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Mathematical Modeling and Fuzzy Adaptive PID Control of Erection Mechanism

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Abstract

This paper describes an application of fuzzy adaptive PID controller to erection mechanism. Mathematical model of erection mechanism was derived. Erection mechanism is driven by electro-hydraulic actuator system which is difficult to control due to its nonlinearity and complexity. Therefore fuzzy adaptive PID controller was applied to control the system. Simulation was performed in Simulink software and experiment was accomplished on laboratory equipment. Simulation and experiment results of erection angle controlled by fuzzy logic, PID and fuzzy adaptive PID controllers were respectively obtained. The results show that fuzzy adaptive PID controller can effectively achieve the best performance for erection mechanism in comparison with fuzzy logic and PID controllers.

Keywords*: erection mechanism, electro-hydraulic system, fuzzy logic control, PID control*

1. Introduction

Erection mechanism of large machineries and armaments is a complex mechanical, electronic and hydraulic integration system. It can be mainly divided into electro-hydraulic part and mechanical part. Electro-hydraulic actuator is widely used in industry such as crane and erection mechanism. Hydraulics has the advantages of producing large force at high speed, high power density and good dynamic performance [1]. Its application can be found in many engineering fields such as automotive industries, machine tools, aircrafts and manufacturing equipment [2-4].

The hydraulic system is a non-linear system which has the dynamic behaviours including friction, fluid compressibility and associated stiffness. It is difficult to control the system with high accuracy [5]. Position tracking performance can be assured when its robustness and tracking accuracy are guaranteed. There are a number of problems such as nonlinearities, uncertainties and time varying parameters which can reduce the robustness and tracking accuracy. The control problems of hydraulic system have been indicated in many researches. Therefore, it is of great significance to design an appropriate controller which possesses strong robustness and high accuracy [6]. In order to improve the position tracking performance, a lot of papers have proposed fuzzy logic control [7], model based control [8] and adaptive control [9-10], which have been proven to be effective in many applications. Jelali and Kroll summarized the application of nonlinear control in electro-hydraulic system. The advanced control algorithm could be used to compensate the nonlinearity of electro-hydraulic control system. It was helpful for achieving good performance tracking control [11]. Proportional-integral-derivative (PID) algorithm has the characteristics of simplicity, good stability and reliability. It could be applied to linear system to achieve high control accuracy. It has been widely utilized in industrial applications. However, the parameters of PID algorithm could not be adjusted according to system changes. It is sensitive to variations of system parameter. Fuzzy logic algorithm depends on the experts' experience which is transformed into control rules. The controller output could be altered according to system parameters. It doesn't need precise system information. It can adapt to the influence of system uncertainty and nonlinearity. The combination of fuzzy logic and PID control algorithm can comprehensively utilize the advantages of two algorithms. Fuzzy PID controller can handle parameter changes and nonlinear effects. It has good stability and has been applied to solve many engineering problems [12]. It can achieve desired goals in control of electro-hydraulic system.

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The structure of this paper is arranged as follows. Model of the electro-hydraulic system is derived in section 2.1. Kinetic analysis of erection mechanism is accomplished in section 2.2. The relationships of erection force, erection angle and the length of hydraulic cylinder are obtained. Fuzzy adaptive PID controller is designed which is composed of fuzzy algorithm and PID control method in section 3. Models of erection system and fuzzy adaptive controller are established in Simulink in section 4. Experimental studies are completed on laboratory equipment in section 5. The control effect of fuzzy adaptive PID controller is validated on erection mechanism based on simulation and experiment.

2. Mathematical Modeling of Erection System

2.1. Model of the Electro-Hydraulic System

Erection mechanism is mainly composed of hydraulic system and mechanical system. Mathematical models of each system are established separately. Hydraulic principle of erection system is shown in Figure 1. It is mainly composed of pump 1, motor 2, relief valve 3, electro-hydraulic proportional valve 4 and hydraulic cylinder 5.

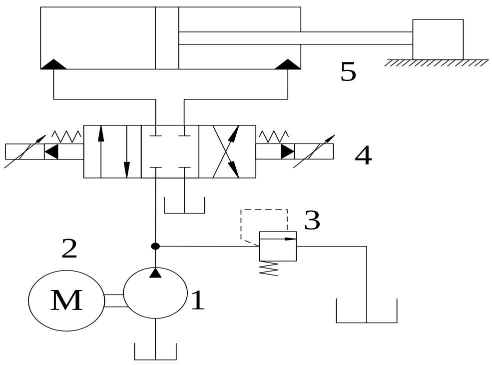


Figure 1. Hydraulic Principle of Erection System

The pump model can be built by the following equation.

$$Q_p = D_p w_p \eta_v \tag{1}$$

Where Q_p is the output flow rate. D_p represents the displacement and η_v is the volumetric efficiency of pump. w_p illustrates the rotational velocity of motor.

The model of electro-hydraulic proportional directional valve is expressed by the following formulas.

$$q_1 = C_d A_x \sqrt{\frac{2(p_s - p_1)}{\rho}} \tag{2}$$

$$q_2 = C_d A_y \sqrt{\frac{2p_2}{\rho}} \tag{3}$$

Where q_1 and q_2 are the flow rate from and to cylinder. C_d represents the flow coefficient of valve. A_x and A_y are the areas of spool valve. p_s is the supply pressure. p_1 is the pressure of piston chamber. p_2 is the pressure of piston rod chamber. ρ represents the density of fluid.

The flow and pressure models of hydraulic cylinder are established using the chamber node method. The dynamic equation is described utilizing the second Newton's Law.

$$q_1 = A_1\frac{dy}{dt} + \frac{(V_{01} + A_1 y)}{\beta_e}\frac{dp_1}{dt} + (C_{ec} + C_{ic})p_1 - C_{ic}p_2 \quad (4)$$

$$q_2 = A_2\frac{dy}{dt} + \frac{(V_{02} - A_2 y)}{\beta_e}\frac{dp_2}{dt} - (C_{ec} + C_{ic})p_2 + C_{ic}p_1 \quad (5)$$

$$p_1 A_1 - p_2 A_2 = m\frac{d^2 y}{dt^2} + F \quad (6)$$

Where A_1 and A_2 represent the area of two chambers in hydraulic cylinder. y illustrates the displacement of piston. β_e illustrates the fluid bulk modulus. V_{01} and V_{02} represent the initial volume of two chambers. C_{ic} and C_{ec} represent leakage coefficients. p_1 and p_2 are the pressure in the forward and return cylinder chambers. m is the mass of load. F represents external force.

2.2. Kinetic Analysis of Erection Mechanism

Mechanical part of erection mechanism is shown in Figure 2. The load revolves around the point P_2 driven by hydraulic cylinder.

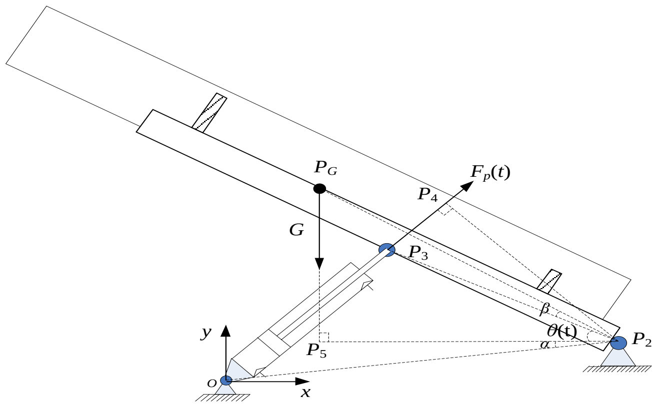


Figure 2. Kinetic Model of Erection Mechanism

The following equation is established to describe the force balance model.

$$J\theta''(t) = F_p(t)\cdot\overline{P_2P_4} - G\cdot\overline{P_2P_5} \quad (7)$$

Where J represents the moment of inertia. $\theta(t)$ is the rotational angle of load. $F_p(t)$ represents the output force of hydraulic cylinder. G illustrates the gravity of load.

$$\overline{P_2P_4} = \overline{OP_2}\sin\angle P_2OP_3 = \overline{OP_2}\frac{\overline{P_2P_3}\sin(\theta(t)+\alpha)}{\overline{OP_3}} \quad (8)$$

$$\overline{P_2P_5} = \overline{P_2P_G}\cos(\theta(t)+\beta) \quad (9)$$

The relationship of erection angle and cylinder length can be acquired by using cosine theorem in ΔOP_2P_3.

$$\cos(\theta(t)+\alpha) = \frac{\overline{P_2P_3}^2 + \overline{OP_2}^2 - \overline{OP_3}^2}{2\overline{P_2P_3}\cdot\overline{OP_2}} \quad (10)$$

3. Fuzzy Adaptive PID Controller Design

The connection of fuzzy logic and PID method can utilized the advantages of the two algorithms. Fuzzy algorithm is utilized to alter the parameters of PID algorithm based on the change of system parameters [13]. It can take full advantage of expert experience and good control effect of PID algorithm. Fuzzy adaptive PID algorithm is adopted in erection process to enhance control performance.

The differential equation of PID control is expressed as follows [14].

$$u(kT) = K_P \left\{ e(kT) + \frac{T}{T_I} \sum e(kT) + \frac{T_D}{T} \left[e(kT - T) \right] \right\} \tag{11}$$

Where $u(kT)$ illustrates the output signal. K_P represents the propotional coefficient. e illustrates the input error signal. k is the sampling number. T is the sampling period. T_I represents the integral coefficient. T_D represents the differential coefficient.

The principle of fuzzy adaptive PID controller is depicted in Figure 3. Fuzzy logic algorithm is utilized to change the parameters of K_P, K_I and K_D. The input parameters of fuzzy logic are $e(t)$ and $ec(t)$. The output parameters of $\triangle K_P$, $\triangle K_I$ and $\triangle K_D$ are obtained by fuzzy inference calculation. The values are respectively added to initial values K_{P0}, K_{I0} and K_{D0}. The adaptive adjustment of PID algorithm is realized.

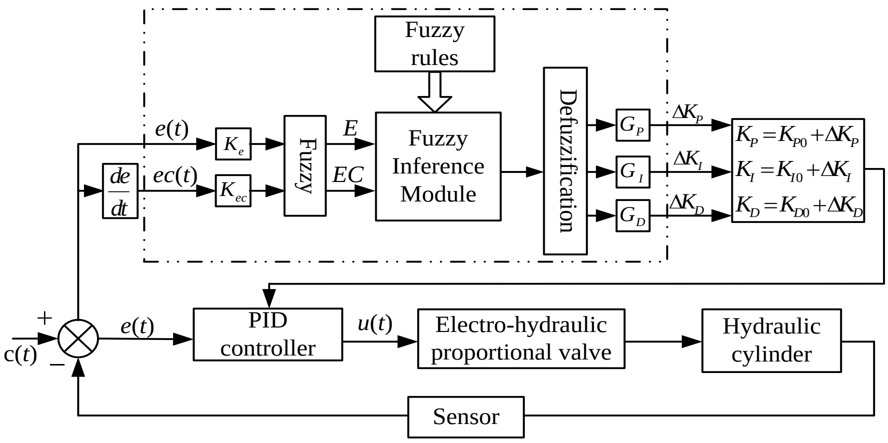


Figure 3. The Principle of Fuzzy Adaptive PID Controller

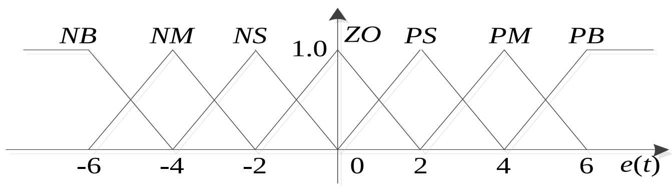


Figure 4. Membership Function Cure of Input

The input and output parameters are transformed into fuzzy logic values and defined as seven values: *NB*, *NM*, *NS*, *ZO*, *PS*, *PM* and *PB* [15]. The domain of error $e(t)$ is defined as [$-x_e$, x_e]. The domain of $c(t)$ is defined as [$-x_c$, x_c]. The domain of fuzzy subset is defined as {-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6}.

Triangular shape function is selected as input membership function, as depicted in Figure 4 [16]. The relationships between $e(t)$, $c(t)$ and K_P, K_I, K_D are summarized through much operating experience [17].

1. When $e(t)$ is relatively large value, K_P should be adopted relatively large value to speed up the system response and reduce time constant with damping coefficient. K_D should be adopted relatively small value to prevent out of range control at the initial stage. In order to prevent overshoot, the integral value should be removed.

2. When $e(t)$ is medium value, K_P should be adopted relatively small value to minimize response overshoot. The value of K_D is important and should be adopted medium value. The integral value is supposed to appropriately increase.

3. When $e(t)$ is relatively small value, K_P and K_I are supposed to adopted relatively large value to have good steady state and avoid oscillation at equilibrium point. K_D is supposed to be adopted appropriate value.

According to above relationships and the impact of $c(t)$, fuzzy logic rules are obtained in Table 1 through theoretical analysis.

Table 1. Fuzzy Control Rules

$\triangle K_P/\triangle K_I/\triangle K_D$		e: NB	NM	NS	ZO	PS	PM	PB
c	NB	PB/NB/NS	PB/NB/PS	PM/NM/PB	PM/NM/PB	PS/NS/PB	ZO/ZO/PM	ZO/ZO/NS
	NM	PB/NB/NS	PB/NB/PS	PM/NM/PB	PS/NS/PM	PS/NS/PM	ZO/ZO/PS	NS/ZO/ZO
	NS	PM/NB/ZO	PM/NM/PS	PM/NS/PM	PS/NS/PM	ZO/ZO/PS	NS/PS/PS	NS/PS/ZO
	ZO	PM/NM/ZO	PM/NM/PS	PS/NS/PS	ZO/ZO/PS	NS/PS/PS	NM/PM/PS	NM/PM/ZO
	PS	PS/NM/ZO	PS/NS/ZO	ZO/ZO/ZO	NS/PS/ZO	NS/PS/ZO	NM/PM/ZO	NM/PB/ZO
	PM	PS/ZO/NB	ZO/ZO/PS	NS/PS/NS	NM/PS/NS	NM/PM/NS	NM/PB/NS	NB/PB/NB
	PB	ZO/ZO/NB	ZO/ZO/NM	NM/PS/NM	NM/PM/NM	NM/PM/NS	NB/PB/NS	NB/PB/NB

Take the first rule as an example, the above rule can be interpreted as "if *e* is *NB* and *c* is *NB*, then $\triangle K_P$ /$\triangle K_I$ /$\triangle K_D$ are *PB/NB/NS*". Mamdani algorithm is introduced as fuzzy implication relation [18].

$$\begin{aligned} R(a,u) &= (A \rightarrow U)(a,u) \\ &= \min(A(a), U(u)) = A(a) \wedge U(u) \end{aligned} \tag{12}$$

Triangular shape function is selected as output membership function, as expressed in Figure 5 [19].

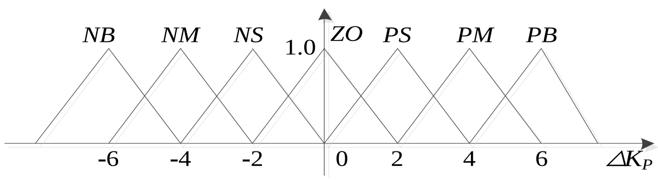


Figure 5. Membership Function Cure of Output

Centroid is chosen as defuzzification method [20], which calculates the area center of the fuzzy set membership function curve surrounded by the horizontal coordinate. The horizontal coordinate value of the center is chosen as the representative value of the fuzzy set. The horizontal coordinate u_0 corresponding to the area center is calculated by the following formula [21].

$$u_0 = \frac{\int_U A(u)udu}{\int_U A(u)du} \tag{13}$$

4. Simulation and Results
4.1. Modeling Erection Mechanism in Simulink
Figure 6 illustrates the model of the erection system in Simulink software. The desired angle signal is simulation input. The pressure and position of hydraulic cylinder are simulation outputs. The entire model is decomposed into several subsystems blocks such as the ‘source’ model, the ‘pump’ model, the ‘valve/cylinder’ model and the ‘fuzzy adaptive PID controller’ model. The output of the controller is derived to control motion of the valve spool. Position of the spool influences the nominal flow into cylinder chamber. The nominal flow change will change the volume and pressure inside both cylinder chambers. Finally, the piston position is affected, therefore the mechanical mechanism moves driven by cylinder piston. Hence, the performance of the system is dependent on the output of the controller.

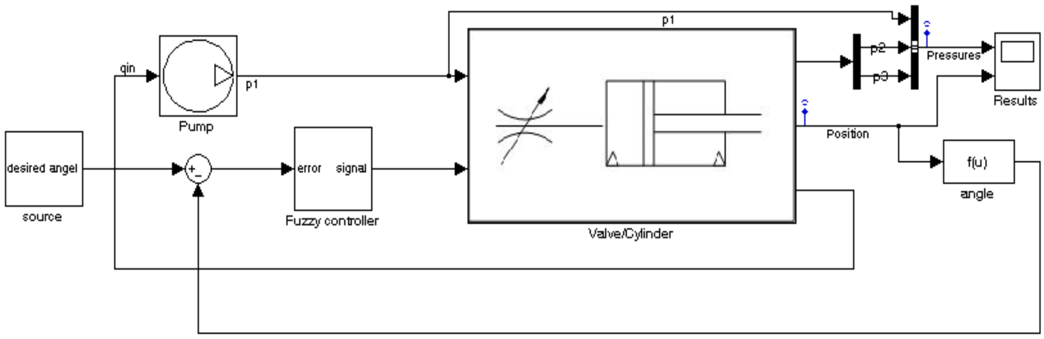


Figure 6. Diagram of Complete Erection Model in Simulink

The ‘source’ model is reference angle trajectory. Composite sine function is selected as reference erection angle trajectory. $\theta(t)$ is expressed by the following equations:

$$\theta(t)=(\theta_1-\theta_0)s(\tau) \quad (14)$$

$$s(\tau)=\begin{cases} \dfrac{k}{4\pi}\left[\tau-\dfrac{\sin(4\pi\tau)}{4\pi}\right] & 0\le\tau<\dfrac{1}{8} \\ \dfrac{k}{4\pi}\left[\tau+\dfrac{2}{\pi}-9\cos(\dfrac{4\pi\tau}{3}-\dfrac{\pi}{6})/4\pi\right] & \dfrac{1}{8}\le\tau<\dfrac{7}{8} \\ \dfrac{k}{4\pi}\left[\tau+\dfrac{4}{\pi}-\dfrac{\sin(4\pi\tau-2\pi)}{4\pi}\right] & \dfrac{7}{8}\le\tau\le 1 \end{cases} \quad (15)$$

Where $k=4\pi^2/\pi+4$. θ_0 is initial value and θ_1 is terminal value of erection angle. T is erection time, $\tau=t/T$.

Figure 7 shows the valve controlled hydraulic cylinder sub-block derived from Equations (1) to (6). The model of control valve is built based on flow equation. The model of hydraulic cylinder is established based on chamber node method. The pressure p_1 and p_2 appearing as derivatives in Equations (4) to (6) are utilized as the states.

Simulink block of fuzzy adaptive PID algorithm is expressed in Figure 8. The model is established in Figure 3 based on the controller structure.

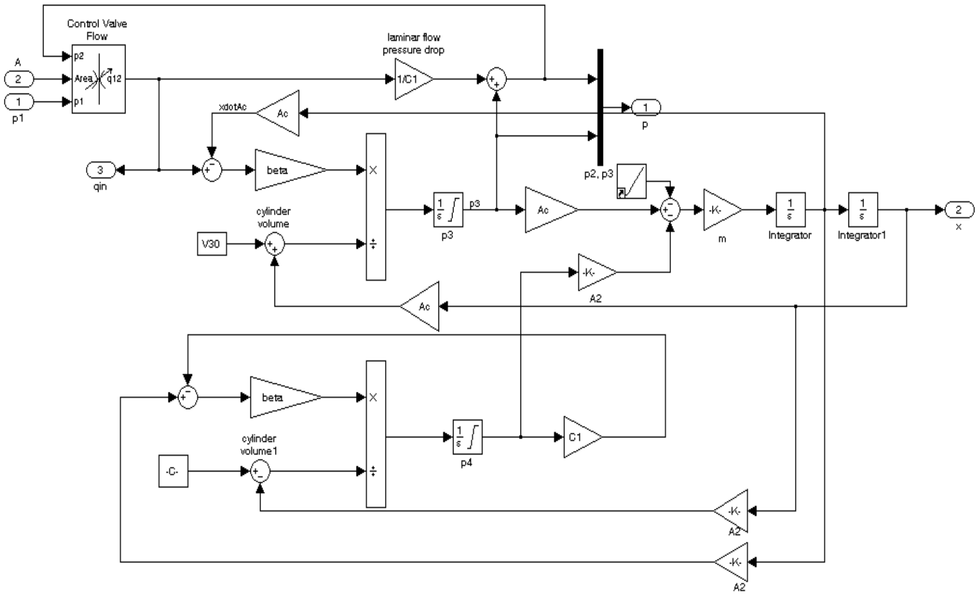


Figure 7. Simulink Block of Hydraulic Cylinder and Valve

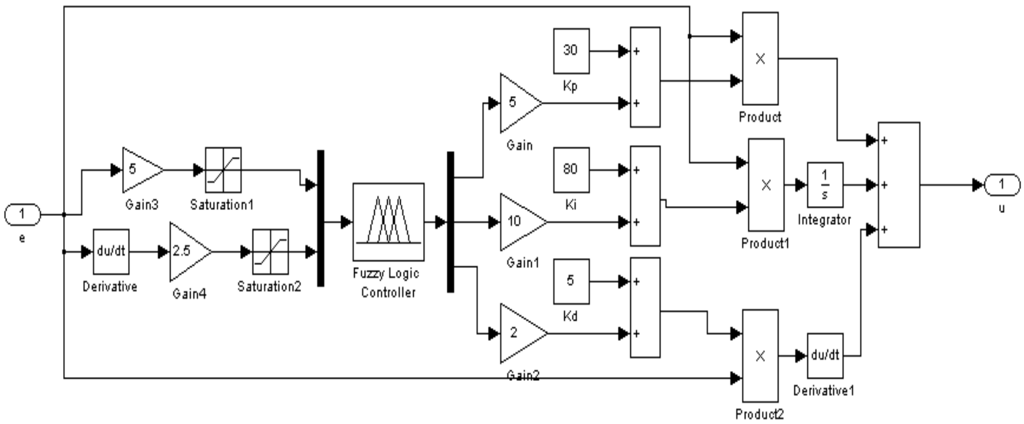


Figure 8. Simulink Block of Fuzzy Adaptive PID Controller

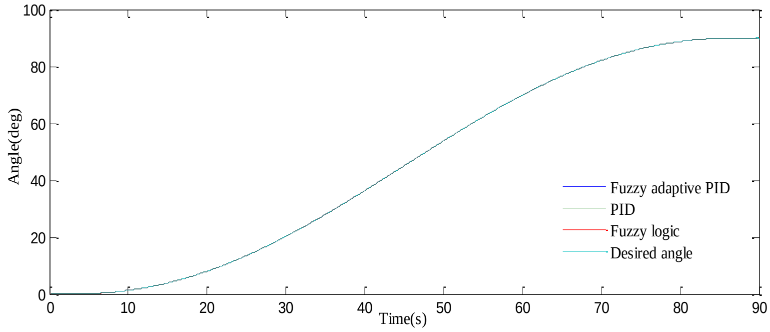
4.2. Simulation Results

The purpose of simulation is to investigate the performance of fuzzy adaptive PID algorithm for erection system. The simulation results are demonstrated by Simulink software in this section. The parameter values of erection system are demonstrated in Table 2.

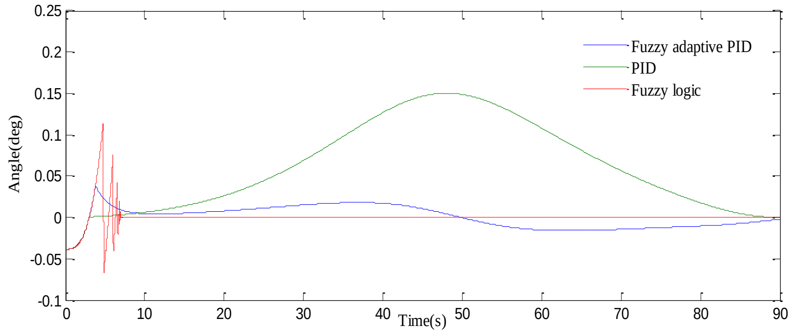
The target of controller is to minimize the error of erection angle and planed angle reference. The control effects of three controllers are compared by simulation. The simulation results of erection angle are demonstrated in Figure 9. Erection angle curves are expressed in Figure 9(a). The error curves of erection angle are expressed in Figure 9(b). The maximum value of angle error controller by PID is 0.15°. The maximum value of angle error controller by fuzzy logic is 0.12°. The maximum value of angle error controller by fuzzy adaptive PID is 0.03°. The angle deviation of fuzzy adaptive PID controller is the smallest obtained from the simulation results. It has the advantage of both PID and fuzzy logic so the control performance is the best. Fuzzy adaptive PID algorithm could be adopted in erection system to achieve high accuracy.

Table 2. Parameter Values for Simulation

Symbol	Explanation	Value
C_d	Discharge coefficient	0.61
β_e	Bulk modulus of hydraulic fluid.	6.85e8 Pa
A_1	Area of piston chamber	1.23e-3 m^2
A_2	Annular area of piston rod chamber	5.91e-4 m^2
ρ	Fluid density	800 kg/m^3
C_{ic}	Internal leakage coefficient	3e-11
C_{ec}	External leakage coefficient	2e-11
m	Equivalent mass	500 kg
P_s	Hydraulic supply pressure	2.5e6 Pa
V_{01}	Initial volume of piston chamber	2.4e-3 m^3
V_{02}	Initial volume of piston rod chamber	0.0089 m^3
$\overline{OP_2}$	The length of $\overline{OP_2}$	3.08 m
$\overline{P_2P_3}$	The length of $\overline{P_2P_3}$	1.11 m



(a) Simulation Erection Angle of Three Controllers



(b) Simulation Erection Angle Error of Three Controllers

Figure 9. Simulation Results of Erection Angle

5. Experimental Verification

Mechanical constitution of laboratory equipment is shown in Figure 10. It is mainly composed of erection arm and erection cylinder. Programs of data acquisition and three designed controllers are completed in LabVIEW software.

The control effects of three controllers are compared by experiment. The experiment results of erection angle are demonstrated in Figure 11. Erection angle curves are expressed in Figure 11(a). The error curves of erection angle are expressed in Figure 11(b). The maximum value of angle error controller by PID is 0.6°. The maximum value of angle error controller by fuzzy logic is 0.2°. The maximum value of angle error controller by fuzzy adaptive PID is 0.1°.From the results we can obtain that the angle deviation of PID is the largest. The angle deviation of fuzzy logic fluctuates wildly. The tracking deviation of fuzzy adaptive PID is small and steady. The experiment results illustrate that fuzzy adaptive PID control could be applied in erection system.

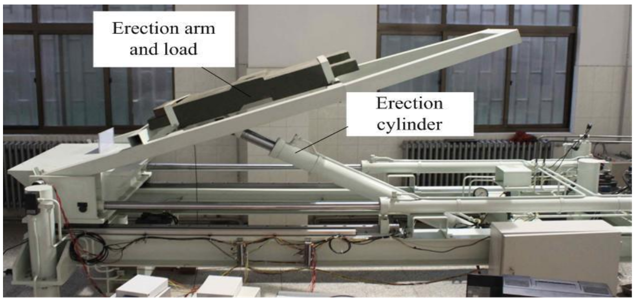
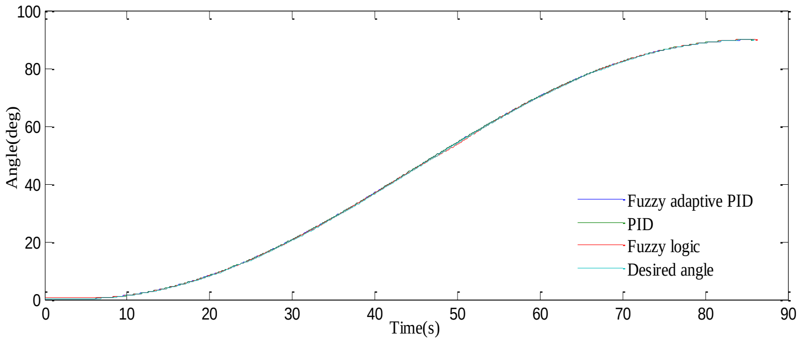
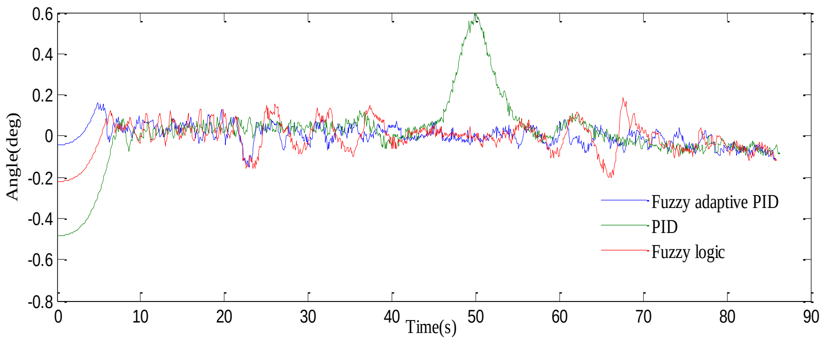


Figure 10. Mechanical Constitution of the Platform



(a) Experiment Erection Angle of Three Controllers



(b) Experiment Erection Angle Error of Three Controllers

Figure 11. Experiment Results of Erection Angle

6. Conclusion

Mathematical model of electro-hydraulic system was derived. Kinetic model of erection mechanism was established. Fuzzy adaptive PID algorithm was applied to erection mechanism in combination of fuzzy logic and PID algorithm. The model of erection system was established in Simulink. The simulation and experiment results of three control algorithms were compared and verified. From the results, it is obviously obtained that fuzzy adaptive PID algorithm has the best control effect. Hence the designed algorithm could be applied in erection system to achieve high control accuracy.

Acknowledgement

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