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Low Complexity Sparse Channel Estimation Based on Compressed Sensing

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Abstract

In wireless communication, channel estimation is a key technology to receive signal precisely. Recently, a new method named compressed sensing (CS) has been proposed to estimate sparse channel, which greatly improves the spectrum efficiency. However, it is difficult to realize it due to its high computational complexity. Although the proposed Orthogonal Matching Pursuit (OMP) can reduce the complexity of CS, the efficiency of OMP is still low because only one index is identified per iteration. Therefore, to solve this problem, more efficient schemes are proposed. At first, we apply Generalized Orthogonal Matching Pursuit (GOMP) to channel estimation, which lower computational complexity by selecting multiple indices in each iteration. Then a more effective scheme that selects indices by least squares (LS) method is proposed to significantly reduce the computational complexity, which is a modified method of GOMP. Simulation results and theoretical analysis show the effectivity of the proposed algorithms.

Keywords*: channel estimation, compressed sensing, computational complexity, index, atom*

1. Introduction

The performance of wireless communication considerably depends on wireless channel. Due to the complexity and variability of geographical environment, the propagation signal is likely to undergo multipath propagation and Doppler shift, generating frequency selective fading and time selective fading which distort the signal severely [1]. In order to obtain accurate receipt signal, it is necessary to acquire the exact channel state information (CSI). Therefore, we need to know the precise channel impulse response (CIR) firstly. So far, channel estimation using reference signals or pilots is the most common method to obtain CIR [2], which transmits a group of known signal, that is, reference signals or pilots, and then carries out channel estimation based on the variation between receiving signal and transmitting signal, lastly obtaining the CIR.

Classical pilots based channel estimation methods include least squares (LS), minimum meansquared error (MMSE), and discrete Fourier Transform (DFT). Noteworthily, traditional methods don't consider the sparsity of wireless channel, which result in huge waste of spectrum resource. With the development of research, a lot of literatures have shown that the wireless channels are generally sparse in practice. The CS based channel estimation utilizes the channel sparsity, reduces the number of pilots and improves the spectrum efficiency finally. Many researchers have proposed a lot of algorithms such as l_1-norm minimization based Basis Pursuit (BP) algorithm [3], greedy iteration based Matching Pursuit (MP) algorithm and Orthogonal Matching Pursuit (OMP) algorithm [4, 5]. BP algorithm has high reconstruction accuracy, but its computation cost is huge. Compared with BP algorithm, MP algorithm improves the computational complexity, but weakens its robustness. OMP is a modified method of MP algorithm, which executes an orthogonalization of the selected atoms. The OMP algorithm can bring a fast convergence and good robustness as well. In addition, OMP algorithm has advantages of high reconstruction accuracy and computation speed.

Recently, most researchers have paid their main attention on the improvement of estimation accuracy. Aiming at Ultra Wideband (UWB) channel, A.H. Muqaibel proposed more practical dictionaries to enhance the sparsity of UWB channel, and then improved the accuracy

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of received signal [6]. S. Pramono has improved the estimation accuracy through applying multiple input multiple output (MIMO) to channel estimation [7]. In literatures [8], a reconfigurable and sparse array antenna was proposed, which can also improve the estimation accuracy. Additionally, many other researchers have pursued high estimation accuracy by optimizing pilots' settings [9, 10].

In fact, the channel estimation accuracy has reached a relatively high level by utilizing the existed algorithms. So the computational complexity of algorithms already becomes the main concern in practice. However, up to now, only a few literatures about reducing computational complexity have been released. A modified OMP (MOMP) was proposed in [11], it not only reduces the computational complexity effectively, but also has a good estimation accuracy. But it assumes all channel taps distributing adjacently, which limits its application in practice. In [12], an expander graph based CS was proposed to sparse channel, which is benefit to reduce the computational complexity to $O((M-N)N)$, where M and N are the length of pilots and channel, respectively. But the method in [12] is sensitive to noise.

Therefore, a further research is needed to solve the problem of high computational complexity. As mentioned previously, OMP algorithm has advantages of high reconstruction accuracy and computation speed, but for practical application, its computational complexity is still too high. Hence, in this paper, we do a further research on the basis of OMP algorithm to reduce the computational complexity and ensure the estimation precision at the same time. Firstly, we apply Generalized Orthogonal Matching Pursuit (GOMP) algorithm [13] to sparse channel estimation, which is a modified method of OMP. Owing to the selection of multiple atoms per iteration, the calculating speed is improved and the estimation accuracy is guaranteed at the same time. Then based on the idea of GOMP algorithm, a better algorithm named M-GOMP is proposed. M-GOMP selects atoms with a new perspective, avoiding repeating iterations of the greedy algorithm, which can greatly reduce the computational complexity. A number of computer simulations are conducted, showing a better computation speed and a good reconstruction accuracy of the both algorithms.

The rest of this paper is organized as follows. Section 2 briefly depicts the CS theory and the problem is stated in Section 3. In Section 4, we propose the efficient schemes of index selection, namely, the GOMP and M-GOMP algorithms. Section 5 presents computer simulations and complexity analysis. The conclusion is drawn in Section 6 finally.

In this paper, bold upper case and bold lower case letters denote matrices and vectors, respectively. $\boldsymbol{A}_T$ denotes the submatrix of $\boldsymbol{A}$ restricted to columns indexed by the set T. $\boldsymbol{A}^H$, $\boldsymbol{A}^T$, and $\boldsymbol{A}^{\dagger}=(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H$ denote the hermitian transposition, transposition, and pseudo-inverse of matrix $\boldsymbol{A}$, respectively. $\|\boldsymbol{t}\|_2$ denotes l_2-norm of $\boldsymbol{t}$. $\langle \boldsymbol{A},\boldsymbol{h}\rangle$ denotes the inner product of matrix $\boldsymbol{A}$ and vector $\boldsymbol{h}$. $\hat{\boldsymbol{h}}$ denotes the estimated value of $\boldsymbol{h}$.

2. Compressed Sensing Based Reconstruction

The proposition of CS is a revolutionary breakthrough in signal processing. It breaks the restriction of the Nyquist sampling theorem and improves the spectrum efficiency greatly [14, 15]. The premise of CS application is that the observed signal is sparse or compressible, which means there are only a few nonzero values or significant values in a group of observed signal. Fortunately, a lot of statistics and observation data indicate that the wireless channel is generally sparse, which lays foundation of the CS application. Standard CS measurement model is given as follows:

$$\boldsymbol{r}=\Phi\boldsymbol{t}+\boldsymbol{\omega} \tag{1}$$

Where $\boldsymbol{r}$ is a $M\times 1$ observed vector, Φ is a $M\times N$ measurement matrix, $\boldsymbol{t}$ is a $N\times 1$ K-sparse signal with K nonzero values, $\boldsymbol{\omega}$ is a $M\times 1$ noise vector whose elements are additive white Gaussian variables. CS is mainly about how to reconstruct original signal $\boldsymbol{t}$ from the known $\boldsymbol{r}$ and Φ. Since $M<N$ for most of the compressive sensing scenarios, reconstructing $\boldsymbol{t}$ directly from (1) is an underdetermined problem. Fortunately, that signal is sparse makes it possible to solve this problem. The literature [16] pointed out that if Φ satisfies Restricted Isometry Property (RIP), then $\boldsymbol{t}$ can be reconstructed accurately. We write RIP as lemma 1.

Lemma 1: For any K -sparse signal $\boldsymbol{t}$, if there exists a constant $\delta_K \in (0,1)$ such that:

$$(1-\delta_K)\|\boldsymbol{t}\|_2^2 \le \|\Phi\boldsymbol{t}\|_2^2 \le (1+\delta_K)\|\boldsymbol{t}\|_2^2 \quad (2)$$

Then Φ satisfies RIP.

In the algorithms proposed in this paper, the number of measurements M is a very important factor. If M satisfies:

$$M \ge \rho(K+1)(\sqrt{K}+1)^2 \log\frac{N}{K+1} \quad (3)$$

Where $\rho \in (0,1)$ is a constant, then Φ obeys the RIP condition with overwhelming probability [17]. This is the lower bound of M on the premise of accurate reconstruction. However, the magnitude ρ is not given in [17], so the exact lower bound of M is not determined either.

With numerous experiment analyses, Tropp and Gilbert pointed out that we can recover $\boldsymbol{t}$ from (1) with large probability if M is expressed as follows [18]:

$$M \approx K \log N \quad (4)$$

3. Problem Statement

Consider a M pilots, N length and K -sparse channel system. The positions of pilots are denoted by $k_0, k_1, \cdots, k_{M-1}$. CS based sparse channel estimation can be expressed as:

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{F}_{M\times N}\boldsymbol{h} + \boldsymbol{\omega} \quad (5)$$

Where, $\boldsymbol{X} = diag[x(k_0), x(k_1), \cdots, x(k_{M-1})]$ is the diagonal matrix of transmitted pilots. $\boldsymbol{y} = [y(k_0), y(k_1), \cdots, y(k_{M-1})]^T$ and $\boldsymbol{h} = [h(0), h(1), \cdots, h(N-1)]^T$ are the $M\times 1$ received pilot vector and $N\times 1$ sparse channel vector, respectively. There are K nonzero values in $\boldsymbol{h}$. $\boldsymbol{\omega}$ is a $M\times 1$ complex additive white Gaussian noise vector. $\boldsymbol{F}_{M\times N}$ is a $M\times N$ partial Fourier matrix, which is obtained by selecting the rows of Fourier matrix with indices $k_0, k_1, \cdots, k_{M-1}$. $\boldsymbol{F}_{M\times N}$ can be expressed as:

$$\boldsymbol{F}_{M\times N} = \begin{bmatrix} f_V^{k_0 0} & f_V^{k_0 1} & \cdots & f_V^{k_0 (N-1)} \\ f_V^{k_1 0} & f_V^{k_1 1} & \cdots & f_V^{k_1 (N-1)} \\ \vdots & \vdots & \vdots & \vdots \\ f_V^{k_{M-1} 0} & f_V^{k_{M-1} 1} & \cdots & f_V^{k_{M-1} (N-1)} \end{bmatrix} \quad (6)$$

Where $f_V^{k_p q} = \exp(-j2\pi k_p q / V)$, V is a positive integer, $0 \le p \le M-1$, $0 \le q \le N-1$. Let $\boldsymbol{A} = \boldsymbol{X}\boldsymbol{F}_{M\times N}$, (5) can be written as:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{h} + \boldsymbol{\omega} \quad (7)$$

Where, $\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_N]$ is a $M\times N$ matrix. $\boldsymbol{a}_i$ is the column of matrix $\boldsymbol{A}$. Comparing (7) with (1), we can see that $\boldsymbol{y}$, $\boldsymbol{A}$, $\boldsymbol{h}$ are the observed vector, measurement matrix and sparse signal in CS, respectively. After obtaining the observed vector $\boldsymbol{y}$, by adopting a certain algorithm, we can reconstruct the channel $\boldsymbol{h}$ from $\boldsymbol{y}$.

The OMP algorithm is a commendable method for channel reconstruction. It is one of the greedy algorithms, which can efficiently and stably recover sparse signal from observed value. The following is the detailed steps of the OMP algorithm.

Algorithm 1: OMP Algorithm

Input: received pilots $\boldsymbol{y}$, measurement matrix $\boldsymbol{A}$, sparsity K , noise variance σ^2 .

Initialize: residual $\boldsymbol{r}_0 = \boldsymbol{y}$, estimated support set $\Lambda_0 = \varnothing$, iteration count $t = 1$.

Repeat: Assume in the t_{th} iteration

1: $k = k + 1$.

2: Identification: $\lambda_t = \arg\max_j \left|\left\langle \boldsymbol{r}_{t-1}, \boldsymbol{a}_j \right\rangle\right|, \boldsymbol{a}_j \subset \boldsymbol{A}$.

3: Support Merger: $\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}$.

4: Estimation: utilize the LS method: $\hat{\boldsymbol{h}}_t = (\boldsymbol{A}_{\Lambda_t}^H \boldsymbol{A}_{\Lambda_t})^{-1} \boldsymbol{A}_{\Lambda_t}^H \boldsymbol{y}$.

5: Residual Update: $\boldsymbol{r}_t = \boldsymbol{y} - \boldsymbol{A}_{\Lambda_t} \hat{\boldsymbol{h}}_t$.

Until: $\|\boldsymbol{r}_t\|_2 < \sigma^2$ or $t \geq K$.

Output: output the estimated value $\hat{\boldsymbol{h}}$.

Carefully studying the OMP algorithm, we can find that the most important part of the whole algorithm is to find the K atoms corresponding to nonzero values in channel $\boldsymbol{h}$, which can be called as matching atoms. As long as the K matching atoms are found, we can achieve the accurate reconstruction of $\boldsymbol{h}$ through LS method given in step 4. At the same time, the most time of the OMP algorithm are spent on seeking matching atoms, namely, the identification step, whose efficiency is quite low. We illuminate it by (8):

$$C = \left\{ \lambda_1, \lambda_2, \cdots, \lambda_N \,\middle|\, \lambda_i = \arg\left|\left\langle \boldsymbol{r}_t, \boldsymbol{a}_i \right\rangle\right| \right\} \quad i = 1, 2, \cdots N \tag{8}$$

Where C is an collection of N indices and one index in C correspond to one atom in $\boldsymbol{A}$. Generally, the value N is large, so obtaining C by step 2 will cost plenty of time. However, even so, OMP algorithm doesn't make full use of C. In each iteration, only one of the biggest indices is selected with the other indices abandoned completely, which not only wastes resource, but also makes the algorithm converge slowly. With the increase of iterations, the disadvantages of the OMP algorithm are more highlighted.

4. Efficient Schemes of Indices Selection

Before giving the proposed schemes, we firstly present two theorems as follows:

Theorem 1: In the algorithm of this paper, the selected indices $\lambda_i, \lambda_i \in \Lambda_t, i \leq N$ will not be selected in the later iteration.

Proof: The theorem 1 shows that when the iteration runs, we don't have to worry about the selected atoms to be selected again, including the non-matching atoms.

For the convenience of description of theorem 2, we give an expression of the location of matching atoms at first.

$$T = \{ i \,|\, h_i \neq 0, i \leq N \} \tag{9}$$

Where h_i is a tap of channel $\boldsymbol{h}$.

Theorem 2: (Assuming there has been executed K iterations in total): The channel $\boldsymbol{h}$ can be perfectly reconstructed when the resulting support set Λ_K and the matching atoms set T satisfy relation as follows:

$$T \subset \Lambda_K \tag{10}$$

Specifically, (10) has two meanings: 1) The support set Λ_K must contain all elements of the matching atoms set T. 2) The support set Λ_K may contain other elements besides the matching atoms set T.

4.1. GOMP Algorithm

The proposition of GOMP algorithm efficiently improves the deficiency of the OMP algorithm. Different from the OMP algorithm, the GOMP algorithm selects $n\,(n \geq 1)$ atoms per iteration, that is, selecting n indices from C. Then GOMP performs sparse estimation and residual update. In this way, we can not only make full use of the indices collection C, but also accelerate the convergence of algorithm, and improve the algorithm efficiency as a whole. We have lemma 2 with respect to the value n.

Lemma 2: (GOMP algorithm): Suppose selecting n atoms per iteration, then the GOMP algorithm. Can realize recovering the original sparse channel under condition:

$$n \leq \min\{K, \frac{M}{K}\} \tag{11}$$

Where K and M are the sparsity of channel and the number of pilots, respectively [13].

The whole procedure of the GOMP algorithm is sketched as follows:

Algorithm 2: GOMP Algorithm

Input: received pilots $\boldsymbol{y}$, measurement matrix $\boldsymbol{A}$, sparsity K, noise variance σ^2, the number of indices selection per iteration n.

Initialize: residual $\boldsymbol{r}_0 = \boldsymbol{y}$, estimated support set $\Lambda_0 = \varnothing$, iteration count $t = 1$.

Repeat: Assume in the t_{th} iteration

1: $k = k + 1$.

2: Identification: $S =$ indices of the n highest amplitude components of $\boldsymbol{A}^T \boldsymbol{r}_{t-1}$.

3: Support Merger: $\Lambda_t = \Lambda_{t-1} \cup S$.

4: Estimation: utilize the LS method : $\hat{\boldsymbol{h}}_t = (\boldsymbol{A}_{\Lambda_t}^H \boldsymbol{A}_{\Lambda_t})^{-1} \boldsymbol{A}_{\Lambda_t}^H \boldsymbol{y}$

5: Residual Update: $\boldsymbol{r}_t = \boldsymbol{y} - \boldsymbol{A}_{\Lambda_t} \hat{\boldsymbol{h}}_t$.

Until: $\|\boldsymbol{r}_t\|_2 < \sigma^2$ or $t \geq K$.

Output: output the estimated value $\hat{\boldsymbol{h}}$.

To make it easier to understand, we give the schematic representation of the GOMP algorithm in Figure 1.

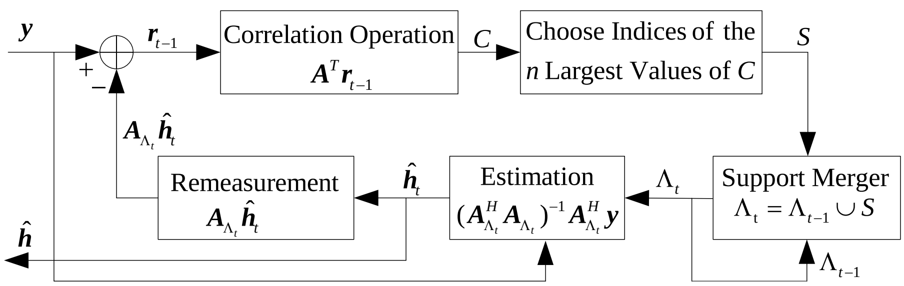

Figure 1. Schematic representation of the GOMP algorithm

Comparing the GOMP algorithm with the OMP algorithm, we find that the most different part is the step 2, which is the core component of the GOMP algorithm. Noteworthily, because more than one atom is selected in each iteration, we inevitably put the non-matching atoms in. Even so, considering theorem 2, we still can recover the sparse channel accurately. In addition, because the GOMP algorithm selects n atoms per iteration, generally, the sparse channel can be reconstructed with K/n loops, which greatly saves the running time of algorithm.

4.2. M-GOMP Algorithm

Based on the idea of GOMP algorithm, we propose a more efficient scheme, that is, M-GOMP algorithm. Comparing the GOMP algorithm with the OMP algorithm, we can see that the key of the two algorithms is to search the matching atoms. As long as all the matching atoms are acquired, we can achieve accurate reconstruction by the step 4 of the OMP algorithm or the GOMP algorithm. Hence, the most critical thing now is how to get all the matching atoms with no plentiful time costing at the same time. Considering the simplicity of LS algorithm, we use the LS algorithm to search atoms, and then estimate the sparse channel by the step 4 of GOMP, which generates the M-GOMP algorithm. The detailed implementation of the M-GOMP algorithm is shown as follows:

Algorithm 3: M-GOMP Algorithm
Input: received pilots $\boldsymbol{y}$, measurement matrix $\boldsymbol{A}$, sparsity K.
Initialize: estimated support set $\Lambda = \varnothing$.
Begin:
1: Identification: solving a simple LS estimation $\hat{\boldsymbol{h}}_0 = \boldsymbol{A}^{-1}\boldsymbol{y}$, let $\Lambda =$ indices of the K highest amplitude components of $\hat{\boldsymbol{h}}_0$.
2: Estimation: utilize the LS method to reconstruct: $\hat{\boldsymbol{h}} = (\boldsymbol{A}_\Lambda^H \boldsymbol{A}_\Lambda)^{-1} \boldsymbol{A}_\Lambda^H \boldsymbol{y}$
End
Output: output the estimated value $\hat{\boldsymbol{h}}$.

Comparing M-GOMP algorithm with the OMP and GOMP algorithms, we can see that the M-GOMP algorithm is quite different from the OMP and GOMP algorithms. The M-GOMP algorithm only has two steps with identification and estimation, and what's more, there is no loops in M-GOMP. For the convenience to understand, we give the schematic representation of the M-GOMP algorithm in Figure 2.

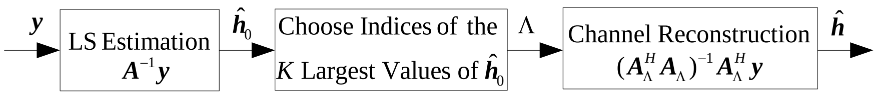

Figure 2. Schematic representation of the M-GOMP algorithm

From Figure 2 we can see the M-GOMP algorithm performs only one iteration from left to right. Hence it can greatly improve the efficiency of reconstruction. In terms of estimation precision, because we can acquire the required K atoms from estimated $\hat{\boldsymbol{h}}_0$, we are able to realize accurate reconstruction by the step 2 in M-GOMP.

In this paper, we also adopt the commonly used LS algorithm and the high accuracy estimated MMSE algorithm as the references of performance analysis. Here, first of all, we put down their expressions as follows:

$$\hat{\boldsymbol{h}}_{LS} = (\boldsymbol{A}^H \boldsymbol{A})^{-1} \boldsymbol{A}^H \boldsymbol{y} \tag{12}$$

$$\hat{\boldsymbol{h}}_{MMSE} = \boldsymbol{R}_{HH} (\boldsymbol{R}_{HH} + \sigma^2 (\boldsymbol{A}^H \boldsymbol{A})^{-1})^{-1} \boldsymbol{A}^{-1} \boldsymbol{y} \tag{13}$$

Where $\boldsymbol{R}_{HH}$ denotes the autocovariance matrix of channel vector $\boldsymbol{H}$, $\boldsymbol{H}$ is the frequency domain representation of $\boldsymbol{h}$ and σ denotes the standard deviation of noise.

5. Simulations and Analyses

In this section, the Mean Square Error (MSE) simulation, running time simulation and complexity analysis are described to verify the good performance of the proposed GOMP and

M-GOMP algorithms. The classical methods such as LS algorithm, MMSE algorithm, OMP algorithm are also applied for comparison at the same time.

5.1. MSE Simulation

In this simulation, we consider a sparse channel with length $N = 496$, sparsity $K = 12$, and utilize Gaussian pilots, whose number is determined by (4). In order to guarantee the accurate reconstruction, we choose $M = 256$. The noise is complex additive white Gaussian noise. Additionally, considering the lemma 2, we choose the number of indices selection with $n = 2,5,9$ in GOMP algorithm. In order to avoid the interference caused by randomicity of signal, we adopt the idea of taking average with multiple loops. In addition, normalized MSE is adopted in the simulation. The simulation results are shown in Figure 3 and Figure 4.

Figure 3 is the MSE simulation under different SNR. As can be seen from Figure 3, the MSE curves of different algorithms decrease when the SNR increases, which means an improvement of channel reconstruction accuracy. Additionally, compared with the classical algorithms, the reconstruction accuracies of the proposed GOMP and M-GOMP algorithms are not only far better, but also very close to the OMP algorithm. Even when $n = 2$, the MSE curve of GOMP algorithm completely catches up with that of OMP algorithm. All of these demonstrate the effectivity of the proposed algorithms.

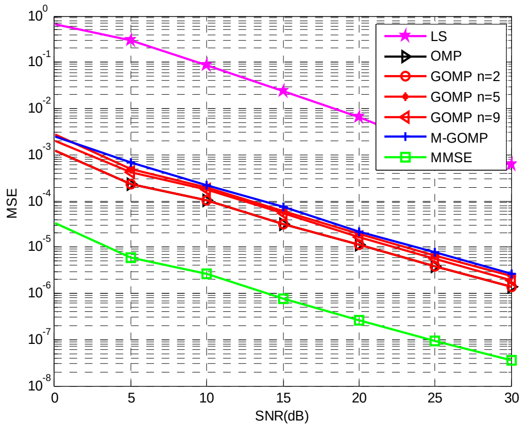

Figure 3. The comparison on MSE with varying SNR

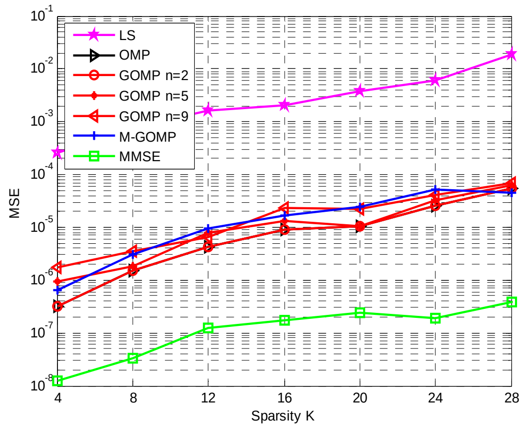

Figure 4. The comparison on MSE with varying sparsity

Figure 4 is the MSE performance simulation with varying sparsity. From Figure 4, we can see that the channel reconstruction accuracy decreases with K increasing. This phenomenon can be explained by the increase of channel density. When the channel density increases, the number of significant taps becomes larger. However, the number of pilots doesn't increase at the same time, which makes us not able to get enough channel information and leads to the increase of estimated errors finally. It's worth noting that with the increasing of sparsity K, although the performance of the proposed algorithms is declined, the MSE curves of the proposed algorithms have followed the OMP well all the time. It implies both of the proposed algorithms have robustness against different channel sparsity. In addition, from Figure 3 and Figure 4, we can see that the estimation accuracy of the MMSE algorithm is the best. However, due to its high computational complexity, we generally can't utilize it in practice. But we can take the MSE curve of MMSE algorithm as the lower bound of estimation accuracy.

Combining Figure 3 with Figure 4, we can conclude that the MSE of the GOMP algorithm becomes worse with the increase of n. This is mainly because when n is larger, the GOMP algorithm will involve more nonmatching atoms. Theoretically, we can utilize (16) to demonstrate. When the value n is larger, the value ϑ_2 in (16) cannot be ignored, whose existence leads to the decline of estimated accuracy. However, it is also likely to show up a special case. When the value n is smaller, the corresponding MSE is higher, as shown in Figure 3. It is because the GOMP algorithm with smaller n executes more times of iteration, and then puts more nonmatching atoms in.

5.2. Running Time Simulation

In this section, we mainly consider the running time simulation with varying channel length. The simulation parameters are set as $\text{SNR} = 5\,\text{dB}$, $M = 512$, $K = 12$, the simulation results are shown in Figure 5. Due to the high complexity of MMSE algorithm, we don't consider the MMSE algorithm in the simulation.

As is shown in Figure 5, compared with the OMP algorithm, the proposed algorithms have good performance. Among them, the GOMP algorithm with $n = 5,9$ cost only about $1/2$ running time as much as that of the OMP algorithm, namely, they can save the running time by more than 50%. What's more, the M-GOMP algorithm can save the running time by more than 85% under different channel lengths, which proves the M-GOMP algorithm to be more superior. These significant improvements mainly owes to the efficient schemes of indices selection.

Besides, the average iterations simulation with varying sparsity is also given in Figure 6. From Figure 6 we can see, the average iterations of the proposed algorithms are less than that of the OMP algorithm under different sparsity. The computational complexity of algorithm is closely related to the number of iterations, so the simulation results of Figure 6 not only verify the superiority of the GOMP and M-GOMP algorithms again, but also demonstrate the robustness of both algorithms at the same time.

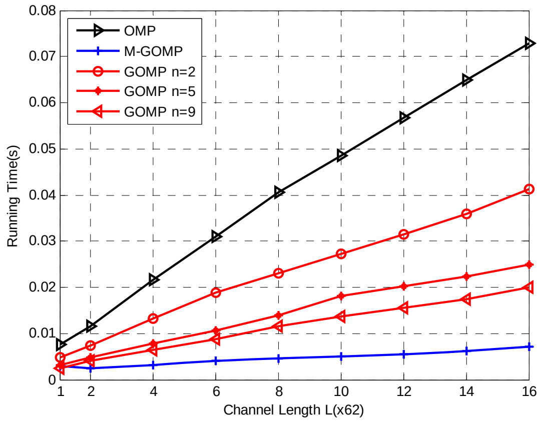

Figure 5. The comparison on running time with varying channel length

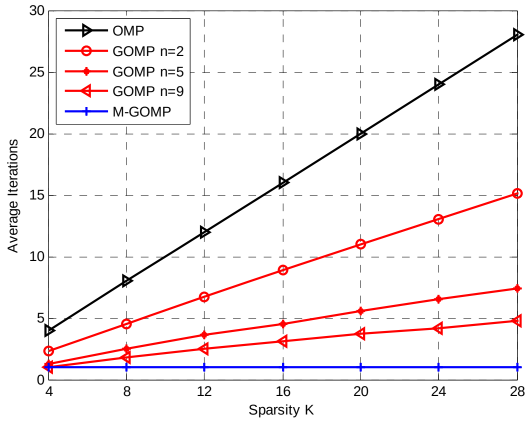

Figure 6. The comparison on average iterations with varying sparsity

5.3. Complexity Analysis

In this section, we analyze the computational complexity of the GOMP and M-GOMP algorithms, to prove the superiority of the two algorithms quantitatively. The literature [13] pointed out that the computational complexity of GOMP algorithm can be approximately expressed as $C_{GOMP} \approx 2sMN + (2n^2 + n)s^2M$, where s is the number of iterations. Because the OMP algorithm is a special case of the GOMP algorithm with $n = 1$, the complexity of the OMP algorithm can be expressed as $C_{OMP} \approx 2KMN + 3K^2M$. In [13], a multiplication and an addition are regarded as a basic operation, respectively. Based on the same principle, we discuss the computational complexity of the M-GOMP algorithm.

Identification: This step has high complexity due to the involvement of matrix inversion, so we figure out the channel frequency response $\boldsymbol{H}$ firstly, and then convert $\boldsymbol{H}$ to time domain by IFFT. By this way, the total number of operations is $2M + (2M-1)N$. In addition, $\hat{\boldsymbol{h}}_0$ needs to be sorted to find K best indices, which requires $(NK - K(K+1))/2$ operations.

Estimation: This step is also involved with matrix inversion, therefore, we utilize the QR factorization and modified Gram-Schmidt (MGS) algorithm to get a fast solution, which leads to a complexity with $2K^2M + 5KM + K^3 + 4K^2 - K$.

The whole M-GOMP algorithm only has one iteration, so its total computational complexity can be approximately expressed as $C_{M-GOMP} \approx 2MN + 2K^2M$.

Now let's discuss the computational complexity of the three algorithms. Noting that the value s and n of GOMP algorithm are small constants, so the complexity of the GOMP algorithm can be expressed as $O(sMN)$, and the complexity of the OMP algorithm can be expressed as $O(KMN)$. As can be seen from the Figure 6, the value s usually tends to be smaller than K, that is to say, the GOMP algorithm has great advantages in terms of computational complexity. Additionally, because $K<M$, $M<N$, the computational complexity of M-GOMP algorithm can be expressed as $O(MN)$. Obviously, the M-GOMP algorithm that we put forward has a better performance.

6. Conclusion

In this paper, in view of the high computational complexity of existing channel estimation methods, we apply GOMP algorithm to sparse channel estimation. Meanwhile, a more effective scheme named M-GOMP algorithm is proposed based on the idea of GOMP algorithm, which selects multiple indices in each iteration. Compared with the OMP algorithm, computer simulations and theoretical analysis show that the proposed two algorithms not only have good estimation accuracy, but also can greatly reduce the running time required for channel estimation. Especially the M-GOMP algorithm, as a result of efficient scheme of indices selection, it can save more than 85% of the running time. Consequently, GOMP algorithm, especially the M-GOMP algorithm is expected to be a competitive candidate scheme for channel estimation in wireless communication.

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