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The Chaos and Stability of Firefly Algorithm Adjacent Individual

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Abstract

In this paper, in order to overcome the defect of the firefly algorithm, for example, the slow convergence rate, low accuracy and easily falling into the local optima in the global optimization search, we propose a dynamic population firefly algorithm based on chaos. The stability between the fireflies is proved, and the similar chaotic phenomenon in firefly algorithm can be simulated.

Keywords: *chaos, firefly algorithm, stability*

1. Introduction

The function optimization problem mainly aims to optimize relatively complex functions. The essence of function optimization is to find optimal solution to an objective function by iteration. The goal of search is a general optimization of the "function" of objective function. [1]. We can calculate the maximum or minimum value of the objective function by using multiple input values from a given range of values. The evolutionary algorithms are one of the most widely used methods for optimization problems. They are used in such algorithms, which are reproduction, mutation, crossover, recombination and so on. Firefly Algorithm (FA) is a new member of this family that is currently an active focus of research, where several modifications and improvements were recorded within the past few years[2-4]. Even though Firefly algorithm have the characteristics of fewer algorithm parameters, simple implementation, n particular, since the optimization ability of the firefly algorithm depends on the interaction and influence of the firefly, and the individual itself lacks the mutation mechanism, it is difficult for the firefly particles to jump out of the local optimum area.The several methods have been used to improve the performance of firefly algorithm, including chaotic maps. Generally speaking, firefly algorithm can be combined with chaos in two ways. On the one hand, some random distributions are replaced by chaotic maps [5-6]. On the other hand, in order to improve performance, the FA parameters are mapped to achieve the purpose of adjusting the intrinsic structural parameters of the firefly through chaos [7-8].

The research focused by scholars is the combination of firefly algorithm with certain chaos. However, the Quasi- chaotic phenomenon in firefly algorithm may be ignored. In this paper, we use mathematical methods to analyze the stability of neighboring individuals in the firefly algorithm.What's more, the Quasi- chaotic phenomenon that adjacent individual contracts each other is simulated with MATLAB in this paper. The paper's structure is as followed: In Section II, the main idea is the brief overview of the firefly algorithm; In Section III, some theorems of stability and Quasi-chaos of adjacent individual are given. In section IV, some experiments and simulations have been carried out to the performance of chaos in firefly algorithm.

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2. The firefly algorithm and chaos

In this section, some involved theories will be introduced in brief.

2.1. Firefly algorithm (FA)

Firefly algorithm (FA), proposed by scholar Yang, is inspired by the phenomenon that the fireflies flash and attract each other in nature[9]. It is one kind of colony searching technology, which simulates the flashing and communication behavior of fireflies. The brightness of a firefly indicates the location is good or bad. Parameters of firefly algorithm are showed in details:

Definition2.1: The relative intensity of fireflies:

$$I = I_o \times e^{-\gamma r_{ij}} \quad (1)$$

Where I_0 is the biggest fluorescence brightness of firefly. γ is light intensity absorbed coefficient and it is constant. It is characteristics of the weak that under the influence of the fireflies emit light in the distance and medium by light intensity absorbed coefficient. For two fireflies i and j, the distance r_{ij} is calculated:

$$r_{ij} = \sqrt{\sum_{k=1}^{d}\left(x_{i,k} - x_{j,k}\right)^2} \quad (2)$$

Equation (2.2) is the distance between any two fireflies and at and, respectively. Definition2. 2: The attractiveness of firefly is:

$$\beta = \beta_o \times e^{-\gamma r^2_{ij}} \quad (3)$$

Where β_o is the attractiveness at $r = 0$. Definition2.3: In every iteration, the fireflies could move to nearby ones with more brightness as determined by Equation (2.4):

$$x_i\left(t+1\right) = x_i\left(t\right) + \beta\left(x_j - x_i\right) + \alpha\left(rand - 0.5\right) \quad (4)$$

Where $x_i(t)$ and $x_i(t+1)$ are the locations of the i-th firefly in last iteration and in this iteration, respectively, so does $x_j(t)$. And α is step factor belonging to [0, 1]; Rand is uniformly distributed random factors belonging to [0, 1]. The process of optimization is followed:

a. Step 1: Initialize the parameters of the algorithm. Define the number of fireflies m, the biggest attraction β_o, the light intensity absorbed coefficient γ, the step length factor α, the maximum number of generations $T_{\max}$ and the search accuracy ε, respectively.
b. Step 2: Calculate the fitness value of every firefly regarded as the respective maximum fluorescence brightness.
c. Step 3: Calculate β used (3).
d. Step 4: Update the locations of fireflies according to (4) and random disturb the firefly in the optimum location.
e. Step 5: Update the fluorescence intensity and attraction after position.
f. Step 6: If the results meet the requirements, break to step 7; else, the number of iterations plus one, back to step 3.
g. Step 7: Output the global extreme point and the optimal individual values. $O(m^2)$ is time complexity of algorithm .

2.2. Chaotic Map (CP)

The chaotic phenomenon refers to a seemingly random irregular movement that occurs in a deterministic system. A system of deterministic theory describes the behavior of the system as unpredictable and unpredictable[10]. In 1975, the word chaos was first used by Li and Yorke in [11].This phenomenon is usually considered as a part of dynamical systems that changed over the time. In this paper, there are some well-known chaotic maps introduced briefly.

1. Sinusodial map

This chaotic map was formally defined by the following equations [12]:

$$x_{n+1} = \sin(\pi x_n), x_n \in (0,1) \tag{5}$$

2. Logistic map

In 1976 , Robert May introduced Logistic map13], and he pointed out that the map led to chaotic dynamics. This map was formally defined by the equations (6):

$$x_{n+1} = \mu x_n (1 - x_n) \tag{6}$$

Where $x_n \in [0,1]$, $\mu \in [0,4]$, $n = 0,1,2....$

3. Tinkerbell map

The Tinkerbell map was a discrete-time dynamical system given by [14]:

$$\begin{aligned} x_{n+1} &= x_n^2 - y_n^2 + ax_n + by_n \\ y_{n+1} &= 2x_n y_n + cx_n + dy_n \end{aligned} \tag{7}$$

Some commonly used values of a, b, c, and d are:

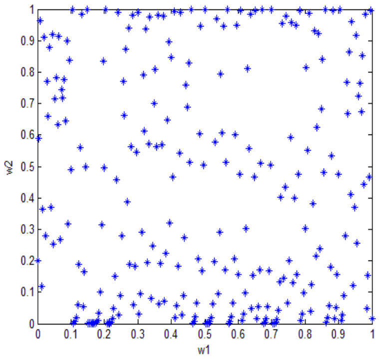
$$\begin{cases} a = 0.9, b = -0.6013, c = 2.0, d = 0.5 \\ a = 0.3, b = 0.6000, c = 2.0, d = 0.27 \end{cases}$$

4. Henon map

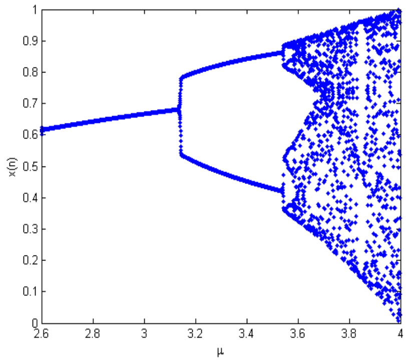
As a two-dimensional map, the Henon map is the simplest non-linear map in the high-dimensional map, which was proposed by French astronomer Michel Henon in 1976. The Henon map takes a point (x_n, y_n)n the plane and maps it to a new point. It can defined as followed [15]:

$$\begin{aligned} x_{n+1} &= 1 - ax_n^2 + y_n \\ y_{n+1} &= bx_n \end{aligned} \tag{8}$$

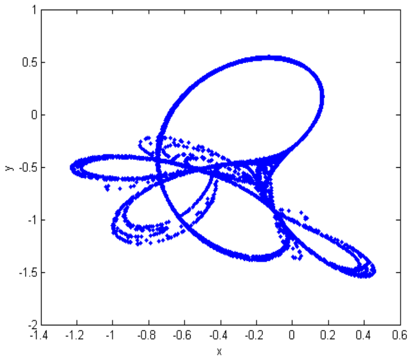
Figure 1 are respectively the chaotic phenomena of Sinusodial map, Logistic map, Tinkerbell map, Henon map and so on. The Sinusodial chaotic map with 251 iterations is showed in Figure 1a, and here the initial point of variable is $x_1 = 0.2$. Figure 1.b. is the result of Logistic map. As we can see, the chaotic phenomenon can be obtained at the point of $\mu = 3.6$ when to increase the value of μ gradually. For Tinkerbell chaotic map, two sets of parameters could be used to realize it: $a = 0.9, b = -0.6013, c = 2.0, d = 0.5$ and $a = 0.3$, $b = 0.6000$, $c = 2.0, d = 0.27$ .In this paper, the former set of parameter has been used to do it, which is showed in Figure 1.c. Like all chaotic maps, the Tinkerbell Map has also been shown to have periods. After a certain number of mapping iterations, every given point showed in the map to the right will find itself once again at its starting location. The chaotic curve of Henon map with 10000 iterations is showed in Figure 1.d, and the initial parameters are: $a = 1.4$, $b = 0.3$, $x_1 = y_1 = 0.4$.



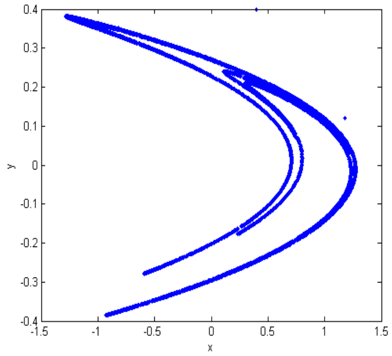
(a). Sinusodial map



(b). Logistic map



(c). Tinkerbell map



(d). Henon map

Figure 1. Chaotic map

2.3. Chaotic-FA (CFA)

In this section, we have used four chaotic maps mentioned above to tune the FA parameters and improve the performance, which will lead to a set of Firefly algorithm, or different variants of the chaotic Firefly algorithm. The flowchart of a schematic chaotic- FA (CFA)[5] is presented in Table 1. The following method describes how parameters can be tuned.

Table 1. Pseudo Code of a Chaotic Firefly Algorithm

Objective function of optimization problem $f(x), x = (x_1, x_2, \cdots, x_d)^T$
Generate initial population of fireflies $x_i\ (i = 1, 2, \cdots, n)$
Dedined search space
Light intensity I_i at x_i is determined by $f(x_i)$
$C_0 = a$ random number
$k = 1$
Tuning of parameters using chaotic maps $((\lambda / \beta) = C_k)$
For $i = 1 : no. firefiles$
For $j = 1 : no. firefiles$
If $(I_j > I_i)$
Move firefly i towards j in d-dimension
End if

Table 1. Pseudo Code of a Chaotic Firefly Algorithm

Attractiveness varies with distance r via Evaluate new solutions and update light intensity
End for j
End for i
Rank the fireflies and find the current best
$k = k + 1$
If $k \le MaximumGeneration$
Iteration continues
If not
Post Process the results

3. The stability and Quasi-chaos of firefly algorithm
3.1. The stability of firefly algorithm

In this section, the stability and chaos of firefly algorithm will be proved in details.

In the process of evolution, brightness is the matter to attract each other, and other fireflies will move towards the brightest one, it will lead to a stabilization of individuals. So we get the following theorem.

Theorem 3.1

If the dynamic behavior of fireflies are stable, it must meet the following conditions:

$$\eta \le -\frac{5\beta}{4\alpha} \tag{9}$$

Where $\eta = \alpha\left(rand - 0.5\right)$

Proof:

The expressions for updating the position of fireflies are:

$$x_i\left(t+1\right) = x_i\left(t\right) + \beta\left(x_j - x_i\right) + \alpha\left(rand - 0.5\right)\left(x_j - x_i\right) \tag{10}$$

$$xbest_i\left(t+1\right) = xbest_i\left(t\right) + \alpha\left(rand - 0.5\right)xbest_i\left(t\right) \tag{11}$$

Assuming $z_1 = x_i\left(t\right) = xbest_i$ and $z_2 = x_j\left(t\right)$,Equations (3.2-3.3) can be represented as

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = M \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{12}$$

Where $M = \begin{bmatrix} 1-\beta-\alpha\eta & \beta+\alpha\eta \\ 1+\alpha\eta & 0 \end{bmatrix}, \eta = \left(rand - 0.5\right)$

Suppose λ_1, λ_2 are the eigenvalue of M ,Then there is an inverse matrix P , such that

$$P^{-1}MP = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{13}$$

Solve the eigenvalue of M,we can get

$$\lambda_{1,2} = \frac{1-\alpha\eta-\beta}{2} \pm \frac{\left(5\alpha^2\eta^2 + 6\alpha\beta\eta + 2\alpha\eta + \beta^2 + 2\beta + 1\right)}{2} \tag{14}$$

According to the theory of stability, the necessary and sufficient condition for the dynamic behavior of fireflies is $\left|\lambda_{1,2}\right| \le 1$, Therefore, the optimal stability condition is $\eta \le -\frac{5\beta}{4\alpha}$.

3.2. Quasi-chaos of firefly algorithm

In the process of iterations, distances will be convergent and fireflies will tend to be stable after a number of iterations or evolutions. Under a certain circumstances, a kind of be similar to chaotic phenomenon will become true, we define it as a Quasi-chaos phenomenon in the paper.

So for firefly i ,we have $$x_i(t+1) = x_i(t) + \beta(x_j - x_i) + \alpha(rand - 0.5) \quad (15)$$

Similarly, for firefly j $$x_j(t+1) = x_j(t) + \beta(x_i - x_j) + \alpha(rand - 0.5) \quad (16)$$

After iterations, we get the propinquity Quasi-chaotic attractor of this map as shown in Figure 2. The simulation is under of circumstance that: For Figure. 2.1, the initial locations of adjacent fireflies are 0.01 and 0.09, the attractiveness of them are 0.001 and 0.08, respectively. However, when these parameters change, the graphics will change greatly. For example, As Figureure 2.2. We changed these parameters separately:0.1,0.1 and 0.00001, 0.00001.

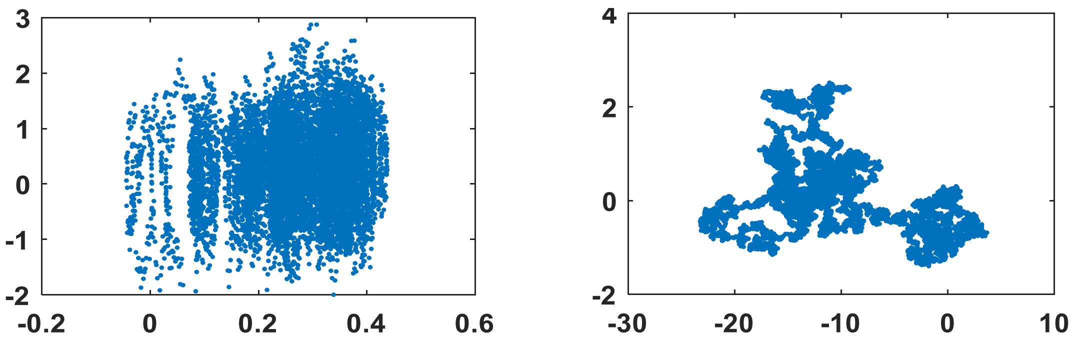


Figure.2 The resemblance Quasi-chaotic phenomenon of firefly algorithm

4. Experiment and results

In this section, three complicated functions will be tested using CFA, which the global optimal point can be found simultaneously. And the space locations of adjacent firefly will be simulated to prove the chaotic Algorithm. Sphere function, rastrigin function, griewank function as shown Figure 3.

Table 2. Test Function

ID	Name	Formula	Dimensions
F1	Sphere	$F_1 = \sqrt{\sum_{i=1}^{n} x_i^2}$	10
F2	Rastrigin	$F_2 = \sum_{i=1}^{n}\left(x_i^2 - 10\cos(2\pi x_i) + 10\right)$ $\lvert x_i \rvert \le 5.2$	10
F3	Griewank	$F_3 = 1 + \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)$	10

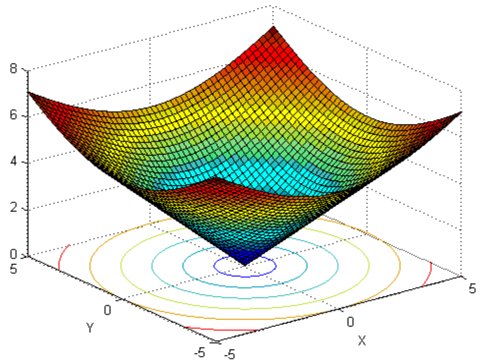


Figure 3.1 Sphere function

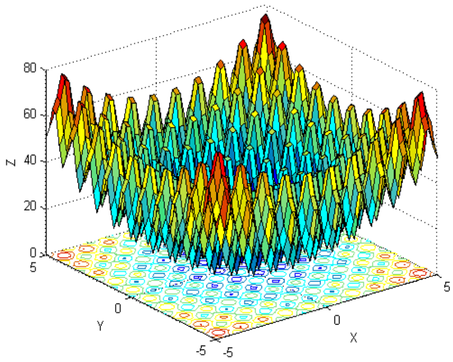


Figure 3.2 Rastrigin function

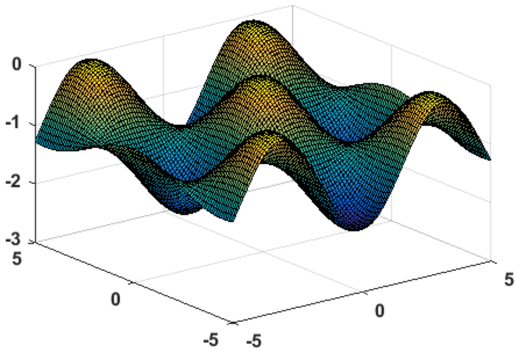


Figure 3.3 Griewank function

The minimum solution of the Sphere function, Rastrigin function, and Griewank function are located at point x=(0, 0, 0, . . . , 0, 0, 0) with an objective function valued equal to F(x)=0.0. When applied the CFA algorithm to the function, the algorithm finds the optimal solution after approximately 38, 15, 9 iterations, respectively as shown in Figure 4.1, Figure 4.2, Figure 4.3.

By testing the four typical chaotic maps in these three complex functions, we are supposed to find that CFA can overcome the defect of the firefly algorithm which slow convergence rate and easily falling into the local optima in the global optimization search,but it still exist the low precision.

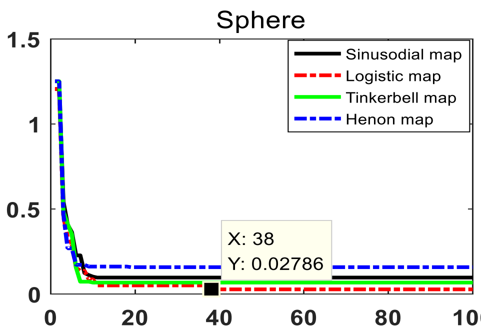


Figure 4.1. Convergence history of Sphere function

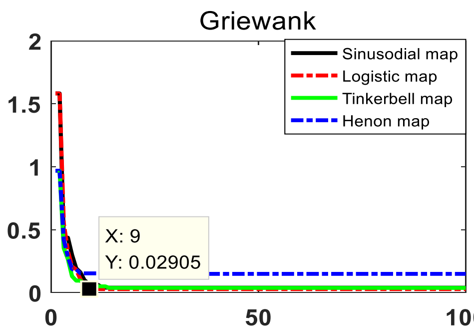


Figure 4.2. Convergence history of Rastrigin function

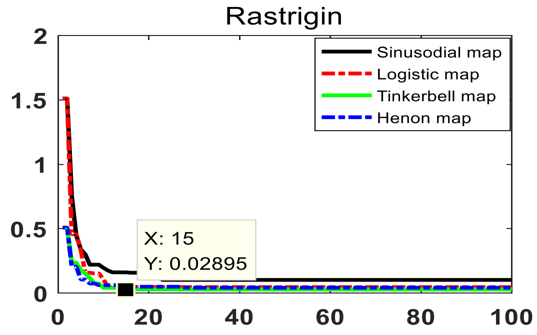


Figure 4.3. Convergence history of Griewank function

5. Conclusion

This paper has focused on the chaos and stability of adjacent fireflies in firefly algorithm. The main contribution is that the chaotic performance and behavior are proved and tested by using theory of stability in firefly algorithm. Through the above experiments and results, CFA can overcome the defect of the firefly algorithm which slow convergence rate and easily falling into the local optima in the global optimization search.

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