

## Chaos-Enhanced Cuckoo Search for Economic Dispatch with Valve Point Effects

M. W. Mustafa, Abdirahman M. Abdilahi\*, M. Mustapha

Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia

\*Corresponding author, e-mail: abdirahman@fkegraduate.utm.my

### Abstract

*Economic dispatch determines the optimal generation outputs to minimize the total fuel cost while satisfying the load demand and operational constraints. Modern optimization techniques fail to solve the problem in a robust manner and finding robust global optimization techniques is necessary for efficient system operation. In this study, the potentiality of introducing chaos into the standard Cuckoo Search (CS) in order to further enhance its global search ability is investigated. Deterministic chaotic maps are random-based techniques that can provide a balanced exploration and exploitation searches for the algorithm. Four different variants are generated by carefully choosing four different locations (within the standard CS) with potential adoption of a candidate chaotic map. Then detailed studies are carried out on benchmark power system problems with four different locations to find out the most efficient one. The best of all test cases generated is chosen and compared with algorithms presented in the literature. The results show that the proposed method with the proposed chaotic map outperforms standard CS. Additionally, the chaos-enhanced CS has a very good performance in comparison with QPSO and NSS.*

**Keywords:** power plants operation, economic dispatch, Cuckoo search, metaheuristic algorithm, chaotic maps

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### 1. Introduction

Despite the global agenda of increasing the share of renewable energy production, thermal power plants contribute predominantly to the global electricity production and will continue to do so in the foreseeable future. This warrants a global concern to lower their operating costs. Allocating power generated in these plants in the least possible operating cost while meeting the system constraints has been one of the main concerns of the utility operators globally. To address this, engineers use the economic dispatch (ED) formulations which is a practical power system optimization problem.

Because of the non-linearity, non-convexity and the multimodal characteristics present in the cost function of the ED, adopting a metaheuristic optimization technique (which is the state-of-the-art global optimization technique [1]) has two major advantages over the usage of the conventional techniques. Firstly, metaheuristic optimization techniques (MOTs) lead to better problem modelling that reduce assumptions related to problem characterization in terms of nonlinearity. Secondly, MOTs have better ability to obtain optimal solutions as compared with a conventional technique. Both of these aspects lead to optimal generator loadings for least cost operation within the power system. As a result, system operators can benefit from significant cost savings over the years.

Earlier formulations of the ED problem were tackled using conventional mathematical techniques such as interior point method, lambda-iteration method and linear programming [2]. However, those methods cannot solve the ED problem when formulated in a non-linear context and they suffer from "curse of dimensionality" problems. In recent times, the knowledge growth of MOTs has given an opportunity to optimize the ED problem in a more practical way than the mathematical techniques. Researchers have implemented many MOTs to solve the problem such as Genetic Algorithm (GA) [3], Evolutionary Algorithms [4], Particle Swarm Optimization (PSO) [5], Evolutionary Programming (EP) [6], Tabu Search (TS) [7], Simulated Annealing (SA) [8], Firefly Algorithm (FA) [9] and Glowworm Swarm Optimization (GWO) [10]. However, many of these techniques and their variants have shown a lack of ability to obtain consistent and robust optimal results reducing their effectiveness in a practical operation.

One significant issue with these diverse MOTs when implemented in the ED problem is their inherent randomness involved in their performance [11]. Unlike the deterministic techniques, most of the methodologies reported in the diverse literatures indicate the inability of the algorithm to achieve absolute robustness to achieve global solutions. To tackle this issue, the development of a robustness-oriented optimization algorithm is necessary. In this paper, an integrated approach that combines an efficient Cuckoo Search (CS) algorithm with deterministic chaotic systems was developed, to solve highly multimodal, nonlinear problems.

CS has shown better characteristics in optimal and efficient global search ability than many MOTs in the literature [12]. The combination of the Levy flight process with standard Gaussian probability distribution process make the algorithm global search technique efficient. With the adoption of chaotic maps, better balance between the diversification and the intensification is achieved with sufficient randomization [13]. In line with this, this paper aims to investigate the effect of introducing chaotic map on the performance of the standard CS algorithm, in the context of optimal ED scheduling. Though, many studies have investigated the improvement of different MOTs using deterministic chaotic signals, however, there are yet any studies on the improvement of the performance of CS algorithm using chaotic maps. Also, another contribution on robust global search techniques for solving ED problems is presented in this work.

## 2. Problem Formulation

The economic dispatch is normally formulated as an optimization problem whereby the operating cost of all the generating plants are minimized subject to physical system constraints. The objective of the problem is mathematically defined by the following cost function:

$$\min \sum_{i=1}^{N_D} F_i(P_i) \quad (1)$$

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad (2)$$

Where  $a_i$ ,  $b_i$ , and  $c_i$  are the cost coefficients of the generation unit  $i$ ;  $P_i$  is the scheduled power of the unit  $i$ ;  $N_D$  is the total number of online generators; and  $F_i$  is the fuel cost.

However, this formulation of the cost function simplifies the generator characteristics to a smooth quadratic function leading to poor problem modelling. To model a more practical cost function, one must consider the valve point effect and the equation is modified as shown below:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \left( \sin \left( f_i \left( P_i - P_i^{\min} \right) \right) \right) \right| \quad (3)$$

Where  $e_i$  and  $f_i$  are the valve point effect coefficients;  $P_i^{\min}$  is the minimum generation limit;

In addition to the above problem formulations, the system is subjected to physical constraints. The system power balance constraint is an equality constraint and can be represented in mathematical form as follows:

$$\sum_{i=1}^{N_D} P_i - P_D - P_L = 0 \quad (4)$$

Where  $P_D$  is the total system demand; and  $P_L$  is the system loss. The limits of each generating unit constitute as the inequality constraint of the problem which can be written mathematically as:

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (5)$$

Where  $P_i^{\min}$  and  $P_i^{\max}$  are the minimum and the maximum limits of the power generation unit.

### 3. Cuckoo Search Algorithm

Cuckoo search (CS) algorithm was developed by Xin-She Yang and Suash Deb in 2009 [14]. The algorithm is based on the theory of Cuckoos, particularly their brood parasitism characteristics in combination with the levy flight concept. Sections 3.1 to 3.4 outlined the stages for implementing CS algorithm.

#### 3.1. Representation and Initialization

The decision variables are represented within the optimization as follows:

$$P_i = P_i^{\min} + rand * (P_i^{\max} - P_i^{\min}) \quad (6)$$

Where  $P_i^{\max}$  is the maximum power output of each unit  $i$ ;  $rand$  is a uniform distributed random generator.

#### 3.2. Fitness Function

The fitness evaluation for all population individuals is performed based on the following fitness equation:

$$Fitness(P_i) = F_i(P_i) + \lambda \left| \left( \sum_{i=1}^{N_D} P_i \right) - P_D - P_L \right| \quad (7)$$

The parameter  $\lambda$  is the penalty factor multiplier to amplify the error values so that it weakens the goodness of the fitness function when there are equality constraint violations [15]. An additional constraint handling module is implemented to cater effectively the constraint violations. In this paper, the method presented in [16] was used to effectively obtain the global optimal values.

#### 3.3. Levy Flight

Within CS, new solutions are generated using the Levy flight process. In this process, the global best ( $P_{best}$ ) index is utilized and the optimal path for the Levy flights. The updating formula for the Levy flight process is given as follows:

$$P_i^{new} = P_i + randn \times Scale \times Levy(\beta) \quad (8)$$

Where  $Scale = \frac{P_i^{\max} - P_i^{\min}}{100}$  and  $rand$  is a normally distributed stochastic number. Additionally, the  $Levy(\beta)$  function for every iteration is determined as follows:

$$Levy(\beta) = \frac{u}{|v|^{1/\beta}} \times (P_i - P_{best}) \quad (9)$$

Where  $u$  and  $v$  are drawn from normal distribution. That is:

$$\mu \sim N(0, \sigma_u^2), v \sim N(0, \sigma_v^2) \quad (10)$$

With,

$$\sigma_u = \left[ \frac{\Gamma(1+\beta) \times \sin((\pi\beta)/2)}{\Gamma((1+\beta)/2) \times \beta \times 2^{((\beta-1)/2)}} \right]^{1/\beta}, \quad \sigma_v = 1 \quad (11)$$

Where  $\Gamma(\cdot)$  is the gamma distribution function.

The main advantage of this Levy flight is to perform a global exploration search by performing occasional long-distance jumps. It is these sudden jumps that might increase the search efficiency of the CS significantly particularly for multimodal, nonlinear problems.

### 3.4. Discovery and Randomization

The second successive updating equation used for the CS algorithm is based on the concept of discovery of the Cuckoo egg within the host nest. This concept brings in a randomization feature and a local search based on a random walk for the algorithm. The new solutions generated as a result of this concept is obtained as follows:

$$P_i^{new} = P_i + rand \otimes H(P_a - rand) \otimes (P_m - P_n) \quad (12)$$

Where  $P_m$  and  $P_n$  are two different solutions selected randomly by permutation;  $H(\cdot)$  is a Heaviside function controlled by a switching parameter  $P_a$ .

### 4. Chaotic Maps (CM)

In this sub-section, a Chebyshev chaotic map is introduced. The CM which will be used in the rest of the experimentations is described and formulated. Equation (2.3) gives the mathematical expression of the implemented Chebyshev chaotic map with the initial value  $x_0 = 0.1$ , and Figure 1 illustrates the visual characteristics of the implemented CM [13].

$$x_{t+1} = \cos(t \cos^{-1} x_t) \quad (13)$$

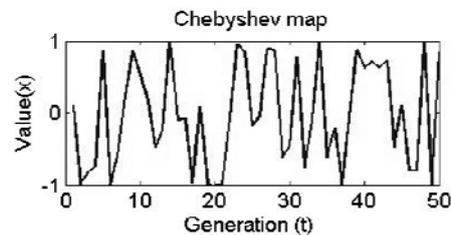


Figure 1. Illustration of chaotic maps used in the study

### 5. Implementation

In this study, four different schemes of integrating a Chebyshev chaotic map in to the standard CS have been investigated, each of them producing a unique variant of CS. Then each variant is exposed to the proposed Chebyshev map explained in the earlier section.

The choice of the favourite locations is directed to the updating equations of the CS algorithm. The standard CS algorithm has two successive updating equations and therefore the modifications are directed to these two equations. The pseudo random generators (particularly the uniform and normal distribution random number generators) and CS's controllable parameter are subjected under this investigation whether the CM can alternatively replace them. Table 1 below summarises the implemented modifications.

The equations in Table 1 are previously formulated in section 3. Last column indicates the corresponding stage name, which are described in sections 3.3 and 3.4. The equations involved in the modifications are equations (2.2) (for variants A and B) and equation (1.9) for variant C. Therefore, the symbolic descriptions of the equations presented in Table 1 remain the same as described in section 3.

Table 1. Illustration of Modifications Implemented for the Different Variants

Variant		Equations Involved	Stage
A	Original	$P_i^{new} = P_i + rand \otimes H(P_a - \varepsilon) \otimes (P_m - P_n)$	Discovery
	Modified	$P_i^{new} = P_i + x_i \otimes H(P_a - \varepsilon) \otimes (P_m - P_n)$	
B	Original	$P_i^{new} = P_i + rand \otimes H(P_a - \varepsilon) \otimes (P_m - P_n)$	Discovery
	Modified	$P_i^{new} = P_i + rand \otimes H(x_i - \varepsilon) \otimes (P_m - P_n)$	
C	Original	$P_i^{new} = P_i + randn \times Scale \times Levy(\beta)$	Levy Flight
	Modified	$P_i^{new} = P_i + (2 \times x_i - 1) \times Scale \times Levy(\beta)$	
D	Modified	Combination of modifications in Variants A and B	Discovery

### 5.1. Experiment Settings

All the experiments implemented in this paper were carried out using MATLAB software. A standard power system test case with thirteen units along with valve loading effects is used to test the algorithm's capability to solve the ED problems. The data of the test system is obtained from [6].

In order to obtain the right parameters of the algorithm, we have carried out a detailed parametric study by varying one parameter at a time. The advantages of the CS algorithm include its nature of having small number of controllable parameters unlike the PSO. The first parameter to be tuned is the fixed number that represents the probability of randomly discovering a Cuckoo's egg in the host nest (donated as  $P_a$ ). Besides, other inherent parameters such as the population size (donated as  $N$ ) and the Levy flight exponent - donated as  $\beta$  (Beta) - are tuned. Table 2 shows the list of parameters being tested and the chosen values for final algorithm experimentation. In all executed experiments, the stopping criterion was the maximum iteration.

Table 2. Parameter Setting in the Algorithm Design

Parameter	Description	Tested Values	13 Unit
$\beta$	Levy Flight Exponent	[0.3, 2.0] in steps of 0.1	0.55
$P_a$	Probability of Discovery	[0, 1] in steps of 0.1	0.90
$N$	Population Size	[10, 100] in steps of 10	50
$T_\phi$	Maximum Iteration	$5 \times 10^3, 10 \times 10^3, 20 \times 10^3, 25 \times 10^3$	$20 \times 10^3$
PF	Penalty Factor	50, 100, 500, 1000	100

## 6. Results and Discussion

This section presents the results of the investigation carried out in this study. The performance of each variant in respect with the standard CS (SCS) are investigated and discussed in the following sub-sections.

### 6.1. Performance of Chaotic Maps within Each Variant

In this sub-section, the performance impacts of a Chebyshev CM on four different locations are looked at, in comparison with the SCS. Each variant is looked at separately. Table 3 summarizes the descriptive statistical parameters of each experiment for all variants. Observe the numbering information of CS-A, CS-B, CS-C and CS-D is in line with the variant lettering given in Table 1. SCS stands for Standard CS.

It can be observed in Table 3 that CS-A performed better than the rest of the test cases when looked at the mean, median, standard deviation (abbreviated as Std here after) and the range. However, in terms of the ability of the algorithm to randomly locate the optimal cost, all the different test cases achieved the desired point. Conclusively, the best case was test case CS-A in which the Chebyshev map replaces the random number generated based on Gaussain probability distribution. This shows that a deterministic chaotic signal (such as Chebyshev map)

might have better ability to stabilize the performance of the algorithm while achieving global optimal solutions.

Table 3. Descriptive statistical results of the various variants (1,800 MW)

Variants	Min Cost	Mean Cost	Median Cost	Max Cost	Std Cost	Range	
Standard CS	SCS	17,963.84	17,969.95	17,969.80	17,990.77	5.50	26.93
	CS-A	17,963.83	17,965.05	17,963.86	17,968.99	2.15	5.16
Chebyshev	CS-B	17,963.83	17,966.34	17,963.91	17,975.91	3.99	12.08
	CS-C	17,963.84	17,970.01	17,969.01	17,985.09	5.55	21.25
	CS-D	17,963.83	17,969.15	17,968.95	18,012.39	6.99	48.56

## 6.2. Algorithmic Robustness of Each Test Case

With statistical performance parameters only, it is difficult to observe the optimal cost distribution of the trial set results. For example, looking at Table 3 one can observe that for test variant B the mean cost is actually higher than the median cost. This indicates that despite the algorithm's ability with half of the total trial set to achieve minimum cost that is close to the desired number, the few high values of optimal cost occurrences of some trials could actually lead to high means. Therefore, the number-oriented table format is not enough to describe the variability and distribution of results including outliers, quartiles and others. To actually show that, an extended analysis of this experiment results are plotted using box-whisker plot. Figure 2 indicates the box plot of a 13 unit test system for each variant implemented.

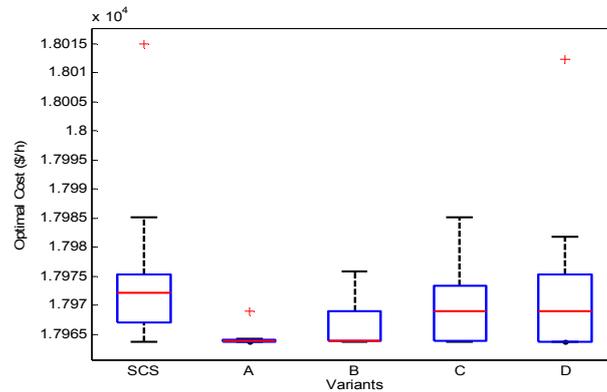


Figure 2. The box plot of five different test cases (1,800 MW)

In Figure 2, the result distribution and the cost range covered for various test cases and variants is presented. The actual performance in terms of robustness is truly evident within this box plot that indicates the optimal results distribution and the consistency level of the implemented variants. The lower the box and the smaller the box size the better the algorithm in achieving optimal and consistent results respectively. Proceeding from the previous observations in sub-section 6.1 to the observations from Figure 2, it can be observed that variant A shows very small optimal result variability in respect to other experimented locations.

## 6.3. Optimal Solutions

After the optimal tuning experiments, another final experiment was run with the recommended optimal values for the parameters as described in the previous section with a maximum iteration of 20,000 for the thirteen unit systems. The output settings of the generators are shown in Table 4. The results of some other methods presented in recent literature are also included for comparison.

Table 4. Best solution output power solution settings for the generators in comparison with other methods in the literature (13-Unit System)

Unit	QPSO [17]	NSS [18]	CCS-A1
1	538.56	448.80	628.32
2	224.70	300.50	222.76
3	150.09	299.20	149.59
4	109.87	60.00	109.87
5	109.87	109.90	109.87
6	109.87	109.90	109.87
7	109.87	61.90	60
8	109.87	109.90	109.87
9	109.87	109.90	109.87
10	77.41	40.00	40
11	40.00	40.00	40
12	55.01	55.00	55
13	55.01	55.00	55
<b>Total cost (\$/h)</b>	<b>17,969.01</b>	<b>17,976.95</b>	<b>17,963.83</b>

#### 6.4. Comparison with the Existing Methods

In order to show CS's effectiveness and suitability in ED problems, results of different methods for both systems with valve-point loading effects are shown in Tables 5. The table summarises the optimal cost result achievement of eleven different methods published in journals that are found in the major energy and engineering databases. In terms of the ability of the method to achieve minimum operating cost out of the set trials, the results of the proposed method are better than those of the methods listed in Table 5. Moreover, the proposed method is the best presented in terms of having both low standard deviation and the global optimum point at the same time.

Table 5. Comparison of results for existing methods for a thirteen unit system with a demand of 1800 MW

Method	Min Cost (\$/h)	Mean Cost (\$/h)	Max Cost (\$/h)	SD (\$/h)
MSL [19]	18,158.68			
IFEP [6]	17,994.07	18,127.06	18,267.42	-
AIS [20]	17,977.09			
NDS [21]	17,976.95	17,976.95	17,976.95	0
HMAPSO [22]	17,969.31	17,969.31	17,969.31	0
FCASO-SQP [23]	17,964.08	18,001.96	-	-
<b>CS-A</b>	<b>17,963.83</b>	<b>17,965.05</b>	<b>17,968.99</b>	<b>2.15</b>

#### 7. Conclusion

Economic dispatch is an important power system optimization problem. Finding elegant techniques that can efficiently tackle this problem brings significant system operation savings. In line with this, this study aims to improve the standard CS by integrating with chaotic maps to develop the new chaotic-enhanced CS. Four different locations of the standard CS have been introduced to Chebyshev CM. The performances of these variants have been investigated. Based on statistical and robustness comparisons, the first location (variant A) produces the best performing algorithm. The results revealed that variant A is the best generated algorithms among all the four test cases investigated in this study. By choosing as CS-A (variant A with Chebyshev) the best generated algorithm in this study, it was further investigated in terms of its performance with established methods in the literature. The proposed methodology proves that it outperforms established methods in terms of robustness and achieving consistent results. Due to high robust results, the methodology is able to eliminate the inherent randomization characteristics related to the heuristic methods when applied in the economic dispatch field.

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