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Reliability Evaluation of Low-voltage Switchgear Based on Maximum Entropy Principle

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Abstract

In this paper, based on the definition of two-parameter joint entropy and the maximum entropy principle, a method was proposed to determine the prior distribution by using the maximum entropy method in the reliability evaluation of low-voltage switchgear. The maximum entropy method takes kinds of priori information as different constraints. The optimal prior distribution was selected by maximizing entropy under these constraints, which not only contains the known prior information but also tries to avoid the introduction of other assumption information. Based on non-parametric bootstrap method, the hyper-parameters of prior distribution is obtained by two-order moment of prior information. Finally, with the bootstrap method, the prior distribution robustness and the posterior robustness were analyzed, and the posterior mean time between failures for the low-voltage switchgear was estimated.

Keywords: *prior distribution, maximum entropy, low-voltage switchgear, reliability evaluation*

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1. Introduction

Low-voltage switchgear is responsible for power control, protection, measurement, transformation and distribution in low-voltage power supply system. For the reliability evaluation of low-voltage switchgear with high reliability and long life, traditional reliability assessment method needs to obtain sufficient data by a large number of sample life tests which is time-consuming, costly and inefficient [1]. Therefore, the rational use of empirical and historical data to determine priori distribution can lay a solid theoretical foundation for the reliability evaluation of low-voltage switchgear.

Bayes method can make good use of not only the field test information, but also priori information, such as historical test information, test information for similar models and the same type products with different conditions, and so on. And the priori information can be used to get the priori distribution which increases the failure data. This method has been applied in most fields, such as medical system [2], web [3], electrical engineering [4], finance [5], and speech recognition [6]. While, there is no discussion about low-voltage switchgear.

In this paper, Bayes method is used to evaluate the reliability of low-voltage switchgear. The priori distribution of low-voltage switchgear is determined by maximum entropy method which avoids the introduction of other assumption information because of the using of priori information. According to maximum entropy principle, priori information can be taken as different contrains, and the optimal prior distribution can be selected by maximizing entropy under these constraints. The non-parametric bootstrap method is used to expand data capacity and then hyper-parameters of priori distribution is eatimated. Finally, with the bootstrap method, the prior distribution robustness and the posterior robustness is analyzed, and the posterior mean time between failures for the low-voltage switchgear is estimated.

2. Maximum Entropy Method

During the practical application of Bayes theory, there is some available prior information, but you would like try not to introduce any other assumptive information except the priori information. This problem can be effectively solved by the concept of entropy [7]. Entropy

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is definited that when Θ is discrete, π is the probability density of Θ, $\varepsilon_N(\pi)$ means the entropy of π. And $\varepsilon_N(\pi)$ is defined as:

$$\varepsilon_N(\pi) = -\sum_{\Theta} \pi(\theta_i) \log \pi(\theta_i) \quad (1)$$

If $\pi(\theta_i)$=0, $\pi(\theta_i)\log\pi(\theta_i)$ can be defined as zero.

It is assumed that part of prior information about θ has been known, then these information can be expressed by some constraints of $\pi(\theta_i)$. The assumption is:

$$E^{\pi}\left[g_k(\theta)\right] = \sum_i \pi(\theta_i) g_k(\theta_i) = \mu_k \left(k = 1,..,m\right) \quad (2)$$

In the premise of meeting the above constraints (including $\Sigma\pi(\theta_i)$=1), the obtained solution is:

$$\bar{\pi}(\theta_i) = \frac{\exp\left\{\sum_{k=1}^{m} \lambda_k g_k(\theta_i)\right\}}{\sum_i \exp\left\{\sum_{k=1}^{m} \lambda_k g_k(\theta_i)\right\}} \quad (3)$$

Where, λ_k is the constant determined by the above-mentioned constraints.

When Θ is continuous, Jaynes defined entropy as [8,9]:

$$\varepsilon_N(\pi) = -E^{\pi}\left[\log \frac{\pi(\theta)}{\pi_0(\theta)}\right] = -\int \pi(\theta) \log\left(\frac{\pi(\theta)}{\pi_0(\theta)}\right) d\theta \quad (4)$$

Where, $\pi_0(\theta)$ is the non-information prior. While people can still use the following definition.

Part of prior information are expressed as follows:

$$E^{\pi}\left[g_k(\theta)\right] = \int_{\Theta} g_k(\theta) \pi(\theta) d\theta = \mu_k \left(k = 1,..,m\right) \quad (5)$$

The prior density that $\varepsilon_N(\pi)$ can be got the maximum is determined by formula (6).

$$\bar{\pi}(\theta) = \frac{\pi_0(\theta) \exp[\sum_{k=1}^{m} \lambda_k g_k(\theta)]}{\int_{\Theta} \pi_0(\theta) \exp[\sum_{k=1}^{m} \lambda_k g_k(\theta)] \mathrm{d}\theta} \quad (6)$$

Where, λ_k is a constant determined by formula (7).

$$\varepsilon_N(\pi) = -E^{\pi}\left[\log \frac{\pi(\theta)}{\pi_0(\theta)}\right] = -\int \pi(\theta) \log\left(\frac{\pi(\theta)}{\pi_0(\theta)}\right) d\theta \quad (7)$$

3. The Maximum Entropy Method for the Prior Distribution of Low-voltage Switchgear

3.1. The Joint Prior Distribution Determined by the Maximum Entropy

The basic viewpoint of maximum entropy principle is that, to accurately infer the system state in the case where there is only part of the information, a reasonable state should be selected to meet the constraint condition and maximize the entropy.

Formula (8) is the expression of the joint prior distribution $\pi(\eta,m)$, and formula (9) to formula (11) are its constraint conditions.

$$\max H(\eta,m) = -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi(\eta,m) \ln \pi(\eta,m) \, d\eta dm \quad (8)$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\pi(\eta,m)d\eta dm=1 \tag{9}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\eta^{i}\pi(\eta,m)d\eta dm=E(\eta^{i})=\eta_{i},i=1,\ldots k \tag{10}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}m^{i}\pi(\eta,m)d\eta dm=E(m^{j})=m_{j},j=1,\ldots n \tag{11}$$

Where, η_i, m_j are the origin moments of i order and j order of two-parameter η and m respectively; k and n is the highest order of two-parameter origin moment respectively.

If one-order moment and two-order moment are exist, when k=n=2, by constructing and solving the auxiliary fonctionelle of joint density function, $\pi(\eta,m)$ can be known [10, 11],

$$\pi(\eta,\ m)=\frac{1}{2\pi\sigma_{\eta}\sigma_{m}}\exp(-\frac{(\eta-\overline{\eta})^{2}}{2\sigma_{\eta}^{2}}-\frac{(m-\overline{m})^{2}}{2\sigma_{m}^{2}}) \tag{12}$$

3.2. Parameter Determination of Prior Distribution

The bivariate normal prior distribution contains hyper-parameters which can be calculated by prior information. So, for the problems of small samples with insufficient historical data, the small sample data can be re-sampled to expand the data capacity by using bootstrap method to simulate the general characteristics and determine the prior information [12, 13].

The methods of determining hyper-parameter by using non-parametric bootstrap method are described as follows [14].

T=(t_1,t_2,…t_n) denotes a set of obtained samples with total failure. The distribution parameter samples can be obtained by the following steps.

1) The self-help sample $T^{*}=(t_{1}^{*},t_{2}^{*},\ldots,t_{\mathrm{n}}^{*})$ can be obtained by carrying out sampling with replacement for T=(t_1,t_2,…t_n);

2) $\hat{m}$ and $\hat{\eta}$ can be got from the maximum likelihood estimation with the self-help sample $T^{*}=(t_{1}^{*},t_{2}^{*},\ldots,t_{\mathrm{n}}^{*})$;

3) Repeat the previous two steps for N times, then the estimated parametric sample can be got:

$$\{(\hat{m}_1,\hat{\eta}_1),(\hat{m}_2,\hat{\eta}_2),\ldots,(\hat{m}_N,\hat{\eta}_N)\} \tag{13}$$

4) The expectations and variances of m and η can be got by the obtained estimation parametric samples.

$\bar{\hat{m}}$ and S_{m}^{2} denote the expectation and variance of the shape parameter m respectively, $\bar{\hat{\eta}}$ and S_{η}^{2} denote the expectation and variance of the scale parameter η. Then:

$$\bar{\hat{m}}=\frac{1}{n}\sum_{\mathrm{j=1}}^{n}\hat{m}_{j},S_{m}^{2}=\frac{1}{n-1}\sum_{\mathrm{j=1}}^{n}\left(\hat{m}_{j}-\bar{\hat{m}}\right)^{2} \tag{14}$$

$$\bar{\hat{\eta}}=\frac{1}{k}\sum_{\mathrm{i=1}}^{k}\hat{\eta}_{i},S_{\eta}^{2}=\frac{1}{k-1}\sum_{\mathrm{i=1}}^{k}\left(\hat{\eta}_{i}-\bar{\hat{\eta}}\right)^{2} \tag{15}$$

By calculating the transcendental moment, the hyper-parameter expressions of the prior distribution can be obtained:

$$\overline{m}=\bar{\hat{m}},\sigma_{\mathrm{m}}^{2}=S_{\mathrm{m}}^{2},\overline{\eta}=\bar{\hat{\eta}},\sigma_{\eta}^{2}=S_{\eta}^{2} \tag{16}$$

The life distribution of a certain low-voltage switchgear obeys Weibull (3600,1.5). 30 samples are randomly generated as the historical failure data, that is the prior information, as shown in Table 1.

Table 1. Historical Failure Data of Low-voltage Switchgear (unit: hours)

950	6439	3432	2201	1273	809	5552	3746	5365	5140
2708	5328	280	3315	1849	6039	4144	3090	2384	2050
1836	7962	3044	3254	1950	2400	2034	1330	1607	840

The parameters of the bivariate normal prior distribution are as follows:

$$\bar{m}=1.69, \sigma_{\mathrm{m}}^{2}=0.0509, \bar{\eta}=3740, \sigma_{\eta}^{2}=180121 \tag{17}$$

Then the prior density function of low-voltage switchgear can be got:

$$\pi(\eta,\ m)=\frac{1}{18336.3178\pi}\exp(-\frac{(\eta-3740)^{2}}{360242}-\frac{(m-\overline{m})^{2}}{0.1018}) \tag{18}$$

3.3. Robust Analysis of Prior Distribution

Before using the prior distribution, it is necessary to analyze its impact on the posteriori statistical inference results, which is known as the robust analysis of prior distributions [15, 16].

A way to deal with priori uncertainty is to try to exclude the priori which should not be considered in the aspect of data from the considered priori. Marginal density $m^*(t/\pi)$ can play a very important role because it is known as the relative likelihood marginal distribution functions. The marginal distribution values of relative likelihood functions represent the relative probability of field test samples with $\pi(\eta,m)$ as the prior distribution.

$m^*(t_i/\pi)$, i=1,…N can be calculated when the field samples ($t_1,t_2,\dots t_N$) are obtained. The prior distribution can be considered conforming to the field test data, which shows the prior distribution $\pi(\eta,m)$ can be determined to be robust, if the value of $m^*(t_i/\pi)$ is not particularly small. For two-parameter Weibull distribution, the relative marginal likelihood function of joint prior distribution $\pi(\eta,m)$ is [17]:

$$m^{*}(t/\pi)=\frac{m(t/\pi)}{\sup_{i} m(t/\pi)}=\frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f\left(t/\eta,m\right)\pi\left(\eta,m\right)d\eta dm}{\max_{t}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f\left(t/\eta,m\right)\pi\left(\eta,m\right)d\eta dm} \tag{19}$$

To ensure the compatibility of prior information and the field testing information, the following test data ($T_1,\dots T_{10}$)=(885, 2165, 3560, 9001, 2036, 1110, 5602, 1630, 7187, 3720) as the field test information will be put into the following formula,

$$\mathrm{m}(T\mid\pi)=\left(\frac{m}{\eta}\right)^{N}\prod_{i=1}^{N}\left(\frac{t_i}{\eta}\right)^{m-1}\exp\left(-\sum_{i=1}^{N}\left(\frac{t_i}{\eta}\right)^{m}\right)\frac{1}{2\pi\sigma_n\sigma_m}\exp\left[-\frac{\left(\eta-\mu_\alpha\right)^{2}}{2\sigma_n^{2}}-\frac{\left(m-\mu_m\right)^{2}}{2\sigma_m^{2}}\right]d\eta dm \tag{20}$$

The relative marginal likelihood function values can be summarized in Table 2.

Table 2. Relative Marginal Likelihood Function Values of Prior Distribution

The first group	The second group	The third group	The forth group	The fifth group
0.5518	7.295	3.536	48.81	0.1241
The sixth group	The seventh group	The eighth group	The ninth group	The tenth group
3.719	5.701	2.568	0.5819	91.73

4. The Posterior Distribution and Reliability Estimation of Low-voltage Switchgear

Based on Bayes principle, it is assumed that the field life data of low-voltage switchgear is $T=(t_1,t_2,\ldots t_n)$, then the likelihood function can be expressed as:

$$p(t_1,t_2,t_3\ldots t_n\mid\eta,m)=\left(\frac{m}{\eta}\right)^n\prod_{i=1}^{n}\left(\frac{t_i}{\eta}\right)^{m-1}\exp\left(-\sum_{i=1}^{n}\left(\frac{t_i}{\eta}\right)^m\right) \quad (21)$$

The joint posterior distribution of parameter η and m can be expressed as:

$$h\left(\eta,m\mid t_1,t_2\cdots t_{\text{n}}\right)=\frac{p\left(t_1,t_2\cdots t_{\text{n}}\mid\eta,m\right)\pi\left(\eta,m\right)}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}p\left(t_1,t_2\ \cdots\ t_n\mid\eta,m\right)\pi\left(\eta,m\right)d\eta dm} \quad (22)$$

Then the point estimate of the posterior distribution can be obtained:

$$\hat{\eta}=E(\eta\mid t_1,t_2\ldots t_n)=3960,\ \ \hat{m}=E(m\mid t_1,\text{t}_2\ldots t_n)=1.5466 \quad (23)$$

Finally, the estimation of posterior mean time between failure of the low-voltage switchgear is:

$$\overline{MTBF}=\hat{\eta}\Gamma\left(1+1/\hat{m}\right)=3562\text{h} \quad (24)$$

5. The Posterior Robustness Analysis of Low-voltage Switchgear

Formula (12) can be rewrittened as follows:

$$\Gamma:\pi\left(\eta,\text{m}\right)\sim N\left(\mu_\eta,\mu_m,\sigma_\eta^2,\sigma_m^2\right) \quad (25)$$

Where,

$$\underline{\mu_\eta}\le\mu_\eta\le\bar{\mu}_\eta,\ \underline{\mu_m}\le\mu_m\le\bar{\mu}_m,\ \underline{\sigma_\eta^2}\le\sigma_\eta^2\le\overline{\sigma_\eta^2},\ \underline{\sigma_m^2}\le\sigma_m^2\le\overline{\sigma_m^2} \quad (26)$$

C_η and C_m are the value intervals of η and m, which is determined by the experience of experts. Also it is assumed that the acceptable maximum posterior expected loss within this confidence interval is γ. The posterior distribution $\pi(\eta,m/T)$ can be obtained according to the Bayes formula after the field date T is obtained. The parametric range of prior distribution whose posterior expected loss is no more than γ can be calculated by:

$$1-\int_{\eta\in C_\eta}\int_{m\in C_m}\pi\left(\eta,m\mid T\right)d\eta dm\le\gamma \quad (27)$$

In fact, it is difficult to calculate the parametric range of prior distribution by sloving the inequality (27). However, prior information can be used to get a set of parameters, based on which it is assumed that some of these parameters are constant, then change other parameters to get the approximate range of parameters.

In addition, by combining with the prior information and self-help sample methods, the sample mean and sample variance of the parametric samples can be obtained:

$$\bar{\bar{m}}=1.44,\ \ \text{S}_m^2=0.025,\ \ \bar{\bar{\eta}}=2962,\ \ \text{S}_\eta^2=1.98\times10^5 \quad (28)$$

Then a set of hyper-parameters of prior distribution can be got:

$$\mu_m=1.44,\ \mu_\eta=2962,\ \sigma_m^2=0.025,\ \sigma_\eta^2=1.98\times10^5 \quad (29)$$

A set of field samples *T* can be obtained by sampling method, as shown in Table 3.

Table 3. A set of Field Samples With Censored Data (unit: hours)

Complete failure data	203	323	934	1583	1947	1961	3206
Censored data	783	1353	1016	3097	3300		

The posterior distribution $\pi(\eta,m/T)$ can be obtained based on the test information of field samples.

To judge the robustness of the posterior distribution, it is necessary to know the parameter range of the prior distribution. The most commonly used method to get the range is by solving marginal distribution of every parametric obtained based on the posterior distribution. When solving, it is adopted to fix three parameters and chang another parameter. Then the parametric range of prior distribution is got by analyzing the obtained posterior expected loss.

According to their experience, experts provide the confidence intervals C_m=(1, 1.7), C_η=(2500, 3500) and the acceptable maximum posterior expected loss γ=0.2. Other parameters in the prior distribution are fixed, and μ_η is calculated through formula (30) with 50 as the unit and the value got from 2500 to 3500 in turn.

$$\int_{\eta\in C_\eta}\left[\int_0^{+\infty}\pi(\eta,m|T)dm\right]d\eta \qquad (30)$$

Where, $\mu_m = 1.44, \sigma_m^2 = 0.025, \sigma_\eta^2 = 1.98\times10^5$. Results are summarized in Figure.1.

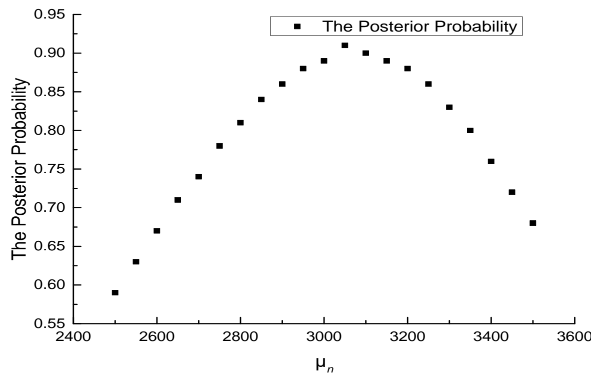

Figure 1. The posterior probability with different μ_η

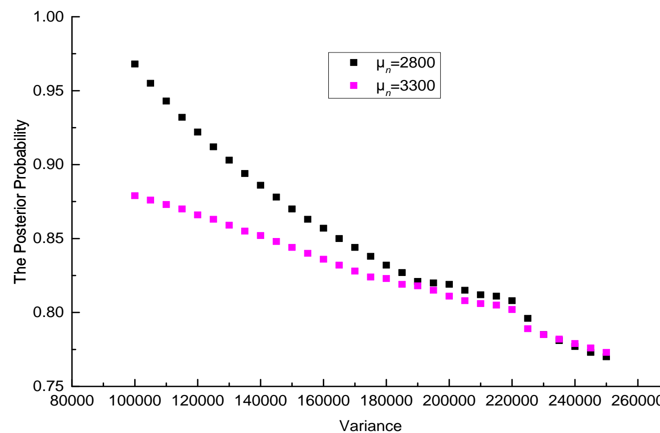

Figure 2. The posterior probability with different variances when μ_η=2800 and μ_η=3300

It can be shown that in the confidence interval C_η=(2500, 3500), when μ_η is changed between 2800 and 3350, it can be sure that the posterior expected loss ρ is less than 0.2.

The posterior probability of σ_η^2 changed from 1.0×10^5 to 2.5×10^5 are showm in Figure 2 when μ_η is 2800 and 3300 respectively. It can be known that the posterior expected loss is less than 0.2 when μ_η is changed between 2800 and 3300, C_η=(2500, 3500), and $\sigma_\eta^2 \le 2.2\times10^5$.

The same method can also be used to calculate the range of μ_m when C_m=(1, 1.7) and $\rho\le0.2$. Then it can be got that the value range of μ_m is changed between 1.1 and 1.54.

Therefore, the prior distribution can be choosed by:

$$\Gamma=\left\{\begin{array}{l}\pi(m,\eta)=\dfrac{1}{2\pi\sigma_m\sigma_\eta}\exp\left(-\dfrac{(m-\mu_m)^2}{2\sigma_m^2}-\dfrac{(\eta-\mu_\eta)^2}{2\sigma_\eta^2}\right)\\ 1.1\le\mu_m\le1.54, 2800\le\mu_\eta\le3300, \sigma_m^2\le0.05, \sigma_\eta^2\le2.2\times10^5\end{array}\right\} \quad (31)$$

It can be sure the posterior expected loss is less than 0.2 within the given confidence intervals if the parameters of prior distribution are taken values in a wider range. Therefore, the prior distribution is robust.

6. Conclusion

According to the maximum entropy principle, it takes kinds of priori information as different constraints. The optimal prior distribution is selected by maximizing entropy under these constraints. The obtained prior distribution not only contains the known prior information but also avoids the introduction of other assumption information. Taking the historical failure data of low-voltage switchgear as priori information, these hyper-parameters of prior distribution were estimated to get the prior density function by two-order moment of prior information. According to the field test data, it can be obtained that the posterior mean time between failures for the low-voltage switchgear is 3562 h by analyzing the reliability of low-voltage switchgear. When given the confidence intervals C_m=(1, 1.7), C_η=(2500, 3500)and the acceptable maximum posterior expected loss γ=0.2 by experts experience, the prior distribution robustness and the posterior robustness were analyzed with the bootstrap method.

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