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Design and Implementation of Efficient Analysis and Synthesis QMF Bank for Multicarrier Cognitive Radio Communication

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Abstract
The present section deals with a new type of technique for designing an efficient two channel Quadrature Mirror Filter Bank with constant phase in frequency. For achieving the Perfect Reconstruction Condition in Filter bank, an attempt has been made to design the low pass prototype filter with its impulse response and frequency response in three regions namely pass band, stop band and transition band region. With the error in terms of Reconstruction and the attenuation in the stop band as seen in the prototype filter response, one can evaluate the performance of the introduced filter with the help of filter coefficients generated in the design examples that affects the quality of filter bank design under the constraints of Near Perfect Reconstruction Conditions.

Keywords: analysis filter bank, synthesis filter bank, quadrature mirror filter, cognitive radio

1. Introduction

The Sensing of the available spectrum is a gigantic task in the cognitive radio systems because of Interference mitigation among different primary and secondary users is only possible by making a sacrifice of a portion from transmission bandwidth. Filter Bank Multicarrier approach can serve as a near optimal tool for spectrum analysis in cognitive radio systems with increased bandwidth efficiency attainable and decreased hardware complexity as well. Some researchers in the past have made an attempt to develop an efficient two channel Quadrature Mirror Filter banks by taking into consideration their magnitude and frequency responses in the passband, stopband and transition band regions [1]. Even, the joint effect of filters and symbols introduce various characteristics in different multicarrier schemes. The survey on wireless multicarrier communications suggests a framework to design and realize newer waveforms that not only lay the basis for further enhancements in wireless radio access techniques but also, it provides a information and ways to address different practical issues involved in the frequency domain synchronization for FBMC based Transmission. Better Spectral Containment is crucial to avoid distortion from asynchronous signals in the bands which are adjacent to each other. Also, higher spectrum is necessary while doing spectrum sensing, that is the fundamental thing in cognitive radio terminology [2].

2. FBMC System

In OFDM approach, the frequency selective channel can be converted into a number of subcarrier channels by using cyclic prefix and as a result, each of these subcarrier channels can be modeled as flat fading channel with constant gain. On the receiver side, the unwanted distortion and ISI can be reduced in each subcarrier band by using the concept of equalization as this holds true in case of Filter Bank based multicarrier communication systems. It has been found that out of all the multicarrier methodologies which are existing, OFDM-OQAM (i.e. FBMC) achieves the highest stop band attenuation [2].The higher attenuation in the stop band of Filter Bank allows the channel selection filtering along with Narrowband Interference attenuation at the analysis bank in receiver, with no pre-processing except for anti-aliasing filter estimated by the sampling rate at input in analysis filter bank. As the Filter Bank gives good frequency selection for the required spectral components, it is genuine to think of an overall

receiver architecture where all the signal processing functions at the baseband are performed in the frequency [1]. FBMC Prototype Filter Design (frequency domain optimized) is an interesting direction of the future studies to investigate the overall system performance. In FBMC system, the pulses that are transmitted are localized in both time and frequency domain. A multicarrier system can be described by a synthesis-analysis filter bank called Trans multiplexer Structure [3]. The synthesis filter bank comprises of parallel transmit filters and analysis filter bank comprises of all the matched receive filters. FBMC waveforms use an advanced filter design to better localize the different subcarriers. The impulse response of the prototype filter is expressed by the following equation.

$$h(t) = G_P(0) + 2\sum (-1)^k G_p(k) \cos([2\pi k/KN](t+1)) \tag{1}$$

where $G_p = [1, 0.97195983, 0.707, 1-G_p(1)^2]$ for Overlapping factor K=4,6,8,10;N is the number of carriers. The higher the overlapping factor is, the more localized the signal is in frequency. In fact, the fragmented spectrum can be observed as the consequence of extension to channel aggregation for mobile communication systems [4].

Here a program is given to design a prototype filter for use in a Quadrature Mirror Filter bank. Here a two-channel filter bank has four filters, each based on a low pass prototype (z). The analysis filters are $H_L(z)$, $H_H(z)$ while the synthesis filters are $G_L(z)$, $G_H(z)$, $H_L(z)=H(z)$; $H_H(z)=H(-z)$; $G_L(z)=G_L(z)$; $G_H(z)=-2H(-z)$ respectively.

The design procedure reduces the ripple energy and the stop band energy of the low pass filter. The design procedure is adopted by Jain-Crochiere [5] and is based on iterative cautious update optimization algorithm. When the configuration of analysis-synthesis filters is done, the overall system has an impulse response with a unit coefficient at a delay of $N-1$ samples. The process explained here is changed to apply a cautious update during the iteration process. This procedure uses Plot Filter command to plot the frequency responses [1].

3. Earlier Related Work Done

Two channesl QMF bank is a multirate filter structure which consists of two decimators in the signal analysis section and two interpolators in the signal synthesis section [5]. QMF banks find applications in automated methods for scoring tissue microarray spots [6], image coding [7], multicarrier modulation systems [8], two dimensional and three dimensional short time spectral analysis of short perfect reconstruction filters [9], antenna systems and MIMO [10], sampling theory techniques [11], biomedical electronics [12], wideband beam forming for sonar and radar [13] and in solving various co-existence problems of wireless communication systems [14, 15]. The earlier related work on the design of Nearly Perfect Reconstruction two channel QMF banks [16-31] can be categorized in different ways.Least Squares [16-18] and Weighted Least Squares [19-21] techniques had been applied for the filter design. Eigen value-Eigen vector approach has been proposed to find the optimum filter coefficients in time [17-18]. Lee and Chan [18] proposed a WLS method on the basis of the linearization of objective function to get optimal filter tapping weights. Several iterative methods [21-26] and genetic algorithm based techniques [27-31] have been suggested for design of two channel QMF banks. In [22], authors have presented a technique by taking into consideration the responses of filter in pass band,stop band and in transition band regions for QMF bank. In the two channel analysis and synthesis section of QMF bank, the discrete input signal x(n) is split into two sub band signals with equal bandwidth using low pass and high pass analysis filters H1(z) and H2(z). The subband signals are downsampled by a factor of 2 to attain signal compression for reducing the complexity during processing. The outcomes of synthesis filters are added to attain the reconstructed signal x(n) which suffers from Aliasing Distortion and Phase Distortion. Hence,the main focus of attention while designing the prototype filter for two channel QMF is the removal of these errors to obtain a Near Perfect Reconstruction [18-23]. In QMF banks,the high pass and low pass analysis filters are linked to each other by the mirror image symmetry i.e H2(z)=H1(-z), around quadrature frequency π/2.Due to this symmetry, the amplitude distortion can be reduced by optimizing the filter tapping weights of the low pass prototype filter. In the present section, we discuss an algorithm to design the two channel QMF bank with no matrix inversion which influences the performance of the optimization method used.

4. QMF Bank Design Methodology

The mathematical expression for the overall system function or distortion transfer function of the alias free two channel QMF bank is expressed as [21-29].

$$T(z)=1/2[H_1^2(z)-H_1^2(-z)] \qquad (2)$$

where for alias cancellation, synthesis filters are defined as:

$$F_1(z)=H_2(-z) \text{ and } F_2(z)=-H_1(-z) \qquad (3)$$

To obtain the perfect reconstruction QMF bank, the overall transfer function T(z) must be a pure delay.i.e.

$$T(z)=(1/2)[H_1^2(z)-H_1^2(-z)]=cz^{-n}_0 . \text{ or } . x(n)=cx(n-n_0) \qquad (4)$$

This equation shows that if the prototype filter $H_1(z)$ is chosen to be linear phase FIR with nil phase distortion, then to ensure linear phase FIR constraint, the impulse response $h_1[n]$ of low pass prototype filter $H_1(z)$ must be symmetrical .i.e. $h_1[n]=h_1[N-1-n], 0<n<N-1$ where N is the filter length [17]. The corresponding frequency response is given by [5] as:

$$H_1(e^{j\omega})=A(\omega)e^{-j\omega(N-1)/2} \qquad (5)$$

where $A(\omega)= H_1(e^{j\omega})$ is the amplitude function.If the prototype flter $H_1(z)$ characteristics are assumed ideal in pass band and stop band regions then the exact reconstruction condition is satisfied for $0<\omega<\omega_p$ and $\omega_s<\omega<\pi$. Here ω_p and ω_s are the passband and stopband edge frequencies. The constraint comes in the transition band($\omega_p<\omega<\omega_s$) where Amplitude Distortion needs to be controlled. It means our motive is to optimize the coefficients of $H_1(z)$ such that the exact reconstruction condition is approximately nearly satisfied.

Design Implementation: A Matlab code has been run to implement the design part of prototype LPF and inference has been drawn on the basis of CPU processing time taken to run that event.The entire process here involves three steps mainly (1). Design Specification for Efficient Two Channel QMF Bank(2). Implementation of Two Channel QMF (3). Evaluation of Designed QMF results.

Case Study (Design Example): We have designed the QMF here for N=24,32,42,48,64,128 with Stopband Frequency Fsb=0.359 for Alpha constant lying between 0 and 1(0.22),ω_s=0.6π,ω_p=0.4π. A Matlab code has been written which implements the design procedure for prototype low pass filter described and tested on a laptop equipped with an Intel Core i5-2410M Processor 2.30GHz with Turbo Boost upto 2.90 GHz with 4GB RAM on Windows 7 (64-bit) Operating system. This section presents a design example to check the effectiveness of the proposed algorithm.The main parameters which govern the performance of the algorithm are First lobe Stop band attenuation,Stop band edge attenuation, A_s=-20$\log_{10}(H_1(\omega_s)$, Maximum Overall Ripple/Passband-Ripple), Prototype QMF Length N, Stopband-Frequency(Fsb),Roll off factor Alpha, Measure of Reconstruction Error(dB)=max(10log(T($e^{j\omega}$))-min(10log(T($e^{j\omega}$)).

The Design Specification: N-Number of coefficients for the lowpass prototype. f_{sb}-Normalized stop band edge frequency for the low pass prototype, where $0.25 < f_{sb} < 0.5$. Alpha-Relative weighting between the stop band energy and the ripple in the overall response. An increase in alpha will lead to greater stop band attenuation in the low pass prototype. A sample filter has been designed with QMF Design (32, 0.3, 1). The performance of the designed algorithm can be analysed through a set of observations in terms of Magnitude w.r.t. Normalized Frequency (π radians/sample), Phase w.r.t. Normalized Frequency plots which also reveal the Phase Distortion occuring at values of Normalized Frequency. The results of the used algorithm have been compared to the results obtainable in case of Jain-Crochiere [5] and S.K. Aggarwal-O.P.Sahu [31]. The following set of filter coefficients have been obtained for (0< n< N/2-1) in case of Filter (Jain & Crochiere) and subsequent set of filter coefficients obtained for O.P.Sahu et al and that obtained in case of our proposed algorithm given in Table 1-4.

Table 1. QMF coefficients generated [Jain & Crochiere]
Optimized Filter Tap Weights in QMF design By Jain and Croichere [N=32]at fsb=0.3;α=1

$h_1(0)=0.46513280$	$h_1(1)= 0.13063700$
$h_1(2)=-0.99656700E-1$	$h_1(3)= -0.41773659E-1$
$h_1(4)= 0.53938050E-1$	$h_1(5)= 0.16805820E-1$
$h_1(6)= -0.33077250E-1$	$h_1(7)= -0.58240110E-2$
$h_1(8)= 0.20216010E-1$	$h_1(9)= 0.71798260E-3$
$h_1(10)= -0.11586330E-1$	$h_1(11)= 0.12928400E-2$
$h_1(12)= 0.58649780E-2$	$h_1(13)= -0.16349580E-2$
$h_1(14)= -0.23388170E-2$	$h_1(15)= 0.12488120E-2$

Table 2. QMF coefficients generated {Sahu et al]
Optimized Filter Tap Weights in QMF design By O.P.Sahu et al [N=24;fsb=0.3;α=1]

$h_1(0)=0.0034$	$h_1(1)= -0.0074$
$h_1(2)=-0.0022$	$h_1(3)= 0.0163$
$h_1(4)= -0.0020$	$h_1(5)= -0.0301$
$h_1(6)= 0.0124$	$h_1(7)= 0.0525$
$h_1(8)= -0.0375$	$h_1(9)= -0.1000$
$h_1(10)= 0.1272$	$h_1(11)= 0.4672$

Table 3. QMF coefficients generated [Kang& Vig-Proposed Algorithm]
Optimized Filter Tap Weights in QMF design by Kang-Vig [N=24;fsb=0.3;α=0.22]

$h_1(0)=0.0036$	$h_1(1)=-0.0072$
$h_1(2)=-0.0023$	$h_1(3)= 0.0161$
$h_1(4)= -0.0018$	$h_1(5)= -0.0300$
$h_1(6)= 0.0122$	$h_1(7)= 0.0524$
$h_1(8)= -0.0373$	$h_1(9)= -0.1000$
$h_1(10)= 0.1270$	$h_1(11)= 0.4673$

Table 4. QMF coefficients generated [Vig& Kang-Proposed Algorithm]
Optimized Filter Tap Weights in QMF design by Vig-Kang [N=32;fsb=0.3;α=0.22]

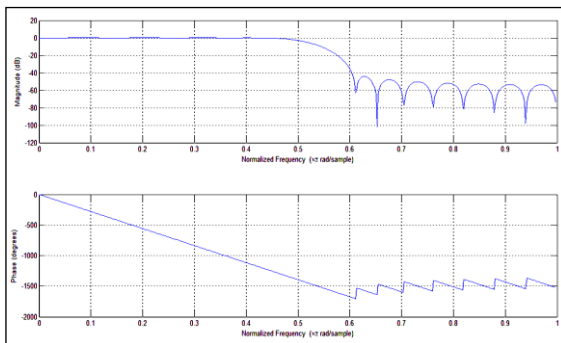|	
$h_1(0)=0.0013$	$h_1(1)= -0.0023$
$h_1(2)=-0.0016$	$h_1(3)= 0.0058$
$h_1(4)= 0.0013$	$h_1(5)= -0.0115$
$h_1(6)= 0.0006$	$h_1(7)= 0.0201$
$h_1(8)= -0.0057$	$h_1(9)= -0.0330$
$h_1(10)= 0.0167$	$h_1(11)= 0.0539$
$h_1(12)= -0.0417$	$h_1(13)= -0.0997$
$h_1(14)= 0.1305$	$h_1(15)= 0.4652$

Figure 1. N=32;Fsb=0.3;Alpha=1 [Jain & Croicere]   Figure 2. N=64;Fsb=0.3;Alpha=1
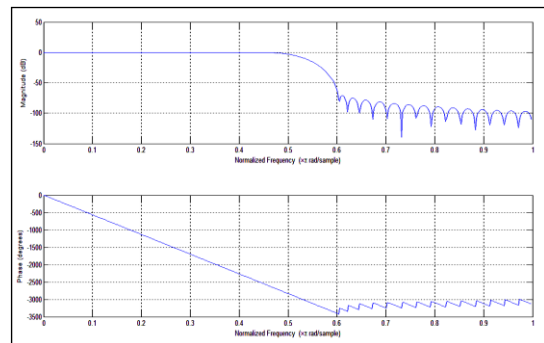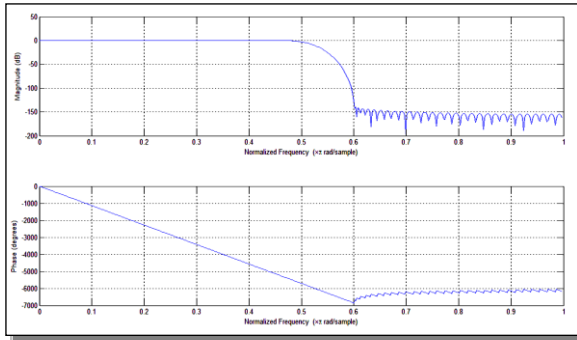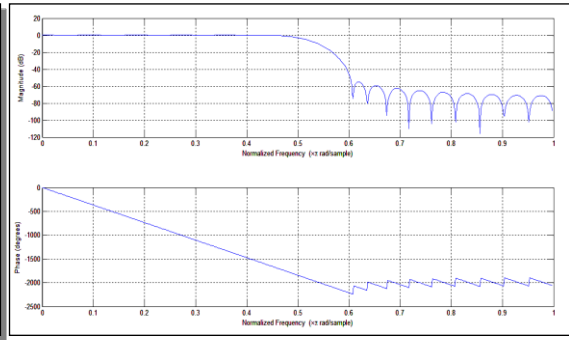
Figure 3. N=128;Fsb=0.3;Alpha=1
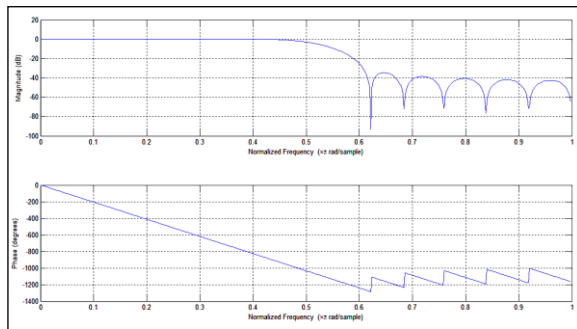

Figure 4. N=42;Fsb=0.3;Alpha=1

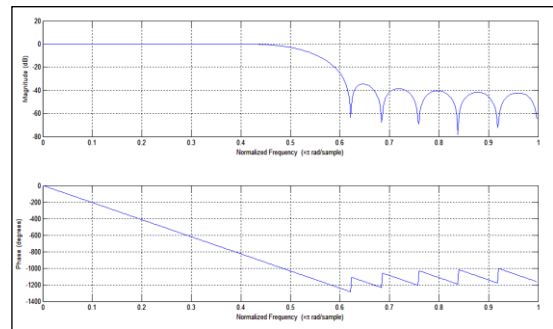Figure 5. N=24;Fsb=0.3;Alpha=1 [Sahu]

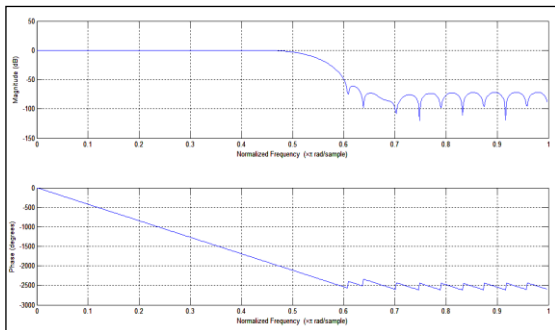Figure 6. N=24;Fsb=0.3;Alpha=0.22
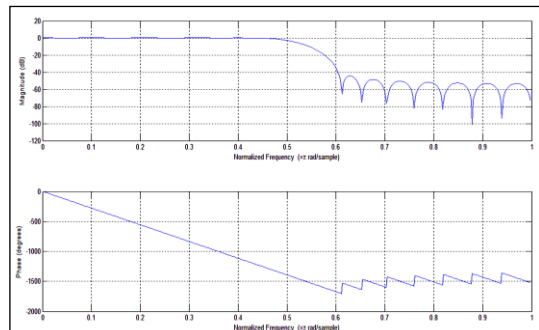

Figure 7. N=48;Fsb=0.3;Alpha=0.22

Figure 8. N=32;Fsb=0.3;Alpha=0.22
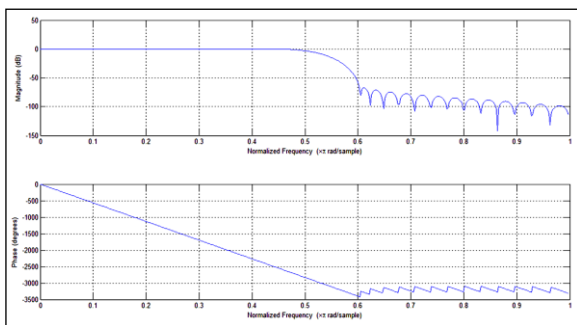

Figure 9. N=64;Fsb=0.3;Alpha=0.22
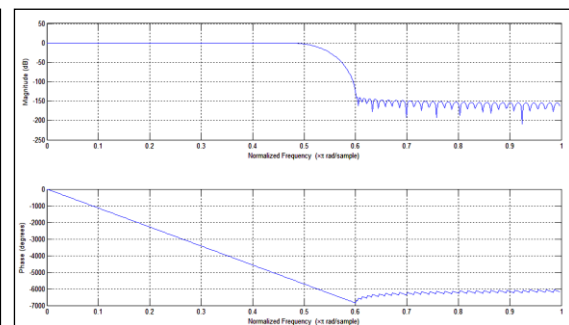

Figure.10 N=128;Fsb=0.3;Alpha=0.22
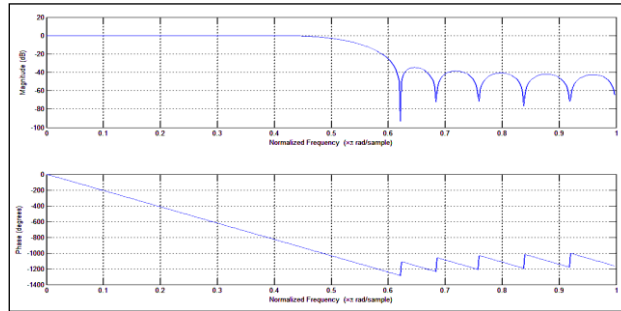
Figure 11. N=24,Fsb=0.3,Alpha=1 [O.P.Sahu et al]

Figure 12. Magnitude wrt Frequency Plot for (a) Gaussian Noise Input (b) Gaussian Noise Output(c)Gaussian Noise Intermediate Signal U0(d) Gaussian Noise Intermediate Signal U1 (Filter Length N=78)
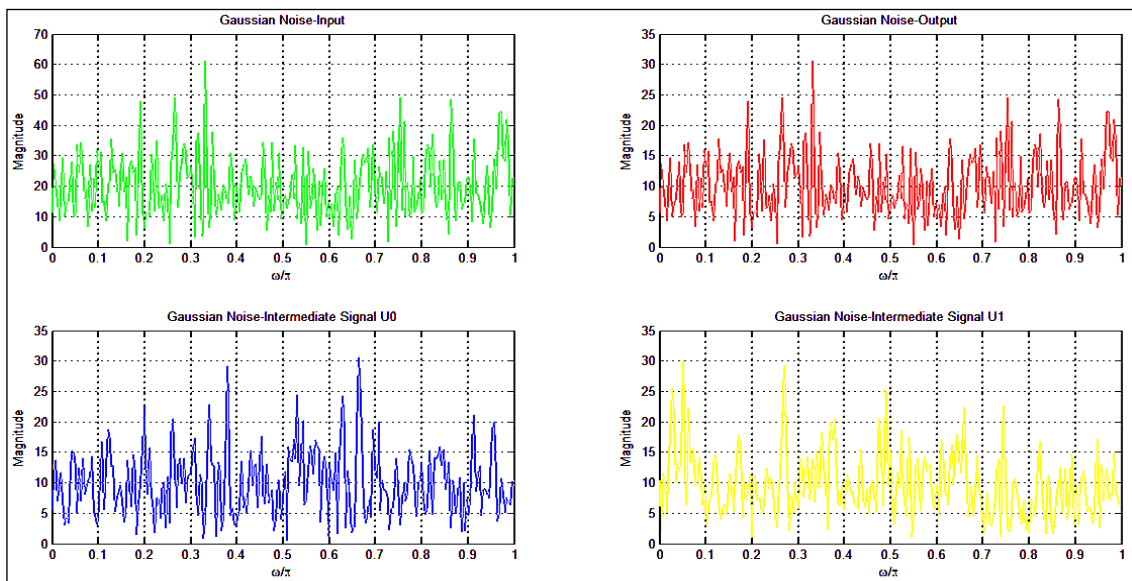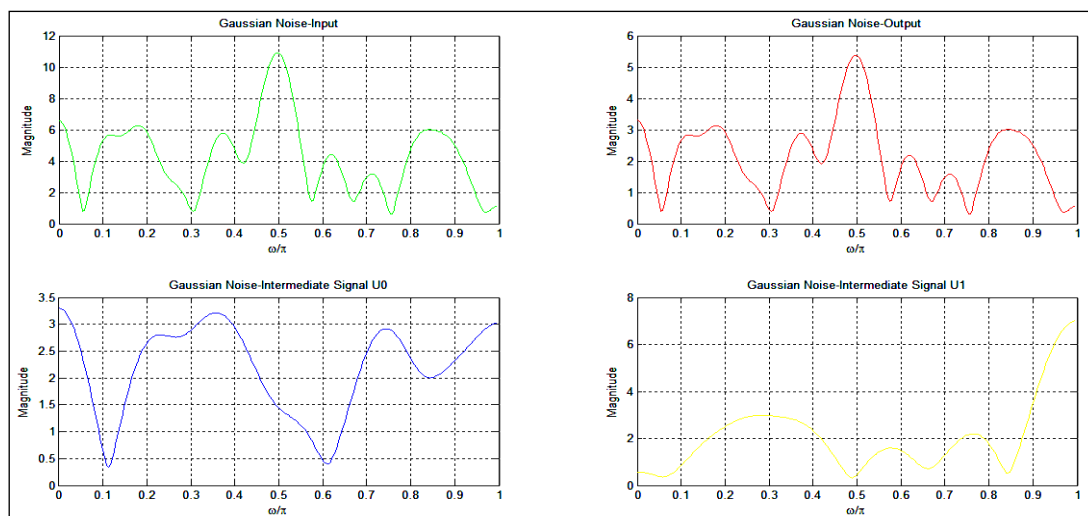

Figure 13. Magnitude wrt Frequency Plot for, (a) Gaussian Noise Input, (b) Gaussian Noise Output, (c) Gaussian Noise Intermediate Signal U0, (d) Gaussian Noise Intermediate Signal U1 (Filter Length N=24)

5. Results and Discussion

Table 5 shows the Performance Evaluation of Proposed Method for QMF design w.r.t QMF design by Earlier workers namely Jain & Croichere [5], Sahu et al [31]. An attempt has been made to minimize the phase distortion occurring w.r.t normalized frequency and the level of stopband attenuation occurring at different instants with variable N, Fsb and Roll off factor α. Beyond N=128 with roll off factor 1, the first lobe attenuation comes around 137.784 with passband ripple of 8.1dB and phase distortion occurring at a normalized frequency of 0.60.

Table 5. Comparatative Performance Evaluation of QMF for Design Specification N=24,32,42,48,64,128 with Stopband Frequency Fsb=0.359 for Alpha constant lying between 0 and 1(0.22),ω_s=0.6π,ω_p=0.4π

QMF Design Method	Filter Length N	Stop band Frequency F_{sb}	Roll Off factor α	Magnitude w.r.t. Normalized Frequency	Normalized Freq value (πradians/sample)at which Minimum Stopband Attenuation occurs	First lobe Attenuation in dB	Stopband Edge Attenuation = -20log10(δs) dB	Passband Ripple/Max Overall Ripple (dB)	Phase Response wrt Normalized Frequency	Normalized Frequency at which Phase Distortion starts occuring
Sahu et al	24	0.3	1.0	-80dB	0.62	34.723	0.30575	0.0334359	Linear	N.A.
Proposed App I	24	0.3	0.22	-63dB	0.83	44.1946	0.30325	0.015315	0 to -1250 degrees	0.63
Jain & Crochiere	32	0.3	1.0	-100dB	0.62	44.1946	0.30325	0.015315	0 to -1650 degrees	0.62
Proposed App II	48	0.3	0.22	-70dB	0.75	61.2333	0.3025	0.00615055	0 to -2600 degrees	0.62
Proposed App III	32	0.3	0.22	-62dB	0.62	44.4244	0.3035	0.0142484	0 to -1700 degrees	0.63
Proposed AppIV	64	0.3	0.22	-75dB	0.86	67.1736	0.30125	0.000810318	0 to -3400 degrees	0.60
Proposed App V	128	0.3	0.22	-165dB	0.63	137.71	0.30025	0.000130643	0 to -6800 degrees	0.58
Proposed App VI	24	0.3	1	-93dB	0.64	34.7233	0.30575	0.0334359	0 to -1250 degrees	0.63
Proposed App VII	42	0.3	1	-72dB	0.86	55.0136	0.302	0.00675901	0 to -2200 degrees	0.61
Proposed App VIII	64	0.3	1	-75dB	1	70.7027	0.301	0.00132929	0 to -3400 degrees	0.61
Proposed App IX	128	0.3	1	-155dB	0.62	137.784	0.30025	8.13492e-05	0 to -6700 degrees	0.60

Figure 14. Magnitude wrt Frequency Plot for (a)Gaussian Noise Input(b)Gaussian Noise Output(c)Gaussian Noise Intermediate Signal U0(d)Gaussian Noise Intermediate Signal U1 (Filter Length N=32)
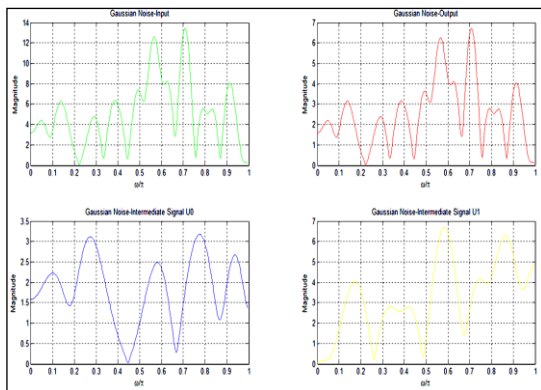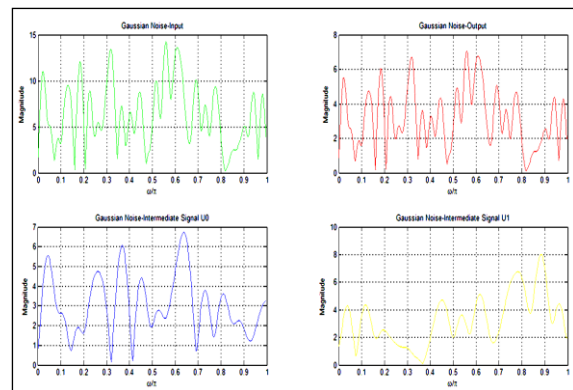

Figure 15. Magnitude wrt Frequency Plot for (a)Gaussian Noise Input(b)Gaussian Noise Output(c)Gaussian Noise Intermediate Signal U0(d)Gaussian Noise Intermediate Signal U1 (Filter Length N=64)

Table 6. Comparison of the Proposed Algorithm with existing optimization algorithms based on computational complexities

| Technique	Filter Length N	Any Matrix Inversion in each Iteration	Computational Complexity Reduced	Selection of $h_1(n)$	Ripple Computation	Phase Response	Perfect Reconstruction Achieved	Aliasing Distortion Reduced	Amplitude Distortion Minimized	Eigen Value Evaluation in each Iteration	Phase Distortion Eliminated	CPU time (sec)
Sahu[31]	24	Yes	Yes	Assumed	3.19056e-06	Linear	Yes, PR	Yes	Eliminated	No	Yes	0.66
Jain & Croichere [5]	32	Yes	Yes	Assumed	5.34059e-07	Linear/ Non Linear	NPR	Yes	Eliminated	Yes	Yes	1.68
Proposed Method I	24	Yes	Yes	Assumed	3.19056e-06	Non Linear	NPR	Yes	Minimized	Yes	Yes	0.62
Proposed Method II	48	Yes	Yes	Assumed	7.23799e-08	Non Linear	NPR	Yes	Minimized	Yes	Yes	0.85
Proposed Method III	32	Yes	Yes	Assumed	4.79034e-07	Non Linear	NPR	Yes	Minimized	Yes	Yes	1.60
Proposed Method IV	64	Yes	Yes	Assumed	1.74207e-09	Non Linear	NPR	Yes	Minimized	Yes	Yes	0.96
Proposed Method V	128	Yes	No	Assumed	4.62215e-11	Non Linear	NPR	Yes	Minimized	Yes	Yes	1.20
Proposed Method VI	24	Yes	Yes	Assumed	3.19056e-06	Non Linear	NPR	Yes	Minimized	Yes	Yes	0.64
Proposed Method VII	42	Yes	Yes	Assumed	1.76966e-07	Non Linear	NPR	Yes	Minimized	Yes	Yes	0.78
Proposed Method VIII	64	Yes	Yes	Assumed	4.90713e-09	Non Linear	NPR	Yes	Minimized	Yes	Yes	0.92
Proposed Method IX	128	Yes	Yes	Assumed	1.71327e-11	Non Linear	NPR	Yes	Minimized	Yes	Yes	1.34
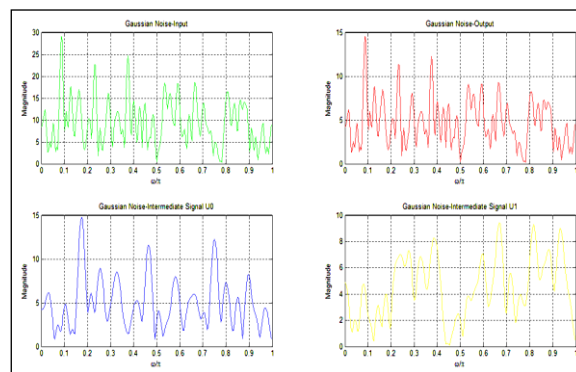
Figure 16. Magnitude wrt Frequency Plot for (a)Gaussian Noise Input(b)Gaussian Noise Output(c)Gaussian Noise Intermediate Signal U0(d)Gaussian Noise Intermediate Signal U1 (Filter Length N=128)

6. Conclusion

The present study on the Design and Implementation of Efficient QMF bank for Cognitive Radio Wireless Communication highlights how the different aspects such as Aliasing Distortion, Computational Complexity, affect the Two Channel QMF bank with Analysis and Synthesis Filters as the discrete input signal x(n) gets split up into two sub-band signals with equal Bandwidth using analysis filters $H_0(z)$ and $H_1(z)$.These filters have been designed to have Near Perfect Reconstruction conditions satisfied. These subband signals are downsampled by a factor of two to achieve signal compression. The decimated signals are coded and then transmitted. At the receiver, the two subband signals are decoded and then interpolated by a

factor of two and ultimately passed through low pass and high pass synthesis filters. The outputs of synthesis filters are combined to obtain the reconstructed signal y(n)=x(n). The reconstructed signal suffers from errors such as Aliasing Distortion, Amplitude Distortion and Phase Distortion because of the fact that these filters are not ideal. So here, the Amplitude distortion has been minimized by optimizing the filter tap weights of low pass analysis filter. The proposed algorithm presents improved performance in terms of reduced computation time when compared to Sahu, Jain.

7. Impact of Study

The present research work has its strong impact on the design of Multirate Filter banks for cognitive radio communication under ubiquitous, pervasive domain. The study can be extended to the wireless networks through the application of efficient filter banks which can be further helpful for analysis and design of future wireless radio technologies [32-35].

References

[1] Stitz TH, Ihalainen Tero. *Practical Issues in Frequency Domain Synchronization for Filter Bank Based Multicarrier Transmission*. Proceedings of the IEEE International Conference ISCCSP 2008. 2008: 411-416.
[2] Sahu OP, Soni MK. Marquardt Optimization Method to design two channels Quadrature Mirror Filter Banks*. Digital Signal Process. 2006;* 16(6): 870-879.
[3] Dore Jean Baptise, Berg Vincent. FBMC receiver for multi-user asynchronous transmission on fragmented spectrum. *EURASIP Journal on Advances in Signal Processing*. 2014: 41: 1-20.
[4] Vaidyanathan PP. Multirate *Digital Filters, Filter Banks, Polyphase Networks and Applications: A Tutorial*. Proc. IEEE. 78(1): 56-93.
[5] Jain VK, Crochiere RE. Quadrature Mirror Filter Design in time domain. *IEEE Trans.Acoust.Speech Signal Process*.1994; ASSP-32(4): 353-361.
[6] Lee TK. *Automated Method for scoring breast tissue microarray spots using quadrature mirror filters and support vector machines.* Proceedings of 15th International Conference on Information Fusion. 2012; 1868-1875.
[7] Xia TJiang Q. Optimal multirate filter banks: Design related symmetric extension transform and application to image compression. *IEEE Trans on Signal Process*.1995; 47(7): 1878-1889.
[8] Chen D, D Qu. Prototype filter optimization to minimize stopband energy with NPR constraint for filter bank multicarrier modulation systems. *IEEE Transactions on Signal Processing*. 2013; 61(1): 159-169.
[9] Wackersreuther G. On two-dimensional polyphase filter banks. *IEEE Transactions on Acoust.Speech Signal Processing*. 1996; ASSP-34; 192-199.
[10] Chandran S. A novel scheme for a sub band adaptive beam forming array implementation using quadrature mirror filter banks. *Electronic Letters*. 2003; 39(12): 891-892.
[11] Sharma KK, Joshi SD. Advances in Shanon Sampling Theory. *Defence Science Journal*, 2013; 63(1): 41-45.
[12] Afonso VX, Tompkins WJ. ECG beat detecting using filter banks. *IEEE Transactions on Biomedical Engg*. 1999; 46(2): 192-202.
[13] Charafeddine H, Groza V. *Wideband adaptive LMS beamforming using QMF subband decomposition for Sonar*. Proceedings of 8th IEEE International Symposium on Applied Computational Intelligence and Informatics. Romania. 2013; 431-436.
[14] Hara S, Masutani H. Filter *bank based adaptive interference canceler for co-existence problem of TDMA/CDMA systems.* Proceedings IEEE VTS 50th Vehicular Technical Conference. 1999; 3:1658-1662.
[15] Tharan L, Yad av RP. Interference Reduction Technique in multistage multiuser detector for DS-CDMA systems. *World Academy of Science, Engg and Tech*. 2008; 2(8): 775-781.
[16] Johnston JD. *A filter family designed for use in quadrature mirror filter banks.* Proceedings of IEEE International Conference. ASSP.1999; 292-294.
[17] Andrew L, Franques VT. Eigen design of Quadrature-Mirror Filters. *IEEE Trans Circuits Systems.II Analog Digital Signal Process.* 1997; 44(9): 754-757.
[18] Chen CK, Lee JH. Design of Quadrature Mirror Filters with Linear Phase in the Frequency Domain. *IEEE Transactions on Circuits Systems.* 1992; 39(9): 593-605.
[19] Xu H, Lu WS, Antoniu A. An improved method for the design of FIR quadrature mirror image filter banks. *IEEE Transactions on Signal Processing*. 1998; 46(6): 1275-1281.
[20] Lu WS, Xu H. A new method for the design of FIR quadrature mirror filter banks. *IEEE Trans.Circuits Syst.II. Analog Digital Signal Processing*.1998; 45(7): 992-927.

[21] Nayebi K, Barnwell TP. Time domain Filter Analysis:A new design theory. *IEEE Transactions on Signal Processing.* 1992; 40(6): 1412-1428.

[22] Sahu OP, Soni MK, Talwar IM. Marquardt optimization method to design two channel quadrature mirror filter banks. *Digital Signal Processing.* 2006; 16(6): 870-879.

[23] Kumar A, Singh GK, Anand RS. An improved method for designing quadrature mirror filter banks via unconstrained optimization. *Journal of Mathematical Modelling and Algorithms.* 9(1): 99-111.

[24] Kumar A, Singh GK. An improved method for the design of quadrature mirror filter bank using the Levenberg-Marquardt optimization, *Signal, Image, and Video Process.* 2011; 7(2): 209-220.

[25] Sahu OP, Soni MK. Designing Quadrature Mirror Filter Banks using Steepest Descent Method. *Journal of Circuits, Systems and Computers.* 2006; 15(2): 29-42.

[26] Swaminathan K, Vaidyanathan PP. Theory and Design of Uniform DFT, parallel QMF banks. *IEEE Transactions on Circuits Systems.* 1996; 33(12): 1170-1191.

[27] Rafi SM, Kumar A, Singh GK. An improved particle swarm optimization method for multirate filter bank design. *Journal of Franklin Institute.* 2013; 350(4): 757-759.

[28] Ghosh P, Das S, Zafar H. Adaptive Differential Evolution based design of two channel quadrature mirror filter banks for sub band coding and data transmission. *IEEE Transactions on Systems, Man and Cybernetics.* 2012; 42(6): 1613-1623.

[29] Upendar J, Gupta CP. Designing of two channel quadrature mirror filter bank using Particle Swarm Optimization. *Digital Signal Processing.* 2010; 20(10): 304-313.

[30] Jou YD. Design of two channel linear phase quadrature mirror filter banks based on neural networks. *Signal Processing.* 2007; 87(5): 1031-1044.

[31] Agarwal SK, Sahu OP. Two Channel Quadrature Mirror Filter Bank: An Overview. *ISRN Signal Processing. Hindawi.* 2013.

[32] Kang AS, Vig Renu. Comparatative Performance Evaluation of Modified Prototype Filter Bank Multi Carrier Cognitive Radio under Constraints of Lp, K, N and D. *The Computer Journal*, *Oxford University Press.* UK. 2016; 59(3): 1-13.

[33] Kang AS, Vig Renu. Comparatative analysis of Energy Detection spectrum sensing of cognitive radio under Wireless Environment using SEAMCAT. *International Journal of Advanced Computer Science and Applications.* USA. 2016; 7(1); 493-498.

[34] Kang AS, Vig Renu. Computer Aided BER Performance Analysis of FBMC Cognitive Radio for Physical Layer under the Effect of Binary Symmetric Radio Fading Channel. *Springer Journal of Wireless Personal Communications, Netherlands.* 2015; 81(2): 1-15.

[35] Kang AS, Vig Renu. Performance Analysis of FBMC-CR through Optimization–Tradeoff between Various Performance Parameters in the Cognitive Radio Environment. *Telecommunication Systems, Springer.* 2016; 63: 1-23.