applied at the hub of link-1 of the manipulator. The same form of signal with amplitude of ±0.35 rad is used as the input signal for link-2. Two system responses namely the hub angular positions and deflections at 10 cm from the hubs of both links with the frequency response of the deflections are obtained and evaluated. Moreover, the effects of varying payload on controller performances are also studied. For these investigations, the system without payload, and the system with payloads of 0.05 kg and 0.1 kg are considered.

To demonstrate the performance of the LMI based robust PID controller with the pole placement parameters α, ρ are the minimum decay rate $\alpha < -1$ and the disk of radius $\rho = 2\pi/10T_s$ respectively. θ is the sector of the centered at the origin and inner angle $\theta = 25$ degree. Inner angle is designed for covering uncertainties parameter of several conditions in this study with varying payload. As comparing the performance of LMI based robust PID controller, a PID controller is designed using Ziegler Nichols method for control of a TLFM. A block diagram is utilised to obtain the proportional gain, K_p, integral gain K_i, and the derivative gain, K_d. In this study, the task of the controller is for input tracking capability of the system. The angular position of link-1 and link-2 are fed back to control of a TLFM with varying payload. Table 2 summaries the PID controller gain using Z-N PID for the TLFM with varying payloads.

On the other hand, by utilising the LMI based robust PID controller that was designed based on the dynamic behaviour of the TLFM with varying payloads. The parameters of the LMI based robust PID controller for a TLFM with varying payloads are $K_{p1}=0.13$, $K_{i1}=0.0343$, $K_{d1}=0.05$ and $K_{p2}=0.090$, $K_{i2}=0.037$, $K_{d2}=0.05$ for link-1 and link-2 respectively.

Table 2. PID parameters using Ziegler Nichols

No	Payload	Link-1			Link-2		
		K_{p1}	K_{i1}	K_{d1}	K_{p2}	K_{i2}	K_{d2}
1	0	0.58	0.07	0.16	0.09	0.12	0.01
2	50 g	0.59	0.07	0.18	0.09	0.13	0.05
3	100 g	0.61	0.07	0.18	0.10	0.15	0.05

System without and with Payload. Figure 1 shows the angular positions of the TLFM without payload for both links. Both using LMI based robust PID control and Z-N PID control results show similar results for link-1 and link-2, where steady state angular position levels of -0.5 rad and 0.35 rad were achieved respectively. The transient response specifications of the angular position for both links without and with payload are summarised in Table 3. Using LMI based robust PID control, the system exhibits lower settling times and smaller overshoots for both links compared using Z-N PID.
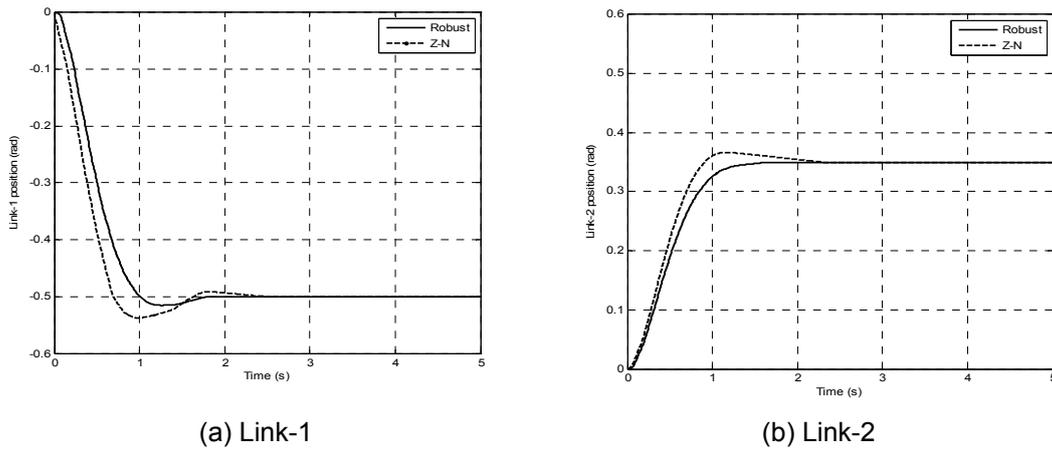
(a) Link-1   (b) Link-2

Figure 1. Angular position of the system.

Table 3. Relation between payloads and specifications of angular position

Payload	Link-1				Link-2			
	Settling time (s)		Overshoot (%)		Settling time (s)		Overshoot (%)	
	LMI PID	Z-N PID	LMI PID	Z-N PID	LMI PID	Z-N PID	LMI PID	Z-N PID
0	1.54	1.70	3.07	7.40	1.18	1.64	0.00	4.86
50 g	1.70	1.77	3.08	11.74	1.29	1.66	0.00	5.91
100 g	1.75	1.81	3.10	12.40	1.35	1.74	0.01	9.40

Figure 2 shows results of the deflection responses of link-1 and link-2. It is noted that the magnitudes of vibration of the deflection responses decrease for both links using LMI based robust PID control compared with PID control. With LMI based robust PID control, the maximum magnitudes of the responses were 3.30 mm and 1.92 mm for link-1 and link-2 respectively. On the other hand with PID control, the maximum magnitudes were 7.02 mm and 4.45 mm. Figure 3 shows the frequency responses of the deflection responses obtained with LMI based robust PID control and PID control exercises. These were obtained by transforming the time response into the frequency domain using Fast Fourier Transform. The results show that controller performances are characterized by the first two modes of vibrations. With LMI based robust PID control show that the vibration occurs at 5.88 Hz and 15.69 Hz, and 3.93 Hz and 25.49 Hz for link-1 and link-2 respectively. Otherwise, the resonance frequencies for link-1 and link-2 were obtained at 7.84 Hz and 23.52 Hz, and 6.05 Hz and 28.49 Hz respectively using Z-N PID.
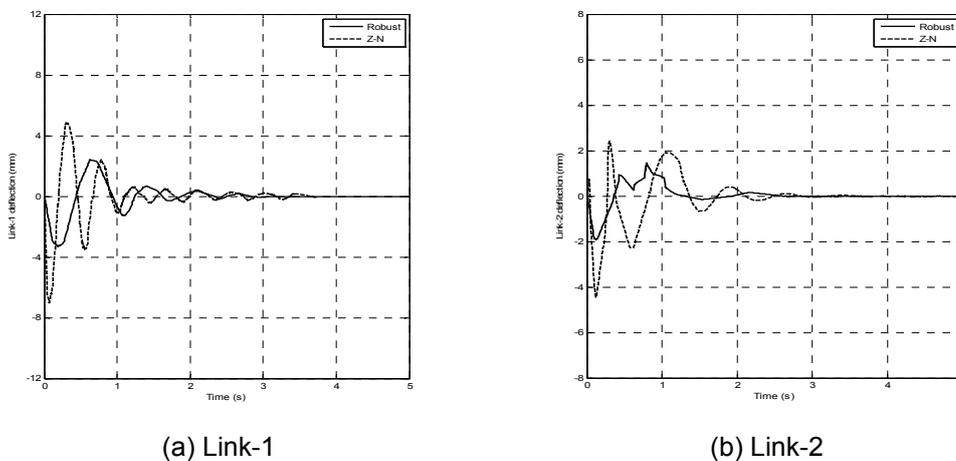

(a) Link-1   (b) Link-2

Figure 2. Deflection response of the system

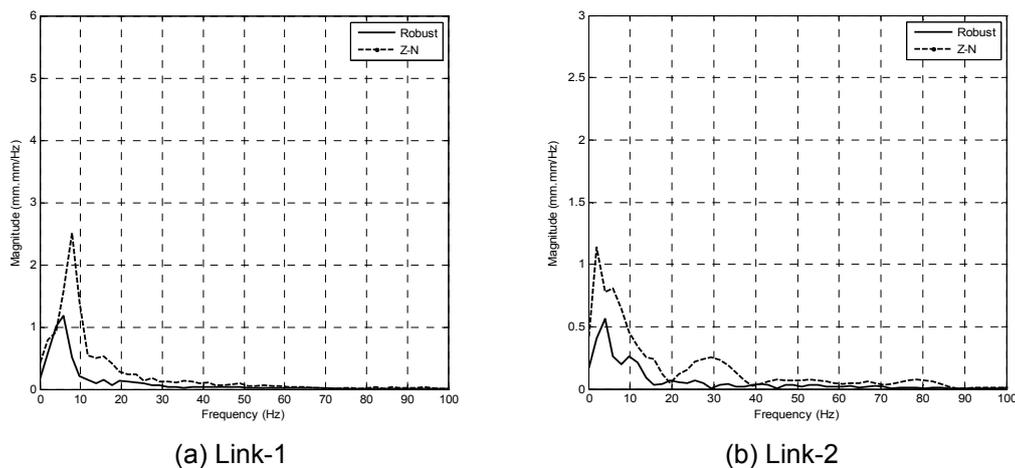(a) Link-1   (b) Link-2

Figure 3. Freq. response of deflection of the system

To investigate the effects of payload on the dynamic characteristics of the system, a TLFM with various payloads was examined. The time response specifications of angular positions have shown significant changes with the variations of payloads. It is noted with LMI based robust PID controller, the system exhibits lower settling times and smaller overshoots for both links compared Z-N PID controller. The results also show that the transient responses of the system are affected by the variations of payload.

It is noted with increasing payloads, the magnitudes of vibration of the deflection increase for both links. However, the magnitudes of vibration of the deflection responses decrease for both links with LMI based robust PID control compared with PID control. Table 4 summarizes the maximum magnitudes of the responses for link-1 and link-2 achieved with LMI based robust PID control and PID control.

Table 4. Effects of payloads on maximum magnitudes of the deflection responses

Payload (g)	Link-1				Link-2			
	Robust PID (mm)		Z-N PID (mm)		LMI PID (mm)		Z-N PID (mm)	
0	-3.30	2.41	-7.02	4.93	-1.92	1.47	-4.45	2.43
50 g	-3.33	2.42	-7.09	7.37	-2.05	1.50	-5.93	4.81
100 g	-3.39	2.94	-7.11	7.69	-3.50	1.62	-7.24	4.85

In this work, the frequency responses of the deflection is utilised to investigate the effects of payload on the dynamic behaviour of the system in the frequency domain. It is noted that the resonance modes of vibration of the system shifts to lower frequencies with increasing payloads. For a payload between 0.05 to 0.1 kg using LMI based robust PID the resonance frequencies for link-1 shifted from 3.92 Hz and 13.74 Hz to 1.96 Hz and 13.71 Hz for the first two modes of vibration respectively. On the other hand, the resonance frequencies of link-2 using LMI based robust PID shifted from 2.02 Hz and 17.65 Hz to 1.98 Hz and 15.69 Hz. Table 5 summarises resonance frequencies of the deflection responses with payloads for link-1 and link-2 with LMI based robust PID control and PID control. This implies that the manipulator oscillates at lower frequency rates than those without payload.

Table 5. Relation between payloads and resonance frequencies of the flexible manipulator

| Payload | Link-1 | | | | Link-2 | | | |
| | Robust PID | | Z-N PID | | Robust PID | | Z-N PID | |
	Mode-1 (Hz)	Mode-2 (Hz)	Mode-1 (Hz)	Mode-2 (Hz)	Mode-1 (Hz)	Mode-2 (Hz)	Mode-1 (Hz)	Mode-2 (Hz)
0	5.88	15.69	7.84	23.52	3.93	25.49	6.05	28.49
50 g	3.92	13.74	5.88	23.49	2.02	17.65	5.89	19.61
100 g	1.96	13.71	5.86	15.68	1.98	15.69	5.83	17.65

5. Conclusion

The development of dynamic model and robust control of a TLFM with varying payload has been presented. A set of linear model has been developed by taking through system identification of a nonlinear TLFM approach. A PID controller has, initially, been developed for control of a TLFM with varying payloads. The LMI is universal and can be adapted for any a nonlinear system. It can be extended by incorporating other design requirement such that it is representing in LMI form. A LMI robust PID controller has been implemented for input tracking control of the TLFM. Performances of the control schemes have been evaluated in terms of the input tracking capability of the system with compared PID controller. Simulations of the dynamic model and LMI based robust PID control have been carried out in the time and frequency domains where the system responses including angular positions and deflection are studied. In term of input tracking, LMI based robust PID has been shown to be more effective technique. These results will be verify on the hardware experimental work for future work.

References
[1] Dwivedy SK, Eberhard P. Dynamic analysis of flexible manipulators, a literature review. *Mechanism and Machine Theory.* 2006; 41: 749-777.
[2] Martins JM, Mohamed Z, Tokhi MO, Sa da Costa J, Botto MA. Approaches for dynamic modelling of flexible manipulator systems. *IEE Proc-Control Theory and Application.* 2003; 150: 401-411.
[3] Zhou Shuhua, Ye Xiaoping, Ji Xiaoming, Zhang Wenhui. Adaptive control of space robot manipulators with task space base on neural network. *TELKOMNIKA.* 2014; 12(2): 349-356.
[4] Dogan M, Istefanopulos Y. Optimal nonlinear controller design for flexible robot manipulators with adaptive internal model. *IET Control Theory and Applications.* 2007; 1(3): 770-778.
[5] De Luca A, Siciliano B. Closed-form dynamic model of planar multi-link lightweight robots. *IEEE Transactions on Systems, Man, and Cybernetics.* 1991; 21: 826-839.
[6] Subudhi B, Morris AS. Dynamic modelling, simulation and control of a manipulator with flexible links and joints. *Robotics and Autonomous System.* 2002; 41: 257-270.
[7] Sellami A, Arzelier D, Mhiri R, Zrida, J. A sliding mode control approach for system subjected to a norm-bounded uncertainties. *International Journal of Robust and Nonlinear Control.* 2007; 17: 327-346.
[8] Gahinet P, Apkarian. A linear matrix inequalities approach to h_∞ control. *International Journal Of Robust And Nonolienar Control.* 1994; 4: 421-448.
[9] Breavani H, Hiyama T. *Robust design of power system stabilizer: an lmi approach.* Proceeding of the IASTED International Conference of Energy and Power System. Thailand. 2006: 70-75.
[10] Rosinova D, Vesely V. Robust decentralised controller design using lmi. *International Journal of Computers Communications and Control.* 2007; 2(2): 195-204.
[11] Goncalves EN, Palhares RM, Takahashi HC. A novel approach for H_2 and H-infinity robust PID synthesis for uncertain systems. *Journal of Process Control.* 2008; 18: 19–26.
[12] Ge M, Ciu MS, Wang QG. Robust PID controller design via LMI approach. *Journal of Process Control.* 2002; 12: 3-13.
[13] Olalla C, Leyva R, El Aroudi A, Garce´s P, Queinnec I. LMI robust control design for boost pwm converters. *IET Power Electron.* 2010; 3(1): 75–85.
[14] Liang Li, Jian Xie, Wei Li. Fuzzy adaptive PID control of a new hydraulic erecting mechanism. *TELKOMNIKA.* 2013; 11(4): 715-724.
[15] Mohammad K, Mohamed Z, Husain AR. Dynamic model and robust control of flexible link robot manipulator. *TELKOMNIKA.* 2011; 9(2) : 279 – 286
[16] Ho MT, Tu YW. Position control of a single-link flexible manipulator using H-infinity based PID control. *IEE Proc.-Control Theory Appl.* 2006; 153(5).
[17] Lewis FL, Syrmos VL. *Optimal Control.* John Wiley & Sons. 1995.
[18] Boyd SP, Ghaoui LE, Feron E, Balkrishnan V. Linear matrix inequalities in systems and control theory. SIAM, Philadelphia. 1994.

[19] Chiliali M., Gahinet P., H design with pole placement constraints an LMI approach. IEEE Trans, Om Auto. Control. 1996; 41: 358-367.
[20] Astrom KJ, Hagglund T. *PID Controller*. 2nd edition. Instrument of society of America. Research triangle park, North Carolina. 1995.