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H-WEMA: A New Approach of Double Exponential Smoothing Method

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Abstract

A popular smoothing technique commonly used in time series analysis is double exponential smoothing. Basically, it's an improvement of simple exponential smoothing which does the exponential filter process twice. Many researchers had developed the technique, hence Brown's double exponential smoothing and Holt's double exponential smoothing. Here, we introduce a new approach of double exponential smoothing, called H-WEMA, which combines the calculation of weighting factor in weighted moving average with Holt's double exponential smoothing method. The proposed method will then be tested on Jakarta Stock Exchange (JKSE) composite index data. The accuracy and robustness level of the proposed method will then be examined by using mean square error and mean absolute percentage error criteria, and be compared to other conventional methods.

Keywords: *Holt's double exponential smoothing, H-WEMA, time series analysis, weighted moving average*

1. Introduction

Based on the definition given by Organisation for Economic Co-operation and Development (OECD) Glossary of Statistical Terms, a time series is a set of regular time-ordered observations of a quantitative characteristic of an individual or collective phenomenon taken at successive, in most cases equidistant, periods/ points of time [1]. To comprehend the characteristics of a time series data, many researchers have developed time series analysis methods with the final aim to find a pattern that can be used to forecast future event or data [2-4]. Some researchers even used the soft computing methods, such as fuzzy, neural networks, or hybrid methods to achieve the same goal [5-9].

Moving average is a popular conventional time series analysis method that has been used widely by people due to its easiness, objectiveness, robustness, and usefulness [10, 11]. It is widely employed within the realm of financial analysis, such as stock market. Clif Droke [12] defines a moving average as an indicator that shows the average value of a security's price over a period of time. There are various kinds of moving average methods, but their underlying purpose remain the same, that is to track the trend determination of the given time series data [10, 13]. The simplest one is simple moving average where each point in time series data is weighted the same regardless of where or when it occurs in the sequence. Weighted moving average is another type of moving average which gives a different weighting factor for each point in time series data. Another type of moving average is exponential moving average which is a variation of weighted moving average that used exponential number as the basis in forming weighting factors in time series analysis. Some other researchers even tried to combine the moving average method with other methods, such as autoregressive and neural networks to represent several types of time series data [14-16].

A new approach of moving average method which combines the weighted moving average and exponential moving average methods to forecast the future data had been introduced in 2013 [17, 18]. However the exponential moving average method used in those researches, also known as the single smoothing method, doesn't excel in time series data when there is a trend [19]. Therefore, there is a need to develop the hybrid moving average method to overcome the limitation of forecasting time series data when there is a trend in it.

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In this paper, we will further develop the hybrid method, by modifying and combining the weighted moving average method with Holt's double exponential smoothing method. Holt's double exponential smoothing method is a varian of exponential smoothing method which been widely used to predict the pattern of a time series data with a trend in it. The proposed method will then be tested on Jakarta Stock Exchange (JKSE) composite index data and be compared with other moving average methods, such as weighted moving average method and Holt's double exponential smoothing method. The results then will be compared by using mean square error and mean absolute percentage error criteria to get the accuracy and robustness level of the proposed method compared to the other moving average methods.

2. The Proposed Method

Basically the proposed method will combine the weighted moving average method with Holt's double exponential smoothing method. Therefore we will begin this chapter with the discussion of weighted moving average method.

2.1. Weighted Moving Average

Weighted moving average (WMA) is an improvement form of simple moving average, which gives a greater weight to more recent data than the older ones [20]. The weighting factors are calculated from the sum of days used in time series data, also known as the 'sum of digits' [11]. The formula of WMA can be described as:

$$WMA = \frac{nP_m + (n-1)P_{m-1} + \cdots + 2P_{(m-n+2)} + P_{(m-n+1)}}{n + (n-1) + \cdots + 2 + 1} \quad (1)$$

Where n refers to the period or span number of forecasting formula and P_m refers to the value of time series data at point m [20].

2.2. Holt's Double Exponential Smoothing

Holt's double exponential smoothing, also known as Holt's linear exponential smoothing, is a type of double exponential smoothing widely used by people. This technique not only smooth the trend and the slope directly by using different smoothing constant, but also provides more flexibility in selecting the rates at which trend and slopes are tracked [21].

There are three equations incorporated in this technique [22, 23]:

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1}) \quad (2)$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \quad (3)$$

$$F_{t+k} = L_t + kT_t \quad (4)$$

Where:

Y_t refers to the actual value in time t
α refers to the process smoothing constant, $0 \le \alpha \le 1$
β refers to the trend smoothing constant, $0 \le \beta \le 1$
L_t refers to the smoothed constant process value for period t
T_t refers to the smoothed trend value for period t
F_{t+k} refers to the forecast value for period $t+k$, where $k > 0$
t is the current time period

As suggested by NIST [19], to set the initial values for L_t and T_t we will use the following equations:

$$L_1 = Y_1 \quad (5)$$

$$T_1 = Y_2 - Y_1 \quad (6)$$

2.3. H-WEMA: Holt's Weighted Exponential Moving Average

In this research, we use weighted moving average's weighting factor calculation and combine it with the Holt's double exponential smoothing method. The proposed method will be called Holt's weighted exponential moving average (H-WEMA). The procedures of the proposed method can be described as following steps.

(1) Calculate the base value, B_t, using equation (1) for a given time series data and periods.

(2) Using the base value obtained, calculate the forecasting value using formula (2) – (4), whereas:

$$L_{t-1} = B_{t-1} \tag{7}$$

$$T_{t-1} = B_t - B_{t-1} \tag{8}$$

Will be used to substitute the initial values for L_t and T_t as stated in the Equation (5) and (6).

(3) Return to step (1) until each data point in the time series data given have ended.

In order to know the accuracy and robustness level of the proposed method against other moving average methods, we use two most common criteria, i.e. mean square error (MSE) and mean absolute percentage error (MAPE).

2.4. Mean Square Error

Mean square error (MSE) is the average of the square of error sum between the forecasted data and the real (actual) data. As described by Lawrence et al [24], the formula can be written as follows:

$$MSE = \frac{\sum_{t=1}^{n} e_t^2}{n} \tag{9}$$

Where n denotes the number of data and e^t denotes the forecasting error from $Y_t - \hat{Y}_t$. Here, Y_t is the actual data and $\hat{Y}_t$ is the forecasted data.

2.5. Mean Absolute Percentage Error

Mean Absolute Percentage Error (MAPE) value gives us an indication about how much the average of absolute error of the forecasted data compare to the actual data, and denotes by the formula [24],

$$MAPE = \frac{\sum_{t=1}^{n} \left| \frac{e_t}{Y_t} \right|}{n} \times 100 \tag{10}$$

Where n denotes the number of data and e^t denotes the forecasting error from $Y_t - \hat{Y}_t$. The actual data is denoted by Y_t, while $\hat{Y}_t$ denote the forecasted data.

3. Results and Discussion

The experiment to test the accuracy and robustness level of the proposed method will be done by implementing the proposed method to forecast Jakarta Stock Exchange (JKSE) composite index data. The number of data been used were 100 JKSE data taken monthly from April 2007 to July 2015 from Yahoo! Finance [25]. The length or span data as well as the initial data can be chosen freely by the user. Mean square error and mean absolute percentage error will be used to calculate and compare the accuracy and robustness level of the proposed method against the other two moving average method, i.e. weighted moving average (WMA) and Holt's double exponential smoothing (H-DES).

The interface of the system is shown by Figure 1. User can choose any value for the initial data to start with and span data as described before. As shown in Figure 1, the initial data been used in the first experiment is 29 and the span data been used is 5, which means the forecast calculation will be started from the 30th data period considering the last 5 data taken successively.

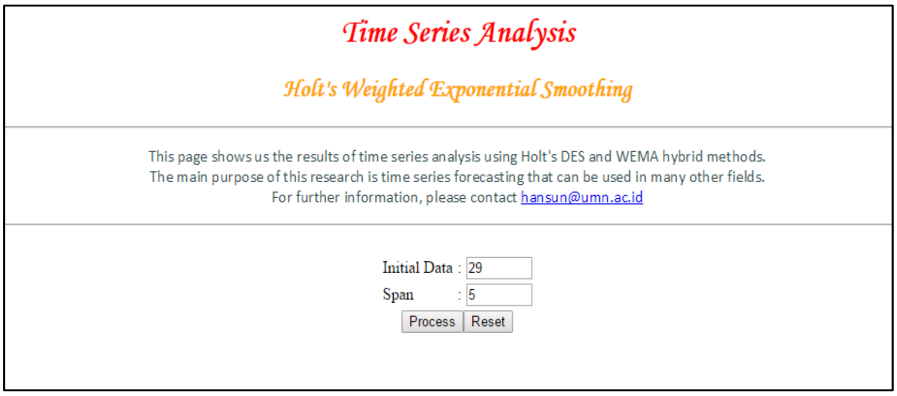

Figure 1. Interface of the system

The graph of forecasted data which had been calculated using weighted moving average (WMA), Holt's double exponential smoothing (H-DES), and Holt's weighted exponential moving average (H-WEMA) are shown on Figure 2, Figure 3, and Figure 4 consecutively. The actual data is denoted by the blue line and the forecasted data is denoted by the red line with a triangle mark on each forecasted point.

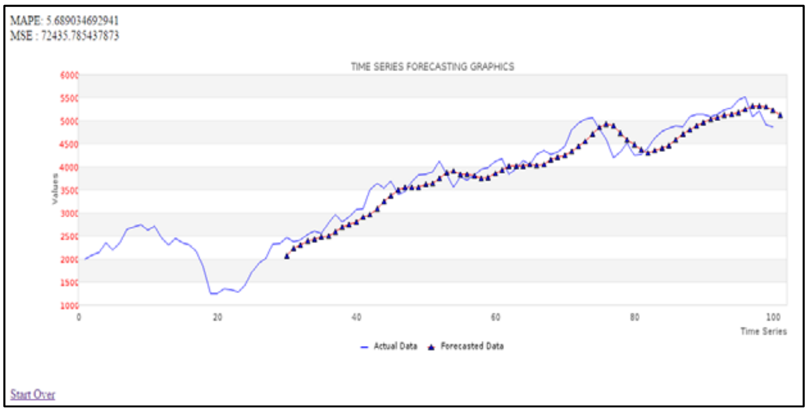

Figure 2. Weighted Moving Average forecasting results

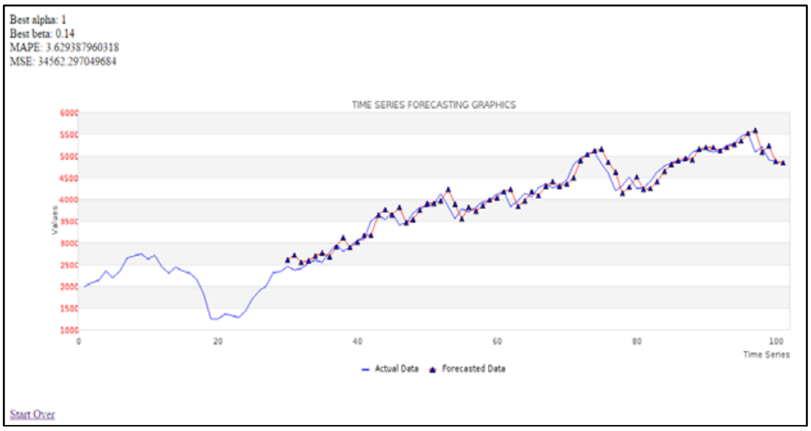

Figure 3. Holt's Double Exponential Smoothing forecasting results

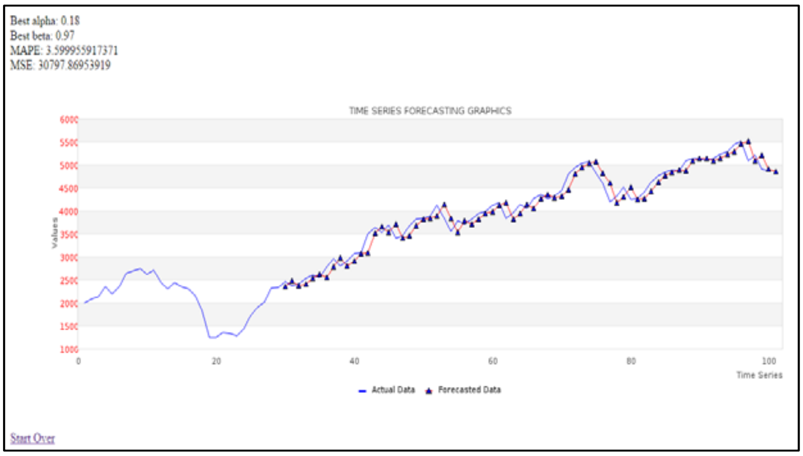

Figure 4. Holt's Weighted Exponential Moving Average forecasting results

The experiments then be continued by using different number of initial data for each moving average method. We will use 10 different numbers of initial data and calculate the accuracy and robustness level using mean square error (MSE) and mean absolute percentage error (MAPE) criteria as can be seen in Table 1.

Table 1. MSE and MAPE comparison of each method

∑ of initial data	MSE			MAPE		
	WMA	H-DES	H-WEMA	WMA	H-DES	H-WEMA
9	92683.89	33536.94	32754.80	8.7223	4.5829	4.7246
14	95814.61	35005.28	32699.05	8.8300	4.8062	4.6059
19	87023.81	38260.21	29973.83	7.8731	4.9383	4.0256
24	83357.75	32254.71	31092.93	6.8425	3.7853	3.8525
29	72435.79	34562.30	30797.87	5.6890	3.6294	3.6000
34	74278.44	33061.38	32411.76	5.5308	3.2849	3.5962
39	75874.49	37851.70	32802.82	5.4092	3.5967	3.4391
44	69022.13	33952.80	31735.53	4.9599	3.0839	3.3439
49	71607.38	36439.93	31230.47	4.9678	3.2198	3.1808
54	71058.42	39563.08	29629.27	4.8121	3.4138	3.0225
Average	**79315.67**	**35448.83**	**31512.83**	**6.36367**	**3.83412**	**3.73911**

Table 1 shows us the different MSE and MAPE values for each method and each experiment. As can be seen on the table, the average MSE and MAPE values of the proposed method are the smallest among the other methods, which means that H-WEMA gives a better forecasting results (better accuracy and robustness) rather than WMA and H-DES method. Therefore, the proposed method can be used as a better forecasting tool in time series analysis rather than the other two moving average methods.

4. Conclusion

In this paper, we develop a new approach of moving average method, which combines the basic formula of weighted moving average (WMA) to get a base value, and use the base value to get the forecasted value using Holt's double exponential smoothing (H-DES) formula.

The experimental results on 100 Jakarta Stock Exchange (JKSE) composite index data show a promising result. The accuracy and robustness level of the proposed method excels both the weighted moving average and the Holt's double exponential smoothing methods, as can be concluded from the small mean square error and mean absolute percentage error values.

For the future research, we can try to take a more comprehensive study to analyze the advantages and disadvantages of the proposed method compare to other hybrid moving average method, such as the weighted exponential moving average (WEMA) method and autoregressive integrated moving average (ARIMA) method. Another study to combine other moving average methods, such as Holt-Winters triple exponential smoothing can also be taken in the future.

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