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A Practical Coordinated Trajectory Tracking for A Group of Mixed Wheeled Mobile Robots with Communication Delays

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Abstract

Coordination between a specific mobile robot type has been widely investigated, e.g coordination between unicycles. To extend the applicability of the system, a coordinated trajectory tracking of mixed type of mobile robots is considered. We prove that if a certain type of wheeled mobile robot is able to individually track its own reference, then coordination in tracking with other type of robots can be achieved simply by sharing individual tracking errors. Using two types of wheeled mobile robots, namely unicycle types (a nonholonomic mobile robot) and omni wheels type (a holonomic mobile robot), a coordinated control algorithm can achieve a global asymptotically stable condition of the error dynamics of the systems. Under bidirectional communication between robots as a constraint, the group is able to maintain individual tracking while coordinating the movements with other robots regardless occurring perturbations in the system and delays in communication channels. Simulation results suggest that information sharing between the robots increase the robustness in coordinating individual trajectories. Results also show that delays cause drop in performance similar to the case of no information sharing.

Keywords: *coordinated trajectory tracking, unicycles, omni wheels robots, Lyapunov function, time delays*

1. Introduction

The ability of a group of wheeled mobile robots in coordinating complex tasks has drawn attention from scientist, hobbyist and practitioner. A group of mobile robots is able to handle more difficult and distributed tasks. The group can achieve a certain formation as well as coordinating movement between the robots so that certain tasks can be completed.

One problem in coordination between robots is the problem of coordinating individual trajectories so that the overall trajectories form a certain spatial patterns [1-7]. In this particular case, a group of unicycle mobile robots is coordinating its member's trajectories such that the coordination forms spatial patterns. The coordination is achieved by exchanging individual position or tracking errors within the group.

Due to the nonholomomic constraints that exist in the systems, unicycle mobile robots is less flexible compared to holonomic mobile robots, e.g. omni wheels mobile robots. In a compact space, the ability of omni wheels to move sideways becomes very important

Omni wheels mobile robots have been widely used in football competition [8-9]. Although it easily suffers from slip, a good control and mechanical design can optimize the ability of an omni wheels mobile robot. Example of trajectory tracking or path following for omniwheel mobile robots can be found for example in [10-12].

In [10], a kinematic control is built for the omnidirectional robot. A detailed kinematic model is implemented to solve trajectory tracking problem. The controller takes into account kinematic model of the wheels to reduce slip effects. In [11], a combination of kinematic and torque control is used to track reference trajectories, while in [12], an omnidirectional robot is used to help patients during rehabilitation process. All algorithms are designed for a single robot.

From the results in [1-3] coordination between robots can be achieved simply via exchanging states of the robots, i.e. the individual tracking errors. The stability proof suggests

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that proper choice of control structure can simplify the mathematical procedure, as well as extending the system with different type of robot.

On the other hand, studies of coordinating trajectories of omni wheels in a simple way are rare subjects. From the results in unicycle case, it can be said that applying similar concept for omni wheels mobile robots will be a valuable result. Motivating by the facts, this paper addresses the problem of coordinating a group of omni wheel mobile robots by means of exchanging individual tracking errors. Furthermore, the group is extended to be a mixed of unicycle and omni wheel mobile robots. The stability of the system is investigated using Lyapunov theorem. The algorithm is structured such that delays in communication channel can be easily compensated.

The contributions of this paper are as follow: i) a trajectory tracking controller for unicycle and omni wheels that able to achieve coordination with either unicycles or omni wheels or both types in the presence of delays in communication channel iii) globally asymptotically stable error dynamics of the overall mixed wheeled mobile robot systems, iv) simulation validation of the overall systems.

The rest of the paper is organized as follows. Section 2 gives the kinematic model of unicycles and omni wheels as well as theories for stability. Section 3 presents the control design process; start with the design of a trajectory tracking for a single omni wheels mobile robot, followed by the extension to *m*-mixed-unicycle-omni wheels mobile robot systems. Stability analysis is presented in this chapter. Section 4 gives the simulation validation and performance analysis of the mixed systems both delayed and non-delayed communicaton channel. Finally, in section 5 the conclusions of this work are given and suggestions for further research are presented.

2. Kinematic Model of Mixed Wheeled Mobile Robots

In this research we consider two types of mobile robots, namely unicycle type and omniwheels type. Unicylcle represents a mobile robot with non-holonomic constraint, i.e. practically cannot move sideways. Omni wheel robot represents mobile robot that belongs to holonomic mobile robots. The kinematic models of each robot are illustrated in Figure 1.

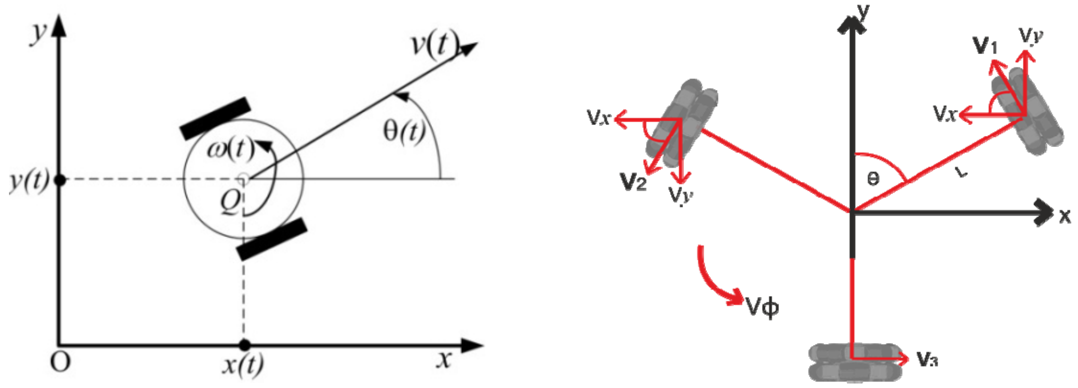


Figure 1. Kinematic Model of an Unicycle (left); Omni Wheels (right)

The kinematic model of an unicycle is given as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (1)$$

With v and ω are the forward and steering velocities of the robots, while the kinematic model of an omni wheels is:

$$\begin{bmatrix} v_x \\ v_y \\ v_\emptyset \end{bmatrix} = \begin{bmatrix} -\cos(\theta) & -\cos(\theta) & 1 \\ \sin(\theta) & -\sin(\theta) & 0 \\ 1/L & 1/L & 1/L \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (2)$$

Where v_1, v_2 and v_3 are the individual wheel speed that drive the robot and are determined by the multiplication of wheel rotation and radii.

It is to be noted that the two models use different input states. At the one hand, the unicycle uses forward and steering velocities as inputs. On the other hand, the omni wheels uses individual wheel speeds as inputs. Athough seem contradictive, the choices help in proving the concept that coordination can be achieved only by exchanging individual state information regardless the model of the robots.

3. Control Design

In this section the control design is explained in detail. For the delay-free case, the controllers for the pure unicycles in the group are obtained from [1-2].

If references tracking for a group of mixed wheeled mobile robots, *i* is etiher an unicycle or omni wheels, are given as $F = \{\boldsymbol{q}_{r1}, \boldsymbol{q}_{r2}, \dots, \boldsymbol{q}_{rm}\}$ and the actual positions are given as $F = \{\boldsymbol{q}_1, \boldsymbol{q}_2, \dots, \boldsymbol{q}_m\}$, and the tracking errors of the group are $\boldsymbol{e_{sys}} = \{\boldsymbol{q}_{r1} - \boldsymbol{q}_1, \boldsymbol{q}_{r2} - \boldsymbol{q}_2, \dots, \boldsymbol{q}_{rm} - \boldsymbol{q}_m\} = \{\boldsymbol{e_1}, \boldsymbol{e_2}, \dots, \boldsymbol{e_m}\}$, the problem of coordinating the trajectory is defined as to make:

$$\boldsymbol{e_{sys}} \rightarrow \boldsymbol{0} \text{ as } t \rightarrow 0 \quad (3)$$

3.1. Coordination without Delays in Communication Channels

The problem if minimizing tracking errors of the group can solved in the following steps. Firstly for the unicycle mobile robot, the trajectory tracking controllers is obtained from [2], i.e. for each unicycle *i,* the controllers are:

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} v_{rui} \cos e_{\theta ui} + k_{xu} e_{xui} \\ \omega_{rui} + k_{yu} v_{ru} e_{yui} \frac{\sin e_{\theta ui}}{e_{\theta ui}} + k_{\theta u} e_{\theta ui} \end{bmatrix} + \begin{bmatrix} \sum_{j=1, j\neq i}^{m} k_x^{ij} e_{xj} \frac{(e_{xui} - e_{xj})}{\sqrt{1 + e_{xui}^2 + e_{xj}^2}} \\ \sum_{j=1, j\neq i}^{m} k_y^{ij} e_{yj} \frac{(e_{yui} - e_{yj})}{\sqrt{1 + e_{yui}^2 + e_{yj}^2}} \end{bmatrix} \quad (4)$$

Where $k_{xu}, k_{yu}, k_{\theta u}$ are control gains for individual tracking, k_x^{ij}, k_y^{ij} are the coupling gains, v_{rui}, ω_{rui} are reference forward and steering velocities of each unicycles, $[e_{xui} \quad e_{yui} \quad e_{\theta ui}]^T$ are individual tracking errors, $[e_{xj} \quad e_{yj}]^T$ are individual tracking errors from other robots that can be either an unicycle or omni wheels.

As for the omni wheels type, by rewriting (2) into $\dot{\boldsymbol{q}}_o = A\boldsymbol{v}$, where $\boldsymbol{v} = [v_1 \quad v_2 \quad v_3]^{\boldsymbol{T}}$, the proposed trajectory tracking controllers are given by:

$$\boldsymbol{v_i} = A^{-1}(\dot{\boldsymbol{q}}_{roi} + \boldsymbol{k}\boldsymbol{e}_{oi}) + \left[\left(\sum_{j=1, j\neq i}^{m} k_x^{ij} e_{xj} \frac{(e_{xoi} - e_{xj})}{\sqrt{1 + e_{xoi}^2 + e_{xj}^2}} \right) \quad \left(\sum_{j=1, j\neq i}^{m} k_y^{ij} e_{yj} \frac{(e_{yoi} - e_{yj})}{\sqrt{1 + e_{yoi}^2 + e_{yj}^2}} \right) \quad \boldsymbol{0} \right]^{\boldsymbol{T}} \quad (5)$$

Where $\boldsymbol{k}$ is the individual tracking control gain vector, k_x^{ij}, k_y^{ij} are the coupling gains, $\dot{\boldsymbol{q}}_{roi}$ is the reference trajectory vector of each omniwheels, $[e_{xoi} \quad e_{yoi} \quad e_{\theta oi}]^T$ are individual tracking errors (omni wheels) $[e_{xj} \quad e_{yj}]^T$ are individual tracking errors from other robots that can be either an unicycle or omni wheels.

For the given controllers and coordinated tracking problems for a group of mixed wheeld mobile robots, the following theorem holds.

Theorem 1. There exist m-mobile robots, which can be either unicycle or omni wheels mobile robots. The robots are tracking the given references trajectories that in overall create a spatial formation pattern. If the control parameters are chosen so that and $\mathrm{k_x^{ij}} = \mathrm{k_x^{ji}}, \mathrm{k_y^{ij}} = \mathrm{k_y^{ji}} \geq 0$,

$\mathrm{k_{xu}, k_{yu}, k_{\theta u}, k_{xo}, k_{yo}} > 0$ and it is assumed that the shared information is received by the corresponding robots without delay, then the controller given in (5) and (6) renders origin of $\mathbf{e_{sys}} = \{\mathbf{q}_{r1} - \mathbf{q}_1, \mathbf{q}_{r2} - \mathbf{q}_2, \dots, \mathbf{q}_{rm} - \mathbf{q}_m\}$ globally asymptotically stable (GAS).

Proof of **Theorem 1.** The stability is proven using Lyapunov functions. To simplify the analysis, the system is categorized into three subsystems. The subsystems represent the types of robots exist in the system. Using the theory that states if a subsystem is GAS using Lyapunov function, then the overall system is also GAS if there is no switching in systems, the error dynamics are analyzed. The subsystem are as follow:

1. A subsystem S1, where $\forall i, j \in m$ is a unicycle type. In this condition the system becomes a homogeny unicycle systems, which is similar as in [2] and its error dynamics is GAS. All stability proof follows the one given in [2]. Thus, stability proof follows the results in [2].
2. A subsystem S2, where $\forall i, j \in m$ is a omniwheels type. In this condition the system becomes a homogeny omni wheels system.
3. A subsystem S3, where $i, j \in m$ is either unicycle or omni wheels., i.e. the case of mixed mobile robot systems.

a. Stability proof for S2

Consider a Lyapunof function $\boldsymbol{V_2} = \boldsymbol{e_{sys}^T e_{sys}}$. For S2, the errors are all coming from omni wheels mobile robots. Thus, the derivative of $\boldsymbol{V_2}$ can be expressed as:

$$\dot{\boldsymbol{V}}_{\mathbf{2}} = \boldsymbol{e}_{sys}^T \dot{\boldsymbol{e}}_{sys} \sum_{i=1}^{m} \boldsymbol{e}_{oi}^T \left(\dot{\boldsymbol{q}}_{roi} - \dot{\boldsymbol{q}}_{oi}\right)$$

$$= \sum_{i=1}^{m} \boldsymbol{e}_{oi}^T \left(\dot{\boldsymbol{q}}_{roi} - \left(A \left(A^{-1}(\dot{\boldsymbol{q}}_{ro} + \boldsymbol{k}\boldsymbol{e}_o + \begin{bmatrix} \sum_{j=1,j\neq i}^{m} k_x^{ij} e_{xj} \frac{(e_{x0i} - e_{xj})}{\sqrt{1+e_{xoi}^2+e_{xj}^2}} \\ \sum_{j=1,j\neq i}^{m} k_y^{ij} e_{yj} \frac{(e_{yoi} - e_{yj})}{\sqrt{1+e_{yoi}^2+e_{yj}^2}} \\ 0 \end{bmatrix} \right) \right) \right)$$

$$= \sum_{i=1}^{m} -\boldsymbol{k}\boldsymbol{e}_{oi}^T \boldsymbol{e}_{oi} + \varphi \tag{6}$$

$$= -\boldsymbol{e}_{oi}^T \begin{bmatrix} \sum_{j=1,j\neq i}^{m} k_x^{ij} e_{xj} \frac{(e_{x0i} - e_{xj})}{\sqrt{1 + e_{xoi}^2 + e_{xj}^2}} \\ \sum_{j=1,j\neq i}^{m} k_y^{ij} e_{yj} \frac{(e_{yoi} - e_{yj})}{\sqrt{1 + e_{yoi}^2 + e_{yj}^2}} \\ 0 \end{bmatrix}$$

φ**= 0**, because of the coupling gain choices (7)

Thus,

$$\dot{\boldsymbol{V}}_{\mathbf{2}} = \sum_{i=1}^{m} -\boldsymbol{k}\boldsymbol{e}_{oi}^T \boldsymbol{e}_{oi} \leq 0 \tag{8}$$

Which is negative definite. This shows that the controller in (6) renders the origin of the error dynamics of subsystem S2, i.e. a group of omni wheels mobile robots, globally asymptotically stable (GAS) because of the choice of $k_x^{ij} = k_x^{ji}, k_y^{ij} = k_y^{ji} \geq 0$.

b. Stability proof for S3

For S3, errors are coming from either unicycle or omni wheels. Suppose, that i is a unicycle and j is an omni wheels. Consider a Lyapunof function $\mathrm{V_3 = e_{sys}^T e_{sys}}$. In this case, the $\mathrm{e_{sys} = [e_{ui} \;\; e_{oj}]^T}$. Using the given controller in (5) and (6), the derivative of the Lyapunov function contains the unicycle part and the omni wheels part. For the omni wheels, the results are similar to the results in the subsystem S2 due to the specific structure of the controller and the coupling gain choices.

Similar condition applies for the unicycle part. As presented in [2], the addition of omni wheels does will not affect the Lyapunov condition due to the specific structure of the proposed controller and coupling gain choices. Thus, the derivative of the Lyapunov function is simply:

$$\dot{\boldsymbol{V}}_{\mathbf{3}} = -\boldsymbol{k}\boldsymbol{e}_{oj}^{T}\boldsymbol{e}_{oj} - \boldsymbol{k}\boldsymbol{e}_{ui}^{T}\boldsymbol{e}_{ui} \leq 0 \tag{9}$$

Which is negative definite. This implies that the controller given in (5) and (6) renders the origin of the error dynamics of the mixed system globally asymptotically stable (GAS).

Since in all subsystems S1, S2, and S3 the controllers in (5) and (6) render the origin o the error dynamics $\boldsymbol{e}_{sys}$ GAS, it can be concluded that the controller in (5) and (6) renders the complete mixed unicycle and omni wheels mobile robot group globally asymptotically stable.

3.2. Coordination with Delays in Communication Channels

The second case considers the presence of delays in communication channel. In this case the shared messages that is to be shared is delayed. The shared signal that is originally $e_{\phi j}\ (t), \phi \in \{x, y\}$ (for simplicity is written as $e_{\phi j}$ as used in the previous section), is delayed for τ periode of time. Thus, the shared information is distributed as $e_{\phi j}\ (t-\tau)$, written as $e_{\phi i}(\tau)$, for $\phi \in \{x, y\}$.

As consequences of the existing delays, the controllers are modified as follows:

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} v_{rui}\cos e_{\theta ui} + k_{xu}e_{xui} \\ \omega_{rui} + k_{yu}v_{ru}e_{yui}\frac{\sin e_{\theta ui}}{e_{\theta ui}} + k_{\theta u}e_{\theta ui} \end{bmatrix} + \begin{bmatrix} \sum_{j=1,j\neq i}^{m} k_x^{ij} e_{xj}(\tau)\frac{\left(e_{xui}-e_{xj}(\tau)\right)}{\sqrt{1+e_{xui}^2+e_{xj}^2(\tau)}} \\ \sum_{j=1,j\neq i}^{m} k_y^{ij} e_{yj}(\tau)\frac{\left(e_{yui}-e_{yj}(\tau)\right)}{\sqrt{1+e_{yui}^2+e_{yj}^2(\tau)}} \end{bmatrix} \tag{10}$$

$$\boldsymbol{v_i} = A^{-1}(\dot{\boldsymbol{q}}_{roi} + \boldsymbol{k}\boldsymbol{e}_{oi}) + \begin{bmatrix} \sum_{j=1,j\neq i}^{m} k_x^{ij} e_{xj}(\tau)\frac{\left(e_{xoi}-e_{xj}(\tau)\right)}{\sqrt{1+e_{xoi}^2+e_{xj}^2(\tau)}} \\ \sum_{j=1,j\neq i}^{m} k_y^{ij} e_{yj}\frac{\left(e_{yoi}-e_{yj}(\tau)\right)}{\sqrt{1+e_{yoi}^2+e_{yj}^2(\tau)}} \\ 0 \end{bmatrix} \tag{11}$$

It is to be noted that the problem formulation remains similar to the case without delays. For the given controllers and coordinated tracking problems for a group of mixed wheeld mobile robots, the following theorem holds.

Theorem 2. There exist m-mobile robots, which can be either unicycle or omni wheels mobile robots. The robots are tracking the given references trajectories that in overall create a spatial formation pattern. If the control parameters are chosen so that and $\mathrm{k_x^{ij}} = \mathrm{k_x^{ji}}, \mathrm{k_y^{ij}} = \mathrm{k_y^{ji}} \geq 0$, $\mathrm{k_{xu}}, \mathrm{k_{yu}}, \mathrm{k_{\theta u}}, \mathrm{k_{xo}}, \mathrm{k_{yo}} > 0$ and it is assumed that the shared information is delayed **uniformly**, then the controller given in (10) and (11) renders origin of $\mathbf{e_{sys}} = \{\mathbf{q}_{\mathrm{r1}} - \mathbf{q}_1, \mathbf{q}_{\mathrm{r2}} - \mathbf{q}_2, \dots, \mathbf{q}_{\mathrm{rm}} - \mathbf{q}_{\mathrm{m}}\}$ globally asymptotically stable (GAS).

Proof of **Theorem 2.** Considering the delay-free case, the most important part is to check the value of φ. In the delay-free case, $\varphi = 0$ in all possible subsystem. Since delay occurs in the communication channel between the robots, only φ changes, the rest (part for individual tracking) is similar, regardless the type of the robot in the system.

Using S2, i.e. all omni wheels, as case study, the analysis of φ is given as follows. If φ in delayed system is denoted $\varphi(\tau)$. Using similar Lyapunov function $\mathbf{V_2} = \mathbf{e_{sys}^T}\mathbf{e_{sys}}$, the derivative of the function is given as follows:

$$\dot{\boldsymbol{V}}_{\boldsymbol{2}} = \boldsymbol{e}_{sys}^{\boldsymbol{T}} \dot{\boldsymbol{e}}_{sys} = \sum_{i=1}^{m} \boldsymbol{e}_{oi}^{T} \left(\dot{\boldsymbol{q}}_{roi} - \dot{\boldsymbol{q}}_{oi}\right)$$
$$= \sum_{i=1}^{m} \boldsymbol{e}_{oi}^{T} \left(\dot{\boldsymbol{q}}_{roi} - \left(A \left(A^{-1}\left(\dot{\boldsymbol{q}}_{ro} + \boldsymbol{k}\boldsymbol{e}_{o} + \begin{bmatrix} \sum_{j=1,j\neq i}^{m} k_x^{ij} e_{xj}(\tau) \frac{\left(e_{x0i} - e_{xj}(\tau)\right)}{\sqrt{1+e_{xoi}^2 + e_{xj}^2(\tau)}} \\ \sum_{j=1,j\neq i}^{m} k_y^{ij} e_{yj}(\tau) \frac{\left(e_{yoi} - e_{yj}(\tau)\right)}{\sqrt{1+e_{yoi}^2 + e_{yj}^2(\tau)}} \\ 0 \end{bmatrix}\right)\right)\right) \tag{12}$$
$$= \sum_{i=1}^{m} -\boldsymbol{k}\boldsymbol{e}_{oi}^{T} \boldsymbol{e}_{oi} + \varphi(\tau)$$

$$\varphi(\tau) = -\boldsymbol{e}_{oi}^{T} \begin{bmatrix} \sum_{j=1,j\neq i}^{m} k_x^{ij} e_{xj}(\tau) \frac{\left(e_{x0i} - e_{xj}(\tau)\right)}{\sqrt{1+e_{xoi}^2 + e_{xj}^2(\tau)}} \\ \sum_{j=1,j\neq i}^{m} k_y^{ij} e_{yj}(\tau) \frac{\left(e_{yoi} - e_{yj}(\tau)\right)}{\sqrt{1+e_{yoi}^2 + e_{yj}^2(\tau)}} \\ 0 \end{bmatrix} \tag{13}$$

In the delay-free case, the value of equation (13) is zero because of the structure of the chosen coupling parameters. Similarly, for the delayed case, the value of $\varphi(\tau) = 0$. This, again, is because of the structure of the controller. For clarification, take a look for the tracking error in *x* direction. Define $\sigma(\tau) = \sqrt{1 + e_{xi}^2 + e_{xj}^2(\tau)}$, the component of $\varphi(\tau)$ in *x* direction is given as follows:

$$\begin{aligned} &\varphi_x(\tau) \\ &= -\sum_{j=1,j\neq i}^{m} \left(e_{xi} k_x^{ij} e_{xj}(\tau) \frac{\left(e_{xi} - e_{xj}(\tau)\right)}{\sigma(\tau)} \right) \\ &= -\sum_{j=1,j\neq i}^{m} \left(\frac{k_x^{ij} e_{xi}^2 e_{xj}(\tau)}{\sigma(\tau)} - \frac{k_x^{ij} e_{xi} e_{xj}^2(\tau)}{\sigma(\tau)} \right) \\ &= 0 \end{aligned} \tag{14}$$

Equation (14) is zero because the **uniform delay** in the system makes the value of $e_{xj}^2(\tau)$ or $e_{yj}^2(\tau)$ in all robot remains the same. Combined with the specific control structure, the computation results in zero.

Thus,

$$\dot{\boldsymbol{V}}_{\boldsymbol{2}} = \sum_{i=1}^{m} -\boldsymbol{k}\boldsymbol{e}_{oi}^{T} \boldsymbol{e}_{oi} \leq 0 \tag{15}$$

Which is negative definite and prove that the trajectory of the origin of the error dynamics is globally asymptotically stable (GAS).

Using similar approach and methodology to prove the stability of S1 and S3, the resulting $\varphi(\tau) = 0$, i.e. the derivative of the Lyapunov functions are similar to the case in S1 and S3 without delay.

Since all subsystems are GAS, the the overall system is also GAS under the condition of bidirectional information and uniform delay in the system.

Some remarks regarding the results:

1. It seems that there is no difference between the delayed and non-delayed systems. However, it is only applied for the stability proof.
2. From the perspective of the performance, the delayed case represents the worst-case scenario, i.e. coordination only by means of individual trajectory tracking.
3. In the proposed controller, the delayed signal means that the system “cannot” exchange information so that coordination is achieved only individually.
4. The assumption of uniform delay is acceptable in practice since in a specific network typically transmission delay is similar in all direction.

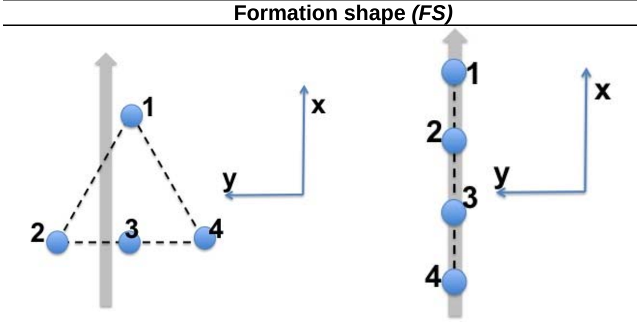
4. Simulation results and analysis
4.1. Control Parameters, Simulation Scenarios and Performance Indicators
The controller is validated by means of simulation. The following control parameters are used:

$$k_{xui} = 0.4, k_{xui} = 100, k_{\theta ui} = 0.5, k_{xoj} = 2, k_{yoj} = 1, k_{\theta oj} = 1,\ k_x^{ij} = k_x^{ji} = 0.06, k_y^{ij} = k_y^{ji} = 10$$

A group of 4 robots is given a task to move in an-8-shape like trajectory as depicted in Figure 3. Different desired formation shape, type of robots in the group and communication topologies are investigated. The summary of parameters choices is given in Table 1.

Table 1. Simulation scenarios

Formation shape *(FS)*



Robot type (*type*)

The sequence indicates the type of the robots in the formation from robot 1 to 4 respectively; 'u' is for unicycle, 'o' is for omni wheels

ID 1: o-o-o-o; ID 2: u-u-u-u; ID 3: u-o-o-o; ID 4: u-o-o-u;
ID 5: u-o-u-o; ID 6: o-u-o-u; ID 7: o-u-u-o

Communication topologies (*com top*)

The bidirectional arros indicates that the robots are communicating.

ID 1: all robots communicates to each other
ID 2: 1 ←→2←→3←→4; ID 3: 1 ←→2←→3←→4←→1
ID 4: 1 ←→2, 1 ←→3, 1 ←→4; ID 5: 1 ←→2, 3 ←→4
ID 6: 1 ←→3, 2 ←→4; ID 7: no communication between robots

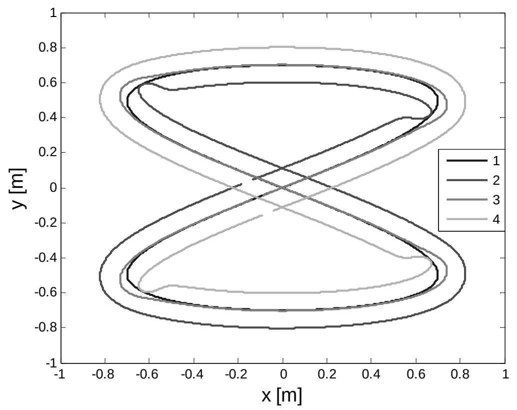


Figure 2. The reference trajectories of the robots

To compare the performance of the controllers in each scenario, a RMS-like performance indicator is used [1, 2]:

$$P_T^{form} = \sum_{i=1}^{m}\sum_{j=i+1}^{m}\sqrt{\frac{1}{l}\sum_{k=1}^{l}\left(\delta_{ij}(k)\right)^2} \quad (8)$$

Where m is the number of robots in the systems, l is the number of data in the simulation/experiments, and $\delta_{ij} = \Delta_{rij} - \Delta_{ij}$ are errors in keeping the relative time varying distance between the robots.

If P_T^{form}=0, it means that all robots maintain the desired relative distances, i.e. the formation is kept. It is to be noted that P_T^{form} can indicate a good formation shape but the one that equals to a rotation mirror of the desired formation shape.

To demonstrate the coordination, during the experiments, at different times, a robot is simulated to drive away from its current position. Thus, the effect of adding coupling gains can be investigated

4.2. Simulation Results and Analysis: Delay-free Cases

Figure 3 shows the example of robot movements in one scenario. The top figure shows the resulting movements where there is no communication between the robots. It can be seen that there is no reaction from other robots, i.e. formation is achieved only by means of trajectory tracking. On the other hand, in Figure 4 it can be observed that one a robot is off the trajectory, other robots reacts to the perturbation in order to keep the overall formation as the references. Less communication sharing means the communication between robots cannot be done peer-to-peer, which means reactions to perturbation maybe delayed or even cannot be executed.

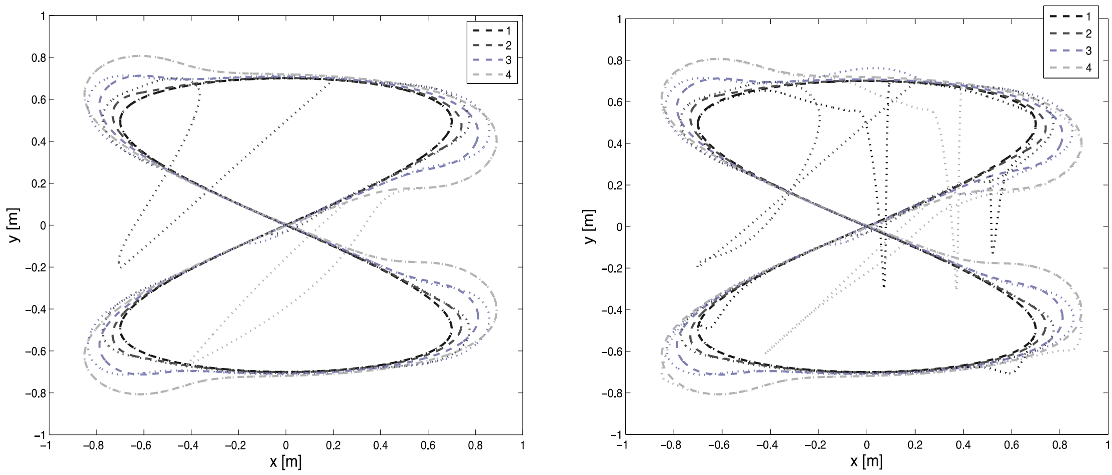


Figure 3. The resulting movements for platoon group 'o-u-u-o'. left: no information sharing; right: fully coupled

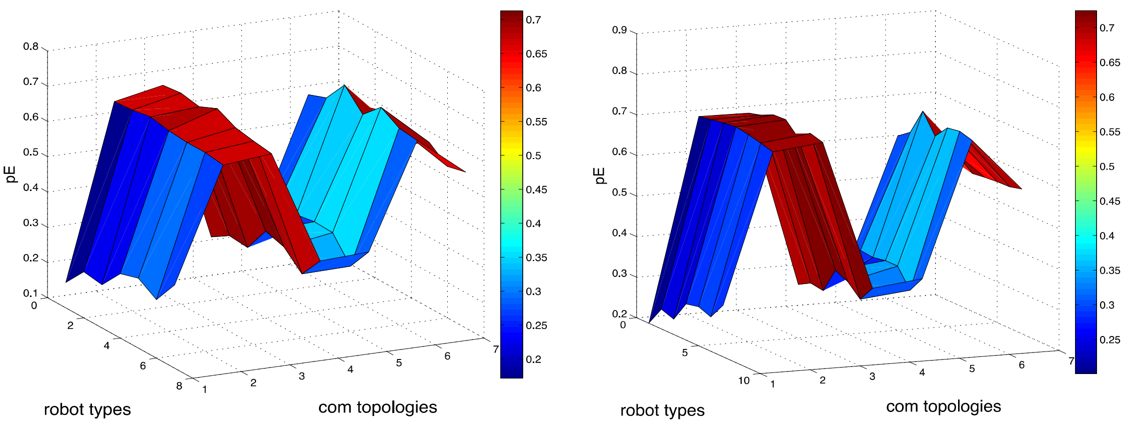


Figure 4. P_T^{form} from simulation, Left: "triangle" formation; right: "platoon" formation

Figure 4 show the values of the performance indicators from the experimental results using the “triangle” and “platoon” formation shape.

The results in Figure 4 shows that for varying robot type in the groups, the controllers also performs differently although the effect cannot be justified clearly. The simulation results sugges that mixed robots tends to perform better compares to all unicycle or all omni wheels robots.

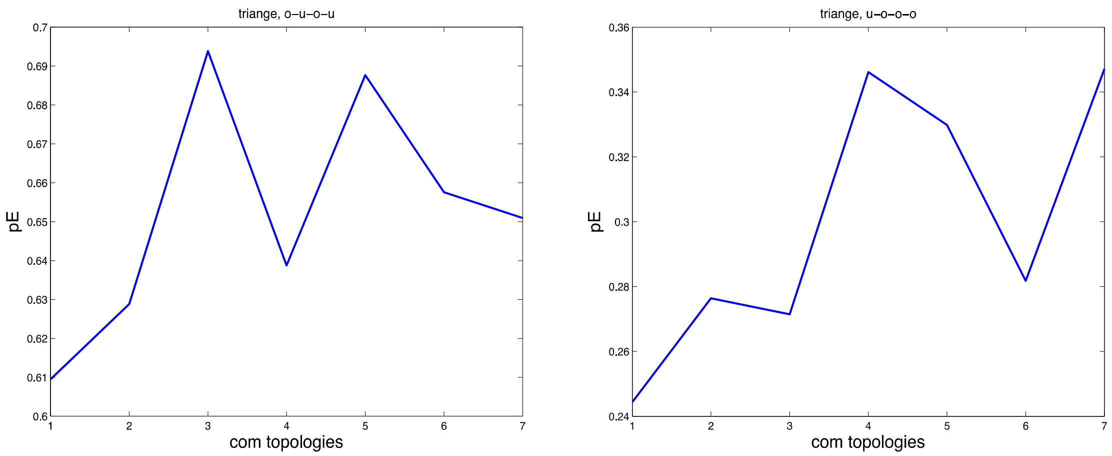


Figure 5. Slices of P_T^{form} from simulation using “triangle” formation. Left: “o-u-o-u”; right: “u-o-o-o”

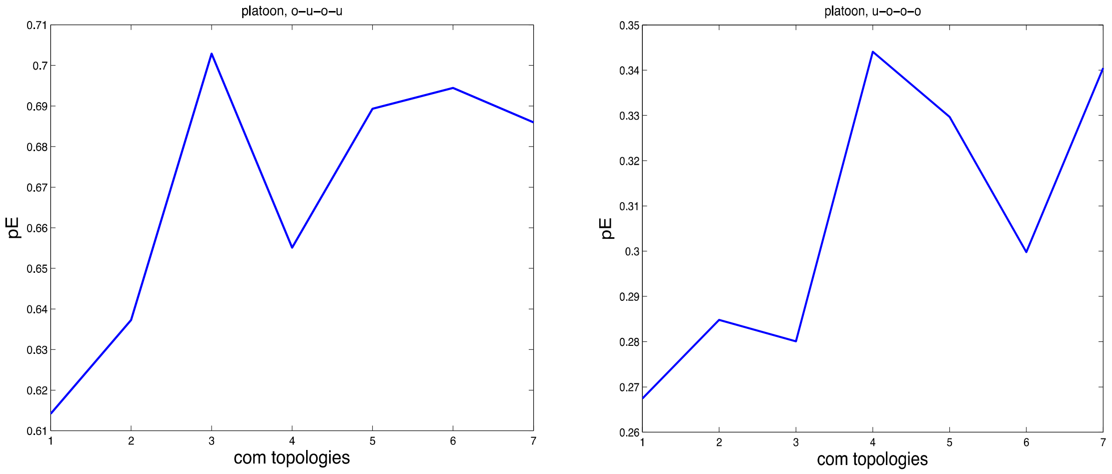


Figure 6. Slices of $P\_T\^form$ from simulation using “platoon” formation. Left: “o-u-o-u”; right: “u-o-o-o”

On the other hand, regardless the robot types and formation shapes, the communication topologies have more influence to the performance. Among all combinations, when all robots communicate, $P\_T\^form$ has the smallest values. At the opposite side, when less robots share individual tracking errors, $P\_T\^form$ tends to grow.

Figure 5 and 6 show some slices of the results shown in Figure 4. These figures indicate that no information sharing does not always means the worst performance. Although adding more information can increase robustness, it has to done correctly according to the desired coordination and type of robot in the group. Thus, to have a strong coordination, it is suggested to have complete communication between the robots. However, this requires a large communication bandwidth.

4.3. Simulation Results and Analysis: Delayed Cases

Figure 7 shows the summary of $P\_T{}^\wedge form$ from simulation using “triangle” and platoon formation with the delay $\tau = 0.5\,T\_s$. The value of T_s represents the sampling time.

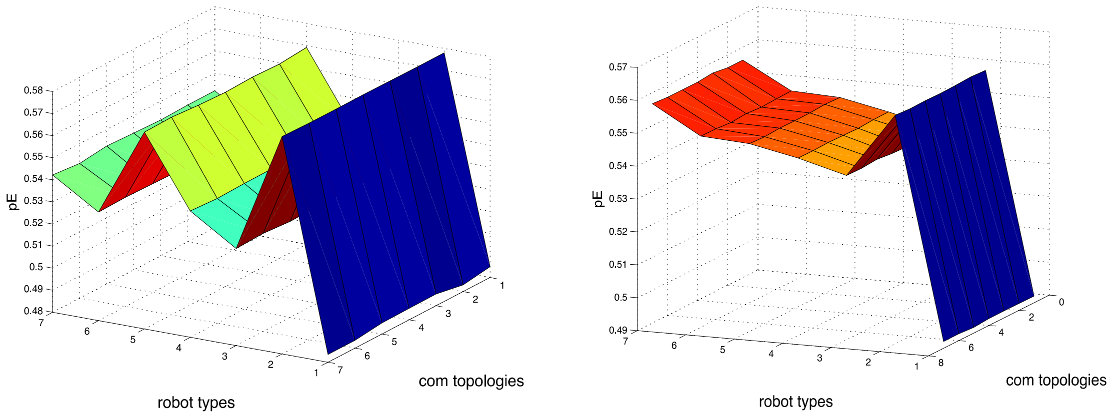


Figure 7. P_T^{form} from simulation using delays in communication channel, Left: “triangle” formation; right: “platoon” formation

The simulation results in Figure 7, both left and right, show that the performance of the system is closed to performance of the systems when there is no information sharing between the robots. As mentioned in remarks of section 3, the specific structure of the controllers and assumptions of uniform delay within the system allows the sytems to be stable. However, with the consequence of having the worst case performance. The simulation results confirm this conclusions.

When communication is delayed, the increasing number of shared information does not increase the performance. The results in Figure 7 indicates that regardless the formation, type of robots, for any communication topologies, the performance is similar to the situation when no information is shared.

It is to be noted that the problem formulation in this research allows the coordination between the robots to be achieved only by means of individual trajectory tracking. The addition of information from other robots is expected to increase robustness againts perturbations.

The results, both for delay-free and delayed cases, show that the proposed control algorithms work well for a group of mixed wheeld mobile robots. Thus, the problem of coordinating individual trajectory tracking is achieved.

5. Conclusion

In this paper we present controllers that achieve globally asymptotically stable of the tracking error dynamics of the mixed group of unicycle and omni wheels mobile robots both in the absent or presence of delays in the communication channel. The coordination between the robots can be achieved by sharing individual tracking errors between the robots. The robots require having a bidirectional communication, i.e. if *i* shares messages to *j,* then *j* has to shares messages to *i.* Simulation results suggest that more information sharing, for delayed-free case, regardless the formation shape and type of robots in the group tends to increase the robustness in coordinating the movements under perturbations. For the delayed case, regardless the formation shape and type of robots, the performance in similar to situation when no robots communicate.

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