

Editorial

## Power Synthesis of Mask-Constrained Shaped Beams Through Maximally-Sparse Planar Arrays

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### Abstract

*A new approach to the optimal synthesis of planar arrays able to radiate mask-constrained shaped beams by exploiting the minimum number of radiating elements is presented. By taking advantage from both the recent theory of Compressive Sensing and the multiplicity of equivalent solutions available for the generation of a unique shaped-beam power pattern, the synthesis results extremely fast and effective. In particular, the overall design is reduced to a Convex Programming optimization, with the inherent advantages in terms of solutions' optimality and computational burden.*

**Keywords:** Array antennas, compressed sensing, power synthesis, shaped beams.

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Maximally-sparse arrays, i.e., array antennas able to fulfill assigned radiation requirements by recurring to the minimum possible number of elements, represent a topic of high interest in many applications (e.g., phased array radars, satellite communications, and multiple-input-multiple-output systems). In fact, they may offer relevant advantages in terms of size, weight, cost, and beam forming network's complexity [1-6].

Recently, in the case of maximally-sparse linear arrays radiating shaped beams, the approach introduced in [1] outperformed all techniques published in [2-5]. This has been possible by jointly exploiting the theory of Compressive Sensing (CS) [7] and the multiplicity of equivalent solutions generally available to generate a single shaped power pattern [8],[9]. Moreover, by avoiding exploitation of Global Optimization (GO) algorithms and relying only to Convex Programming (CP) tools, the approach in [1] provided relevant advantages in terms of computational times.

Due to the relevance of the outcomes achieved by the procedure in [1], the present work is aimed at extending it to the case of planar arrays. To present the resulting approach, it is worth first recalling the procedure in [1], which is composed by the following steps:

1. Assign the power mask, i.e., the lower and upper bound functions required to shape as desired the radiated power pattern;
2. Synthesize, by means of the approach presented in [9], a 'virtual' linear array radiating a power pattern fulfilling the mask of step 1. Notably, such a step provides a number of different possible field solutions [8];
3. Identify, amongst all the different far-field distributions coming out from step 2, the one leading to the minimum  $\ell_1$ -norm of the 'virtual' array excitations. By virtue of the theory in [7], this far-field distribution is the one associated to maximum amount of 'sparsity';
4. Synthesize a 'new' equispaced linear array by performing a CS-based reconstruction. This step consists in solving the following CP problems in the unknown  $\mathbf{b}=[b_1, \dots, b_N]$ , which represents the vector of the 'new' array's excitations:

$$\text{Minimize: } \|\mathbf{b}\|_{\ell_1} := \sum_{n=1}^N |b_n| \quad (1)$$

$$\text{subject to: } \begin{cases} |f_b(\alpha)|^2 \leq g(\alpha) & \forall \alpha \in \tau_1 \\ |f_b(\alpha) - f_r(\alpha)|^2 \leq \varepsilon & \forall \alpha \in \tau_2 \end{cases} \quad (2)$$

$$\text{with: } f_b(\alpha) = \sum_{n=1}^N b_n e^{j\beta x_n \sin \alpha} \quad (3)$$

Wherein  $N$  is the number of elements,  $f_r$  is the far field coming out from step 2,  $\alpha$  is the elevation angle with respect to boresight,  $\beta$  is the wavenumber, and  $\mathbf{x}=[x_1, \dots, x_N]$  is the vector containing the 'new' array's equispaced locations. By so doing, while the objective function (1) will induce a minimization of the active elements number, convex constraints (2) will ensure that the synthesized power pattern:

- a. Fulfills an user-defined upper bound constraint in the sidelobes region denoted with  $\tau_1$  ( $g$  being an arbitrary real and positive function);
- b. Fits the reference field  $f_r$  with a precision  $\varepsilon$  in the region denoted by  $\tau_2$ ;
5. Further reduce the overall number of array elements by:
  - a. Discarding the elements having a negligible excitation amplitude;
  - b. Recursively substituting each couple of remaining elements whose distance is lower than a threshold  $\sigma$  with a single element placed in the middle point between them and excited with the sum of the two original excitations;
6. Perform a refinement of the solution by means of a local optimization procedure: identify slight shifts on array locations and excitations so as to recover from possible losses on the radiation performance induced by step 5.

In order to extend the above procedure to the planar arrays case, we tried to synthesize a beam having a flat-top behavior along the azimuth angle and a square-cosecant behavior along the elevation angle (see Figure 1), which is of interest in radar applications as well as for radio-base stations. In so doing, we took as a reference the pattern generated from the factorization of the fields respectively depicted in blue color in subplots (b) and (c) of Figure 1.

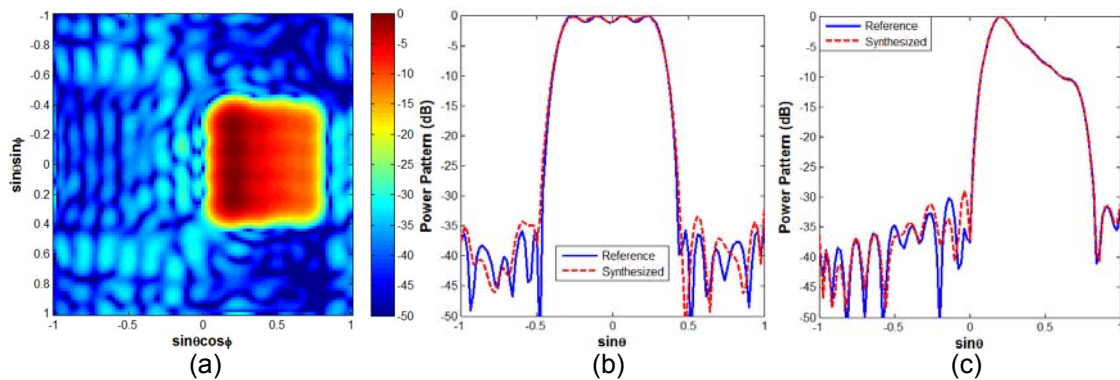


Figure 1. Power pattern of the array view in the spectral plane [subplot (a)], and plot of the main plane [subplot (a)] and the abscissa [subplot (b)]. The reference and synthesized distributions are respectively depicted in blue color and red color

Then, the procedure in [1] has been extended to the two-dimensional case as follows.

**First**, we factorized the separate one-dimensional solutions achieved by applying steps 1-6 above to each of the reference power pattern's main cuts.

**Second**, we discarded those elements having an excitation amplitude lower than  $u=0.04$  and achieved the array layout depicted in Figure 2 [subplot (a), blue circles], which is composed by 94 elements.

**Third**, by adapting step 6 above to the two-dimensional case (in such a way to *jointly* refine *both* the  $x$  and  $y$  array locations), we finally achieved the layout depicted in Figure 2 [subplot (a), red dots] and the corresponding optimal excitations depicted in Figure 2 [subplot (b)].

The achieved power pattern is shown in Figure 1, wherein  $\theta$  and  $\phi$  respectively represent the elevation and azimuth angles. As it can be seen, a very good agreement is achieved between the reference field and the main cuts of the synthesized pattern.

Notably, the overall approach allowed saving the 58% of the elements with respect to a fully-populated array with an uniform  $\lambda/2$  spacing, and the 34% of elements with respect to a simple factorization of the solutions in [2]. This circumstance is coherent with the fact that, as long as a pattern is factorable, the elements' number reduction in the planar array is roughly doubled with respect to the one experienced in the two underlying one-dimensional arrays.

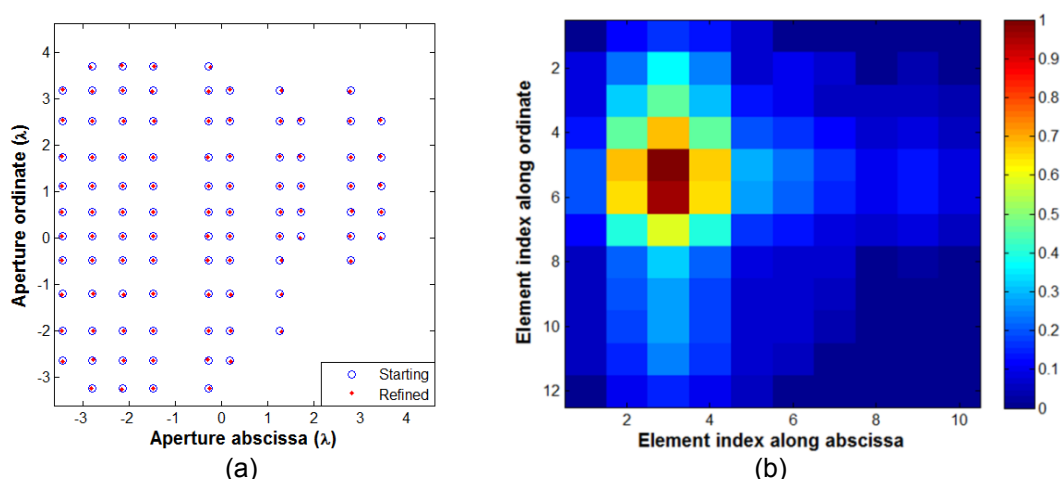


Figure 2. Subplot(a): Array layout achieved by factorizing the one-dimensional reference solutions and discarding the elements having a negligible excitation amplitude. Locations achieved before (blue circles) and after (red dots) the refinement step. Subplot (b): Final excitation amplitudes

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