

Particle Swarm Optimization Performance: Comparison of Dynamic Economic Dispatch with Dantzig-Wolfe Decomposition

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Abstract

Economic Dispatch (ED) problem, in practice, is a nonlinear, non-convex type, which has developed gradually into a serious task management goal in the planning phase of the power system. The prime purpose of Dynamic Economic Dispatch (DED) is to minimize generators' total cost of the power system. DED is to engage the committed generating units at a minimum cost to meet the load demand while fulfilling various constraints. Utilizing heuristic, population-based, and advanced optimization technique, Particle Swarm Optimization (PSO), represents a challenging problem with large dimension in providing a superior solution for DED optimization problem. The feasibility of the PSO method has been demonstrated technically, and economically for two different systems, and it is compared with the Dantzig-Wolfe technique regarding the solution quality and simplicity of implementation. While Dantzig-Wolfe method has its intrinsic drawbacks and positive features, PSO algorithm is the finest and the most appropriate solution. Conventional techniques have been unsuccessful to present compatible solutions to such problems due to their susceptibility to first estimates and possible entrapment into local optima which may complicate computations.

Keywords: *particle swarm optimization (PSO), Dantzig-Wolfe decomposition, problem formulation, dynamic economic dispatch (DED)*

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1. Introduction

Dynamic economic dispatch (DED) problem is one of the essential operational matters in power system. The main objective of employing optimization technique is to minimize the total operation costs of the generation units while fulfilling all required constraints. Several traditional methods are used to solve this problem like Lagrangian and gradient methods [1-2]. Solving DED problem using these classical methods essentially assumes a monotonic increase in the curves representing the incremental cost of generators with a function characterized by a piecewise linearization. However, in practical system, the nonlinear characteristics of these generation units may not comply with such linearization assumption and resulting in an infeasibility status [3]. Large-scale power system involving large number of generating units with contradicting constraints and possible non-smooth cost function requires longer solution time. In Addition, various constraints may play a causative role in increasing the dimension of the DED problem. Therefore, requiring large computational resources, excessive numerical iterations, and enormous calculation efforts [4]. To make more convenient formulation to the numerical methods for solving the DED problem optimally, Dantzig-Wolfe DW is used to solve DED problems for units with quadratic functions for the fuel cost and considering various equality and inequality constraints. Although the DW method have been utilized to find solution for complex DED optimization problems, some problems have been identified in DW due to complicated problem formulation [1-2].

Dynamic economic dispatch (DED) is a technique to dispatch the generating units to the anticipated demands for electrical power over a specified time interval at minimum operational

cost while satisfying equality and inequality constraints [5-6]. DED problems are non-linear, complicated, dynamic optimization problem. Many methods have been developed to find optimum solution for the dynamic economic dispatch problem, such as linear programming [7], Lagrange relaxation method [8], dynamic programming [9]. Nevertheless, the discontinuity and nonlinearity of the search domain, to obtain the optimal solution lead to sub-optimal solution due to the entrapping in a local optimum [7].

Kennedy and Eberhart [11] introduce Particle swarm optimization (PSO) as a modern heuristic technique which mimics the behavior of birds flock or fish school. The PSO algorithm can lead to a higher quality solution with time and secure convergence in comparison with other stochastic methods. DED is solving the economic dispatch in every time increment power variation.

In the recent years, new meta-heuristic optimization approaches and methods are being significantly utilized as an alternative to the traditional methods to address the DED problem regarding quality, speed, and efficiency, due to their favorable search characteristics as population-based. PSO technique was adopted to address nonlinearity and complexity issues of the optimization problem [10-16].

PSO has been characterised with several advantages of crucial importance over current optimization methods on their speed of convergence, robustness, and distinctive simplicity [12]. Because the established process of PSO involves two basic updating rules only, to implement in computer simulations using basic logic and mathematical operations is easy. Furthermore, PSO can be compliant when hybridized with other optimization techniques because it has a fewer number of operators to conform to other techniques in the implementation process [12-13].

PSO shows that the particles' motion is regulated by its previous velocity, besides two other elements of acceleration, namely cognitive component and social component. Cognitive and social components depend on the acceleration coefficients and the uniformly distributed random numbers associated with PSO variants. The behavior of the particles is highly dependent on the relative values of these components. In case the cognitive component has a higher value compared to the social component, it will result in aimlessly unrestrained motion of a particle through search space. On the contrary, particles may results in an untimely advance towards local optima and assume it as the required solution when the social element has a relatively high value, in other words, it is more susceptible to be entrapped into local optima.

Exploring search space and exploiting local domain highly rely on the values of the particles velocity where every dimension's particles velocity is ensured to have a maximum velocity V_{max} . If this maximum velocity assumes high value initiates global exploration, conversely, low value motivates local exploitation. For this reason, Shi and Eberhart [17] propose the inertia weight concept to efficiently manipulate exploration and exploitation and attaining enhanced quality of the optimal solution and minimizing convergence time.

Unlike alternative and similar modern optimization techniques like genetic algorithms which have exorbitant evolutionary operations regarding computational resources such as mutation and crossover, PSO facilitates a better performance and expedite convergence [10]. The mechanism of PSO makes it a derivative-free algorithm unlike the classical optimization techniques and this feature especially makes it suitably effective in handling nonlinearity and complex problems. PSO shows more robustness to deal with such problems because it is less susceptible to the objective function nature regarding continuity and convexity [14] to the optimizer parameters [9]. Inherently, the inner working mechanism of PSO assists in breaking free from local optima.

In this paper, the particle swarm optimization (PSO) algorithm was proposed to deal with the DED problem considering various equality and inequality constraints. Compared with the Dantzig-Wolfe decomposition technique. The performance of the proposed optimization method was tested on a real data system with 20 and 100 generation units as a test system.

2. Problem Formulation

The DED problem is to assign each committed generating unit with a portion of the system load demand over the program time horizon achieving the main objective of minimizing the operation cost while taking physical constraints into consideration through satisfying their limits in addition to other operational matters in the form of specified requirements. Mathematically, this optimization problem can be formulated as a nonlinear programming

problem. The objective function to be optimized (minimized in this case) is the Total generation cost:

$$\text{Min } \sum_{i=1}^N F_i(P_i) \quad (1)$$

In practice, usually, $F_i(P_i)$ is expressed in form of a quadratic function as follows:

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad (2)$$

Where, a_i , b_i and c_i represent the cost coefficients of the generator, N is the number of generators, P_i is the power produced by the i^{th} generator (MW), $F_i(P_i)$ is the operating cost of the generation unit i (\$/h). The optimization is subject to the following constraints.

2.1. Equality Constraints

The power balance which is defined as the equality constraints, where the total generated power must meet the total power demand requirement and the power loss which may include the spinning reserve, formulated as in the following equation:

$$\sum_{i=1}^N P_i - D - P_L = 0 \quad (3)$$

2.2. Inequality Constraints

Taking the operational limits on physical devices to ensure safe and stable production of power in addition to guarantee system security dictates the consideration of another set of constraints referred to as the inequality constraints, which are represented by the following formula:

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad \text{for } i = 1, 2, 3 \dots NG$$

$$P_L = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j + \sum_{i=1}^N B_{0i} P_i + B_{00} \quad (4)$$

Ramp rate limit, or the loading and deloading rate limits of the generator are defined based on practical aspects and operational considerations of the generators such as mechanical stresses and load, Figure 1. Therefore, the capacity of generating units requires a finite time to change the capacity of a specified thermal plant.

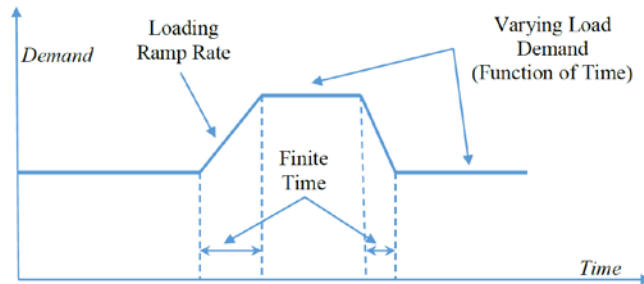


Figure 1. Typical load profile unit variations with time

2.3. Additional Constraints

In addition to equality and inequality constraints, dynamic economic dispatch addressed in this paper also considers an additional constraint which is spinning reserve and group constraints. Spinning reserve generates the extra capacity to handle failure, unscheduled interruption, and unexpected load variation. Spinning reserve of generators is proportional to the generation level below a defined output known as the Spinning Reserve Level (SL) and equal to the spare capacity above SL. To formulate this constraint mathematically, we have the following equation:

$$S_i = \begin{cases} k_i G_i & \text{for } G_i^{min} \leq G_i \leq SL_i \\ G_i^{max} - G_i & \text{for } SL_i \leq G_i \leq G_i^{max} \end{cases}$$

Another type of constraint, referred as the group constraint, wherein different generators combined output is limited by certain boundaries. The causes may include regulatory restrictions, limitations in transmission line conducting capacity, and area security considerations.

3. Particle Swarm Optimization with Dantzig-Wolfe Method

Particle swarm optimization (PSO) is a population-based continuous optimization technique and one of the gradually developed modern optimization method, proposed by Kennedy and Eberhart (1995). A population of random solutions is used to initialize the system for iterative searches for optima by continuously updating the generation levels [13]. PSO is characterized by advantageous features, unlike other similar evolutionary technique like the Genetic Algorithm GA with no costly evolutionary operators like mutant and crossover which makes PSO suitable for providing better performance and expediting convergence. The particles in the PSO represent the potential solutions; these particles change their positions through the search space by traveling towards the present particles.

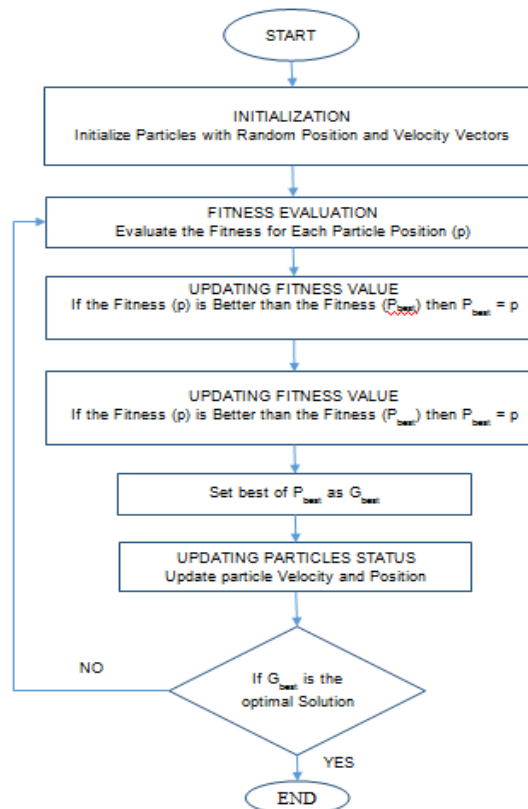


Figure 2. Particle Swarm Optimization Generic Flowchart

Particle Swarm Optimization can be utilized to address several problems associated with other similar modern algorithms. PSO features group interaction which provides a pool of shared information which acts as a memory that facilitates to progress toward the optimal solution. Within the set of the population, each particle keeps track of 'memorizing' the best solution referred to as the P_{best} , which is associated with its coordinates in the hyperspace. Following this trend and tracking another best value until the 'Global' version of the PS optimizer

is reached through the tracking of the overall best value, which is called Gbest. The concept of PSO involves varying the velocity of each particle toward its P_{best} and G_{best} at each step as illustrated in, Figure 2. Acceleration toward P_{best} and G_{best} is being weighted according to a random term [14, 15]. In the beginning, the research is aiming at model the social behavior of bird flocks, fish schools, and animal herds graphically. Nevertheless, the original version is capable of addressing nonlinear continuous optimization problems only. Further developments in this PSO algorithm have enabled the search for global optimal solutions of complex engineering and sciences problems.

Comparing particle swarm optimization PSO with Dantzig-Wolfe D-W regarding the following points:

1. Implementation: D-W method is difficult and cumbersome. Constructing the block diagonal structure for decomposition requires an enormous manual work, observation and decision. On the other hand, the PSO can be more easily implemented both for constraints and objective function to be optimized.
2. Addition or removal of constraints can be achieved easily with PSO, while in the D-W, this process needs to be analyzed from the beginning.
3. The D-W method takes less time to find the solution in comparison with PSO method. However, this problem can be greatly alleviated by adopting hybridization to reach the required optimal solution.

Code debugging of unintentional errors in the implemented block diagonal structure of the D-W method is tiresome and susceptible to further mistakes while PSO code can be debugged easily.

4. Optimization Results and Discussions

The performance of the developed algorithm of particle swarm optimization PSO is tested using real data of power system in two case studies; with a primary target of minimizing the cost function for both case studies. The first case study involves twenty generation units with twenty-four periods, for this problem, the equality constraints, inequality constraints, ramp rate limits, spinning reserve, and bound generation limits have been defined for each generation units. The optimizer has to find an optimum solution to a problem with 480 variables which is considered a high dimensionality problem.

The second case study involves one hundred generation units with five periods with a similar type of constraints as in the first case study in addition to the group constrained which is associated with this problem of 500 variables, which means another high-dimensionality problem.

The simulation results cover two cases; an optimal case which involves minimizing the cost function subject to the constraints of total periods taken into consideration at the same time. While, the sub-optimal case involves finding the solution period by period subject to the constraints, thus reaching a sub-optimal solution, but in sub-optimal case the required computational resources are far less than in the optimal case. The input data for both cases are given in the Appendix A.

4.1. Case Study 1

The first case study examined the performance of the PSO algorithm for the 20 generators with 24 periods. This result is an optimization problem with a dimension of 500 (20x25, including the initial conditions) variables to be evaluated and optimized accordingly. In addition to the ramp rate constraints, this problem had spinning reserve constraints. The 500 variables problem was solved using the PSO method. For 24 period's case, the total cost using PSO method was minimized to 98836.58 units as the optimal solution compared to 99545.54 units using Dantzig-Wolfe method. As for the sub-optimal solution where the problem was solved period by period, the total cost for PSO method was found to be 98843.06 units compared 99595.35 units using Dantzig-Wolfe method. The results were found for different periods for this problem, the periods were 6, 12, 18, and 24 and the costs are shown in Table 1. It summarizes the findings for this case of 20 generator and different periods and the results of Dantzig-Wolfe Solution for both optimal and sub-optimal are taken from Ab Ghani (1989), [1]. PSO method gives better cost value for each optimal and sub-optimal case with different periods.

Table 1. Simulation results of 20-generator with 24 periods system
Comparison between the Dantzig-Wolfe and PSO methods for DED

No.	Generators / periods	Case	Cost (Unit Cost)	
			PSO	D-W
1	20/ 6	Optimal	27000.78	27028.43
2		Sub-Optimal	27033.60	27031.22
3	20/12	Optimal	52207.89	52652.89
4		Sub-Optimal	52210.53	52653.22
5	20/18	Optimal	76416.21	76982.39
6		Sub-Optimal	76455.49	77000.74
7	20/24	Optimal	98836.58	99545.54
8		Sub-Optimal	98843.06	99595.35

D-W: Dantzig-Wolfe decomposition method

PSO: Particle Swarm Optimization method

Optimal: Solution over the entire periods

Sub-Optimal: Solution period-by-period

4.2. Case Study 2

The second case study examined the performance of the PSO algorithm for the 100 generators with five periods. This result is an optimization problem with a dimension of 600 (100x6, including the initial conditions) variables to be evaluated and optimized accordingly. In addition to the ramp rate constraints, this problem had spinning reserve constraints and group constraints. The 600 variables problem was solved using the PSO method. The total cost was minimized to 659395.10 units as the optimal solution compared to 663017.40 units using Dantzig-Wolfe method. As for the sub-optimal solution where the problem was solved period by period, the total cost was found to be 659430 units compared 663099 units using Dantzig-Wolfe method.

Table 2 summarizes the findings for this case of 100 generators with five periods (with 22 group import-export constraints) and shows the results of Dantzig-Wolfe simulation results for both optimal and sub-optimal [1-2]. For both case studies, the particle swarm optimization PSO outperforms the Dantzig-Wolfe D-W method in finding the minimum cost by solving the optimization problem subject to similar constraints for both approaches. In addition to the advantages and beneficial features that characterize PSO over the classical technique D-W as explained in this paper, the achieved solution for both situation optimal and sub-optimal results in a better cost value when solved by PSO compared to D-W method.

Table 2. Simulation results of 100-generator with five periods system
Comparison between the two methods for 100-generator system (with 22-group import-export constraints)

No.	Generators / periods	Case	Cost (Unit Cost)	
			PSO	D-W
1	100/5	Optimal	659395.10	663017.40
2		Sub-Optimal	659430.00	663099.00

5. Conclusion

In this work, the formulation and implementation of solution methods to obtain the optimum solution of dynamic economic dispatch using PSO method and compared with the solution results of Dantzig-Wolfe decomposition method is carried out. PSO technique can be utilized to solve similar problems as Dantzig-Wolfe decomposition. However, PSO method does not suffer from some of D-Ws difficulties: interaction within the group reinforces progress toward locating the optimal solution. Moreover, the memory acquired by the PSO technique, which is lacking in the Dantzig-Wolfe, increases the probability of attaining the global solution. Change in Dantzig-Wolfe constraints or objectives results in nullifying of all previous structure of the problem, except when a minor amendment is required, in which case usually a small number of individual parameters retain their "values." In particle swarm optimization, particular particles that fly past optima points are compelled to return toward them; through the social component, the knowledge of good 'best' solutions is possessed at all time by all particles. PSO has also

been demonstrated to perform well compared to the D-W algorithm on real data network problems, and it appears to attain a better minimum cost value. The effectiveness of the developed PSO program is tested for twenty generators with 24 periods and 100 generators with 5 periods experimental data. The results obtained by these methods are compared with each other. It is found that PSO optimization algorithm is giving better results than D-W decomposition optimization techniques.

Acknowledgements

The authors gratefully acknowledge the funding provided by the Ministry of Electricity of Iraq (MOE) and their continuous support and encouragement. The authors also would like to gratefully acknowledge the funding support provided by the Ministry of Higher Education Malaysia under the research grant No: FRGS/2/2013/ TK02/FKE/01/F00167.

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Appendix A (Ab Ghani, M. R., [1])

Table A1. Input parameters for the 20 generators DED problem

Generator No.	Generation limit (MW)		Loading rate (MW/hr)	De-loading rate (MW/hr)	Cost (units)	SL (MW)
	maximum	minimum				
1	430	360	300	600	1.0000	0
2	410	360	300	600	1.0063	0
3	82	50	180	600	1.0111	77
4	82	50	180	600	1.0124	77
5	82	50	180	600	1.0137	77
6	82	50	180	600	1.0150	77
7	430	250	300	900	1.0629	411
8	430	300	300	900	1.0636	411
9	430	140	300	900	1.0643	411
10	102	70	240	360	1.1304	92
11	483	130	180	600	1.1318	463
12	483	130	180	600	1.1325	463
13	483	130	180	600	1.1332	463
14	483	130	180	600	1.1339	463
15	102	70	240	360	1.1464	92
16	102	70	240	360	1.1512	92
17	56	30	120	600	1.1548	51
18	56	30	120	600	1.1565	51
19	57	30	300	360	1.2327	52
20	28	15	120	120	1.4457	26

Table A2. Demand and Spinning Reserve Data (20 generators)

Period	Demand (MW)	Required reserve (MW)	Period	Demand (MW)	Required reserve (MW)
0	4346	80	13	3825	80
1	4240	80	14	3776	80
2	4214	80	15	3847	80
3	4124	80	16	3859	80
4	4097	80	17	3778	80
5	4074	80	18	3567	80
6	4173	80	19	3335	80
7	4267	80	20	3220	80
8	4147	80	21	3247	80
9	3918	80	22	3418	80
10	3690	80	23	3856	80
11	3769	80	24	3983	80
12	3851	80			

Table A3. Input parameters for the 100 generators DED problem

Generator No.	Generation limit (MW)		Loading rate (MW/hr)	De-loading rate (MW/hr)	Cost (units)	SL (MW)
	maximum	minimum				
1	60	10	120	180	19	55
2	60	10	120	180	19	55
3	60	10	120	180	20	55
4	60	10	120	180	20	55
5	60	10	120	180	20	55
6	100	20	120	360	20	90
7	100	20	120	360	20	90
8	500	50	1000	1500	15	0
9	500	50	1000	1500	15	0
10	500	50	1000	1500	15	0
11	500	50	1000	1500	15	0
12	60	10	120	300	19	55
13	60	10	120	300	19	55
14	60	10	120	300	19	55
15	100	20	120	300	20	90
16	100	20	120	300	20	90
17	100	20	120	300	19	90

18	100	20	120	300	19	90
19	100	50	120	300	20	94
20	100	20	120	300	20	90
21	100	20	120	300	20	90
22	100	20	120	300	20	90
23	60	10	30	180	21	55
24	50	20	30	180	22	48
25	40	10	30	180	22	38
26	60	30	30	180	21	56
27	50	10	60	180	20	46
28	50	10	60	180	20	46
29	60	10	120	300	19	55
30	60	10	120	300	19	55
31	60	10	120	300	19	55
32	100	20	120	300	20	90
33	100	20	120	300	20	90
34	100	20	120	300	20	90
35	100	20	120	300	20	90
36	100	20	120	300	19	90
37	50	10	60	180	19	46
38	50	10	60	180	19	46
39	50	10	60	180	20	46
40	50	10	60	180	20	46
41	50	10	60	180	20	46
42	50	20	60	180	19	46
43	50	10	60	180	19	46
44	50	10	60	180	19	46
45	50	20	60	180	21	48
46	50	20	60	180	22	48
47	60	10	60	180	19	55
48	60	10	60	180	19	55
49	60	10	60	180	19	55
50	60	10	60	180	19	55
51	30	5	30	180	22	28
52	30	5	30	180	22	28
53	30	5	30	180	22	28
54	30	5	30	180	22	28
55	30	5	30	180	22	28
56	60	10	60	180	20	55
57	60	10	60	180	20	55
58	60	10	60	180	20	55
59	50	20	60	180	21	48
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61	50	20	60	180	21	48
62	50	30	60	180	21	48
63	50	20	60	180	21	48
64	50	10	60	180	21	48
65	50	20	60	300	21	48
66	100	20	60	300	18	46
67	100	20	60	180	18	48
68	60	20	60	180	20	90
69	60	10	60	180	20	90
70	60	10	60	180	20	55
71	60	10	60	180	20	55
72	60	10	60	180	20	55
73	50	10	60	180	19	46
74	50	10	60	180	21	46
75	50	10	60	180	21	46
76	50	10	60	180	21	46
77	50	10	60	180	21	46
78	60	20	60	180	20	55
79	60	20	60	180	19	55
80	50	10	60	180	15	46
81	500	50	1000	1500	16	0
82	400	40	1000	1500	15	0
83	500	50	1000	1500	20	0
84	50	10	60	180	19	46
85	50	10	60	180	19	46
86	50	10	60	180	19	46
87	50	10	60	180	19	46
88	50	10	120	180	19	46
89	40	10	120	180	19	38

90	60	20	120	180	20	55
91	60	20	120	180	20	55
92	50	10	120	180	20	46
93	60	20	120	180	20	55
94	50	10	120	180	22	46
95	50	10	120	180	22	46
96	50	30	120	180	21	48
97	50	20	120	180	22	48
98	50	20	120	180	22	48
99	50	20	120	180	22	48
100	50	20	120	180	22	48

Table A4. Demand and Spinning Reserve Data (100 generators)

Period	Demand (MW)	Required reserve (MW)
0	6464	240
1	7000	240
2	7500	240
3	7250	240
4	7700	240
5	7100	240

Table A5. Group Constraints Data (100 generators)

Group limits		Generators in group
Lower	Upper	
40	250	1,2,3,4,5
40	200	6,7
100	1500	8, 9, 10, 11
20	160	12, 13, 14
140	750	15, 16, 17, 18, 19, 20, 21, 22
40	200	23, 24, 25, 26
20	2000	27, 28
10	450	32, 33, 34, 35, 36
10	190	37, 38, 39, 40
10	150	45, 46
40	200	47, 48, 49, 50
10	160	51, 52, 53, 54, 55
30	200	56, 57, 58
100	300	59, 60, 61, 62, 63, 64, 65
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