

## Unambiguous Acquisition for Galileo E1 OS Signal Based on Delay and Multiply

Deng Zhongliang<sup>\*1</sup>, Xi Yue<sup>2</sup>, Jiao Jichao<sup>3</sup>, Yin Lu<sup>4</sup>

Laboratory of Intelligent Communication, Navigation and Micro/Nano-Systems, Beijing University of Posts and Telecommunications, Beijing100876, Beijing, P. R. China

\*Corresponding author, e-mail: dengzhl@bupt.edu.cn<sup>1</sup>, xiyue0430@gmail.com<sup>2</sup>, jiaojichao@gmail.com<sup>3</sup>, inlu\_mail@163.com<sup>4</sup>

### Abstract

*Galileo E1 Open Service (OS) signal is transmitted with the modulation of Composite Binary Offset Carrier (CBOC). CBOC has a main drawback that is the autocorrelation function has multiple side-peaks, which will lead to ambiguous acquisition. The high rate of data bit and secondary code makes it very difficult to increase coherent integration time. This paper will propose a new scheme based on the delay-and-multiply concept. And also this scheme combines the data channel and pilot channel. Finally, the theoretical results will be given to prove that the new scheme will accomplish unambiguous acquisition and also eliminate the influence of bit transition.*

**Keywords:** Galileo E1 OS signal; CBOC; unambiguous acquisition; delay-and-multiply

### 1. Introduction

European Galileo Satellite Navigation System have been supporting Galileo-only autonomous position fix for an aeronautical user since 2013 [1]. And there will be more satellites launched in the near future. The exploitation plan is to provide early Galileo services by early 2015 and come to hand-over exploitation phase by the end of 2016. And by 2020 Galileo System will be of full operational capability. Therefore, Galileo System can play a crucial role among all of the navigation satellite systems, which also includes Chinese Beidou, American GPS and Russian GLONASS [2].

CBOC has been chosen as the final choice of Galileo OS Service. CBOC is a result of multiplexing BOC (6,1) with BOC (1,1) and the proportion of the former to the latter is 10%. Therefore, the maximum degradation on the detection probability when acquiring CBOC signals like a BOC (1,1) is less than 0.8dB [3]. With the property of splitting spectrum, CBOC can reduce the intra-system interference and improve code delay tracking.

Nevertheless, BOC-modulated signal will lead to a main drawback that is the autocorrelation function has multiple side-peaks, which will probably result in possible false acquisition. Several techniques have been proposed in the literature.

The Sub Carrier Phase Cancellation (SCPC) method generates an in phase and quadrature sub-carrier signals, getting rid of the sub carrier. Therefore, this method doubles the number of correlators because it is necessary for two channels wiping off the carrier to generate two kinds of sub carrier signal [4]. Autocorrelation Side-Peak Cancellation Technique (ASPeCT) method combines two kinds of autocorrelation functions to formulate a new one. After that, the new autocorrelation function still has small side peaks [5].

The modernized navigation satellite system has a characteristic that there is not only data channel like traditional GPS L1 signals but also pilot channel without data modulation. Several possible joint data/pilot acquisition strategies accompanying the non-coherent combination technique are analyzed. But the ambiguous problem caused by subcarrier is neglected through the article.

The rate of data bit and secondary code is equal with the rate of spreading codes in data channel and pilot channel for Galileo E1 OS signal. It is very likely that bit transition will occur if the coherent integration time is longer than one period of spreading code, which will degrade the power of the coherent integration. Double Block Zero Padding Transition Insensitive (DBZPTI) presents a new method based on Double Block Zero Padding (DBZP). This method is capable of being insensitive to bit transition during one period of coherent

integration time. But when it comes to longer coherent integration time, the problem caused by bit transition still exists [6].

In this paper, we will focus on a new acquisition scheme based on the delay-and-multiply concept, which also combines data channel and pilot channel both. At first, the signal module will be introduced, and the problem of Galileo E1 OS signal acquisition will be discussed. Then, the new scheme will be proposed. Finally, the simulation results of detection probability will be given.

## 2. Signal Model

The Galileo E1 OS signal can be expressed as

$$s_{E1}(t) = \frac{\sqrt{C}}{2} (e_{E1-B}(t)SC_{E1-B}(t) - e_{E1-C}(t)SC_{E1-C}(t)) \quad (1)$$

Where

$$SC_{E1-B}(t) = \sqrt{\frac{10}{11}}sc_a(t) + \sqrt{\frac{1}{11}}sc_b(t) \quad (2)$$

$$SC_{E1-C}(t) = \sqrt{\frac{10}{11}}sc_a(t) - \sqrt{\frac{1}{11}}sc_b(t) \quad (3)$$

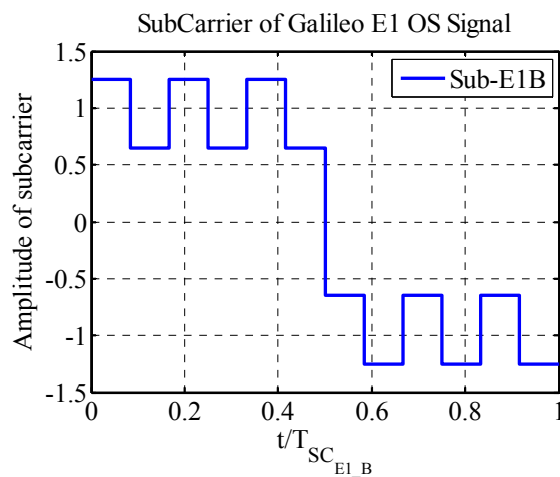
$$sc_X(t) = \text{sgn}(\sin(2\pi R_X t)) \quad (4)$$

$X$  is the kind of subcarrier, including BOC(1,1) noted as  $a$  and BOC(6,1) noted as  $b$  ;

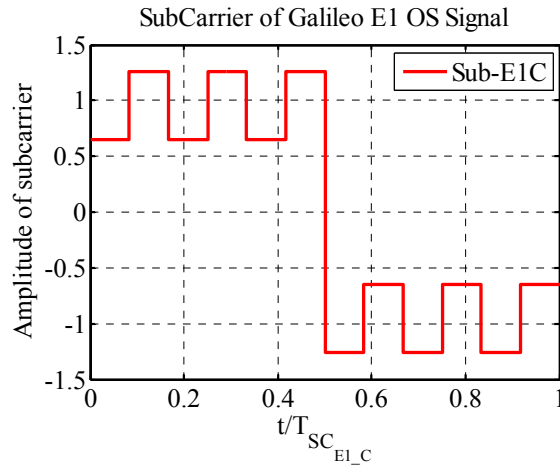
$e_{E1-B}(t)$  includes the data stream and primary code of data channel;

$e_{E1-C}(t)$  includes the data stream and primary code of pilot channel.

Figure 1 shows one period of the sub-carrier function for the  $E1-B$  signal component and one period of the sub-carrier function for the  $E1-C$  signal component.



a)



b)

Figure 1. One period of the CBOC sub-carrier for a) the E1-B signal component and b) the E1-C signal component

**3. Problem In E1 Acquisition**

A traditional acquisition scheme is shown in Figure 2[7]. The receiver will launch a two-dimensional search for one satellite by generating local signals with different Doppler frequencies and code phases.

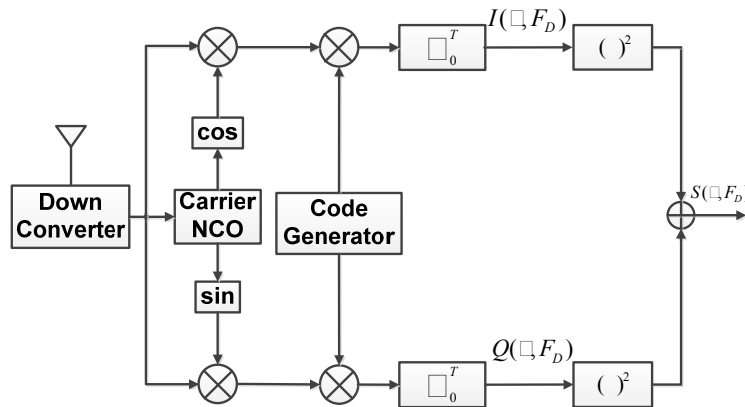


Figure 2. Traditional acquisition scheme

Therefore, a two dimensional  $S(\tau, f_D)$  will be obtained. When one of  $S(\tau, f_D)$  is above threshold, the acquisition will be finished. The acquisition process must detect the incoming signal energy and estimate the signal Doppler and code delay. The detection criterion can be expressed as

$$S(\tau, f_D) = I^2(\tau, f_D) + Q^2(\tau, f_D) \tag{5}$$

Where

$$I(\tau, f_D) = \frac{\sqrt{C}}{4} R(\tau) \text{sinc}(\Delta f_e T_{coh}) \cos(\phi) + n_I(\tau, f_D) \tag{6}$$

$$Q(\tau, f_D) = \frac{\sqrt{C}}{4} R(\tau) \text{sinc}(\Delta f_e T_{coh}) \sin(\phi) + n_Q(\tau, f_D) \quad (7)$$

$I(\tau, f_D)$  and  $Q(\tau, f_D)$  are the correlator outputs with certain code delay and Doppler frequency.  $R(\tau)$  is the final correlation function  $\Delta f_e$  is the difference between the real Doppler frequency and the local Doppler frequency  $f_D$ ,  $\phi$  is the error on the carrier phase,  $n_I(\tau, f_D)$  and  $n_Q(\tau, f_D)$  are the in-phase and quadrature correlator output noise.

### 3.1 Ambiguous problem

CBOC autocorrelation function can be expressed as: [8]:

$$R_{CBOC(-)} = V^2 R_{BOC(1,1)}(\tau) + W^2 R_{BOC(6,1)}(\tau) - 2VWR_{BOC(1,1)/BOC(6,1)}(\tau) \quad (8)$$

$$R_{CBOC(+)} = V^2 R_{BOC(1,1)}(\tau) + W^2 R_{BOC(6,1)}(\tau) + 2VWR_{BOC(1,1)/BOC(6,1)}(\tau) \quad (9)$$

Where

$$V = \sqrt{\frac{10}{11}} \quad (10)$$

$R_{CBOC(+)}$  is the autocorrelation function of data channel and  $R_{CBOC(-)}$  is the autocorrelation function of pilot channel. Two kinds of autocorrelation functions are shown in Figure 3. It can be seen that there is not only one peak any more in the autocorrelation function of CBOC. There are two more side peaks.

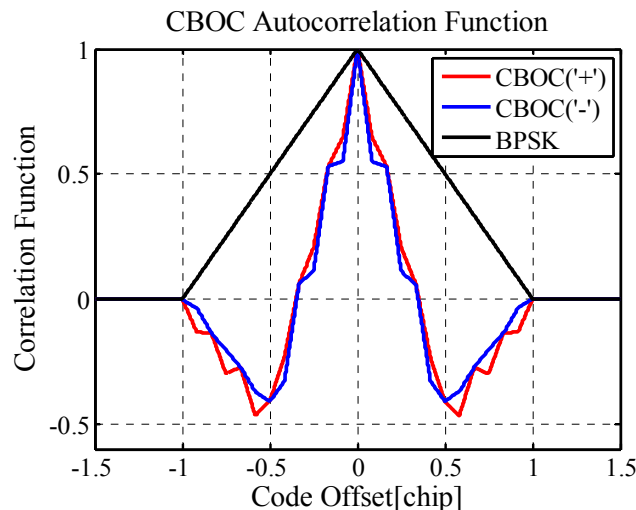


Figure 3. Autocorrelation function of CBOC and BPSK

If the traditional scheme is used,  $S(\tau, f_D)$  will be obtained. Under the influence of noise, the side peaks are either higher or lower than that of auto-correlation function, and the correlation outputs are not symmetric about the center any more. Once the side peak is above

the threshold, it will lead to a false acquisition. After a false acquisition, the code tracking loop will lock on the side peak.

### 3.2 Data transition problem

Modernized navigation satellite systems has not only data channel, which is modulated with data, but also pilot channel, which is modulated without data and with secondary code. Secondary code, as the name implies, is a second code, which multiplies the primary code to form a longer code, called tiered code. The technique characteristics of the Galileo E1 OS signal, GPS L1C signal and GPS L1C/A signal are given in Table 1.

Table 1. Characteristics of Galileo E1 OS signal, GPS L1C and GPS L1C/A

GNSS system	Galileo		GPS		GPS
Signal Type	E1 OS		L1C		L1C/A
Spreading modulation	CBOC(6,1,1/11)		TMBOC(6,1,4/33)		BPSK
Primary code frequency	1.023MHz		1.023MHz		1.023MHz
Primary code length	4092		10230		1023
Primary code period	4ms		10ms		1ms
Signal component	Data	Pilot	Data	Pilot	Data
Data rate	250bps	-	100bps	-	50bps
Secondary code rate	-	250bps	-	100bps	-
Secondary code length	-	25	-	1800	-

As it is shown in Table 1, the spreading codes' periods have the same duration as a data or secondary code bit, which makes it difficult to perform the acquisition using multiple periods of primary codes for Galileo E1 signal and GPS L1C signal, which is different from GPS L1C/A signal. For GPS L1C/A signal, a data bit transition can occur every 20ms, compared to 4ms for Galileo E1 OS signal and 10ms for GPS L1C signal. This characteristic will lead to short integration time for Galileo E1 OS signal and GPS L1C signal if the data bit transition occurs. When a bit transition occurs, it may lead to a high losses, which will result in the failure of acquisition as shown in Figure 4.

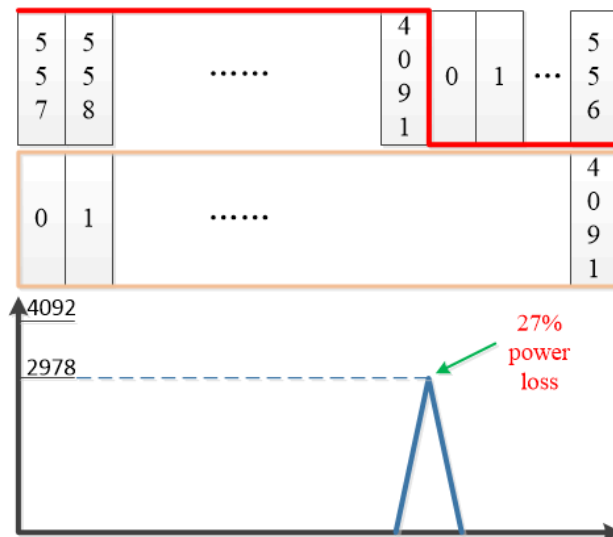


Figure 4. Data transition problem

### 3.3 Large number of Doppler bins

As we can see from above, the final correlator output contains a term of  $\text{sinc}(\Delta f_e T_{coh})$ . Then the correlator output will undergo a power loss if the difference between local carrier frequency and the Doppler frequency of input signals is big. For Galileo E1 OS signal, the coherent time is longer than that of GPS L1 C/A when one primary code period is used to perform acquisition. It is shown in Figure 5. When the difference between the Doppler frequency of input signal and local carrier frequency, the difference of power loss between Galileo E1 OS and GPS L1C/A signal is -3.22dB. Therefore, for the sake of reducing the loss due to the difference between local carrier frequency and the Doppler frequency, smaller Doppler search step will be chosen if the coherent integration time is longer. Consequently, there will be more Doppler bins and longer searching time for Doppler frequency.

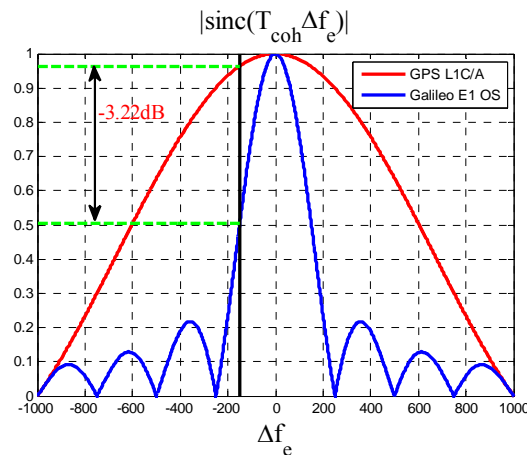


Figure 5. The degradation due to frequency difference

### 4. Proposed Technique

In this paper, a new technique will be proposed. The scheme of proposed technique is shown in Figure 6.

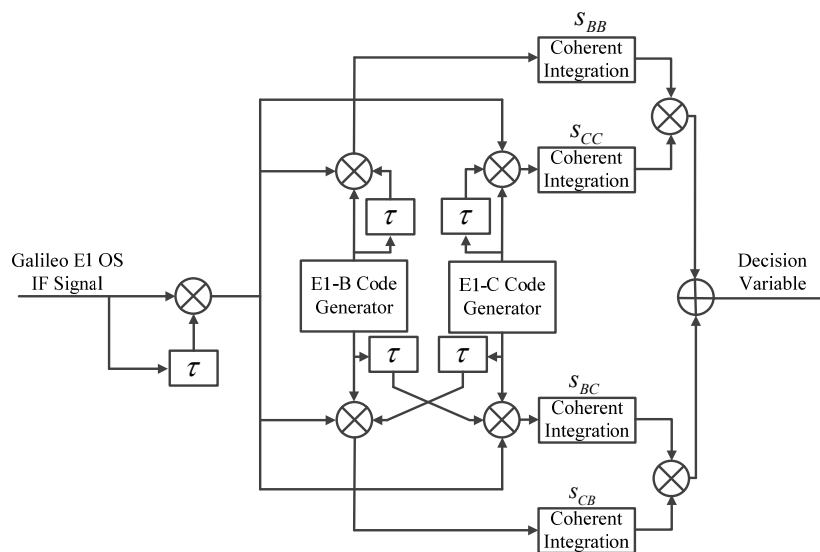


Figure 6. Proposed acquisition scheme of Galileo E1 OS signal

The band-pass signal at the output of RF front end can be expressed as

$$r(t) = s_{E1}(t)\cos(2\pi(f_{IF} + f_D)t) + n(t) \quad (11)$$

where  $n(t)$  is the zero-mean, additive white Gaussian noise with variance.

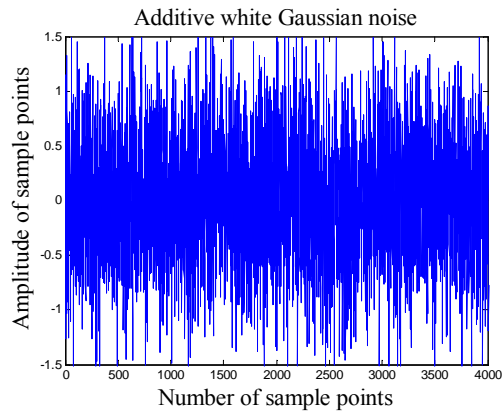
$$\sigma_n^2 = E\{n - E\{n}\}^2 = 2 \cdot E\{n_I^2\} = 2 \cdot E\{n_Q^2\} = 2N_0 \cdot B \quad (12)$$

$f_{IF}$  is the intermediate frequency and  $f_D$  is the Doppler frequency.

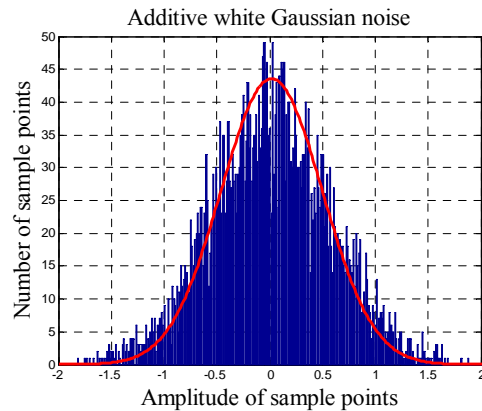
According to the scheme,

$$\begin{aligned} & r(t)r(t+\tau) \\ &= (s_{E1}(t)\cos(2\pi(f_{IF} + f_D)t) + n(t)) \\ & (s_{E1}(t+\tau)\cos(2\pi(f_{IF} + f_D)(t+\tau)) + n(t+\tau)) \\ &= \frac{1}{2}s_{E1}(t)s_{E1}(t+\tau)\cos(2\pi(f_{IF} + f_D)\tau) \\ & + s_{E1}(t)n(t+\tau)\cos(2\pi(f_{IF} + f_D)t) \\ & + n(t)s_{E1}(t+\tau)\cos(2\pi(f_{IF} + f_D)(t+\tau)) \\ & + n(t)n(t+\tau) \end{aligned} \quad (13)$$

As it is shown above, there will four term in the result. Only the first term is useful. Then, the property of second and third term will be analyzed. Figure 7(a) shows the zero-mean additive white Gaussian noise and Figure 7(b) shows the distribution of the noise. Then the noise will be multiplied by Pseudo Random Noise (PRN) codes of Galileo E1 OS signal. Figure 7(c) shows the product of zero-mean additive white Gaussian noise and PRN codes. Figure 7(d) shows the distribution of the product. Comparing the results before and after the multiplication, we can see that the multiplication by PRN codes will not change the distribution of the noise. The red line in Figure 7(c) and (d) shows the Normal distribution.



a)



b)

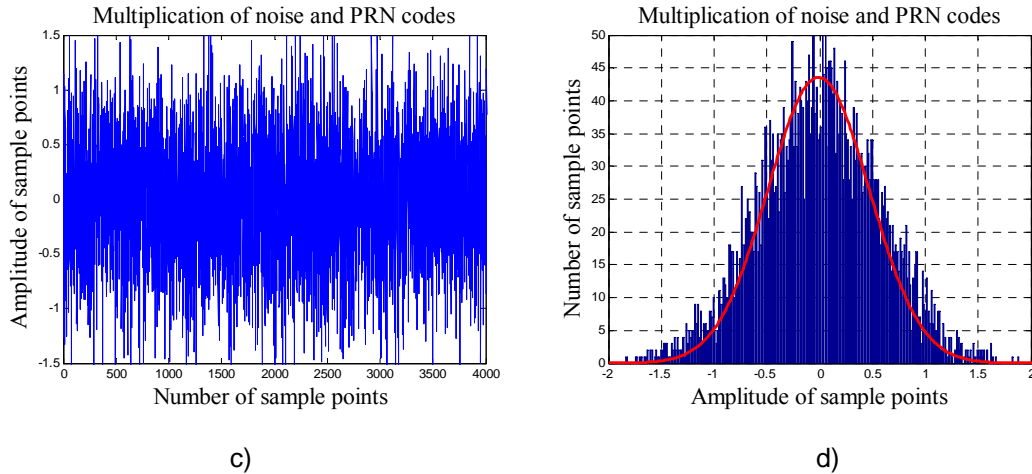


Figure 7. Noise and the distribution

The fourth term is the product of two zero-mean complex Gaussian distributions. The product of two independent normally distributed variates  $x$  and  $y$  with zero means and variances  $\sigma_x^2$  and  $\sigma_y^2$  obeys a normal product distribution [9].

$$p_{xy}(u) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-\frac{x^2}{2\sigma_x^2}}}{\sigma_x \sqrt{2\pi}} \frac{e^{-\frac{y^2}{2\sigma_y^2}}}{\sigma_y \sqrt{2\pi}} \delta(xy - u) dx dy = \frac{K_0\left(\frac{|u|}{\sigma_x \sigma_y}\right)}{\pi \sigma_x \sigma_y} \tag{14}$$

where  $\delta(x)$  is the Dirac distribution and  $K_0(x)$  the modified Bessel function of the second kind and zero order, which is one of two solutions for the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - x^2 y = 0 \tag{15}$$

The solution is

$$K_0(x) = \int_0^x \cos(x \cdot \sinh(t)) dt = \int_0^{\infty} \frac{\cos(x \cdot t)}{\sqrt{t^2 + 1}} dt \tag{16}$$

The normal product distribution is presented in Figure 8.



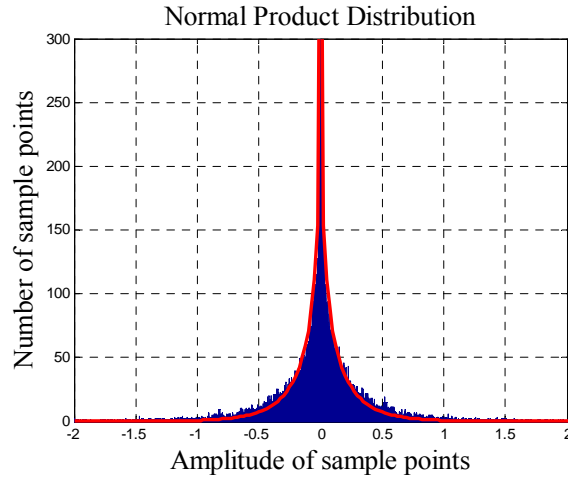


Figure 8. Normal product distribution

The useful term

$$\begin{aligned}
 & s_{E1}(t)s_{E1}(t + \tau) \\
 &= \frac{C}{4} (e_{E1-B}(t)e_{E1-B}(t + \tau)SC_{E1-B}(t)SC_{E1-B}(t + \tau) \\
 & - e_{E1-C}(t)e_{E1-B}(t + \tau)SC_{E1-C}(t)SC_{E1-B}(t + \tau) \\
 & - e_{E1-B}(t)e_{E1-C}(t + \tau)SC_{E1-B}(t)SC_{E1-C}(t + \tau) \\
 & + e_{E1-C}(t)e_{E1-C}(t + \tau)SC_{E1-C}(t)SC_{E1-C}(t + \tau))
 \end{aligned} \tag{17}$$

We can see that the primary codes in data channel and pilot channel will be multiplied by the delay of them. Figure 9 shows the autocorrelation of primary codes and the correlation between primary codes and the delay of them. If  $\tau$  is chosen above one code chip, the product of primary codes will remain the autocorrelation property of primary codes.

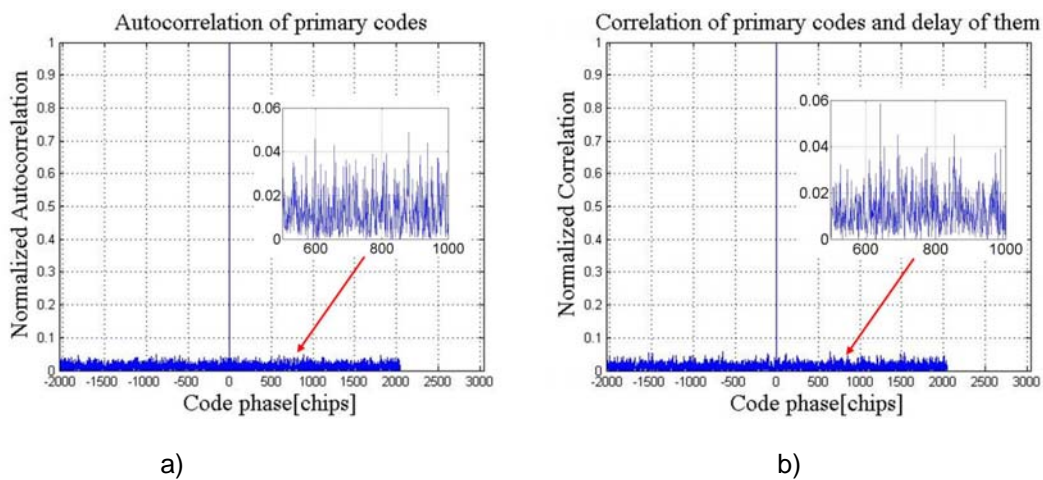


Figure 9. Correlation of primary codes

Through delay-and-multiply, the subcarrier will be wiped off also. There are two kinds of product. One is that of the subcarrier and delay of it. The other is that of the subcarrier and delay of the other kind of subcarrier. Figure 10 shows the autocorrelation and correlation of subcarriers.

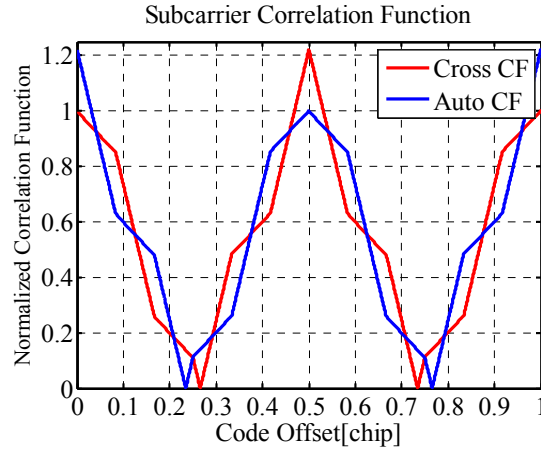


Figure 10. Autocorrelation and correlation of subcarriers

Figure 10 shows that the autocorrelation function and cross-correlation function will both have a peak if the right  $\tau$  is chosen. It is noted that  $\tau$  is chosen to satisfy  $\cos(2\pi(f_{IF} + f_D)\tau) \approx 1$ . Furthermore, if  $f_{IF} \ll f_D$ , just consider  $\cos(2\pi f_{IF}\tau) \approx 1$ . In this way, the effect of Doppler frequency is removed. Finally,  $\tau$  is chosen to satisfy

$$\tau_0 \approx \begin{cases} n+0.5, & n \geq 1 \\ \frac{2k\pi}{2\pi f_{IF}}, & k = 0, 1, 2, 3 \end{cases} \tag{18}$$

**5. Performance Analysis**

After the coherent integration, four terms will be obtained:

$$s_{BB} = \frac{C}{4} R_{BB}(\tau) \cos(2\pi(f_{IF} + f_D)\tau_0) \alpha_{sc} + n_{sn} + n_{ns} + n_{nn} \tag{19}$$

$$s_{CC} = \frac{C}{4} R_{CC}(\tau) \cos(2\pi(f_{IF} + f_D)\tau_0) \alpha_{sc} + n_{sn} + n_{ns} + n_{nn} \tag{20}$$

$$s_{BC} = \frac{C}{4} R_{BC}(\tau) \cos(2\pi(f_{IF} + f_D)\tau_0) \beta_{sc} + n_{sn} + n_{ns} + n_{nn} \tag{21}$$

$$s_{CB} = \frac{C}{4} R_{CB}(\tau) \cos(2\pi(f_{IF} + f_D)\tau_0) \beta_{sc} + n_{sn} + n_{ns} + n_{nn} \tag{22}$$

As it is discussed above,  $n_{sn}$  and  $n_{ns}$  is the product of signal and noise, which is still the zero-mean additive white Gaussian noise with variance

$$\sigma_{sn}^2 = \sigma_{ns}^2 = \sigma_n^2$$

$n_{mn}$  is the product of two additive white Gaussian noise. By applying [10]

$$\int_0^\infty t^\mu \cdot K_\nu(t) dt = 2^{\mu-1} \cdot \Gamma\left(\frac{\mu+\nu+1}{2}\right) \cdot \Gamma\left(\frac{\mu-\nu+1}{2}\right) \quad (23)$$

where  $\Gamma(x)$  denotes the Gamma function and  $K_\nu(t)$  the modified Bessel function of second kind and  $\nu$ -th order, the variance of each zero-mean, normal product distributed summand is defined by

$$\sigma_u^2 = E\{u^2\} = \int_{-\infty}^\infty u^2 \cdot p_u(u) du \quad (24)$$

$$p_u(u) = \frac{K_0\left(\frac{|u|}{\sigma_n^2}\right)}{\pi \cdot \sigma_n^2} \quad (25)$$

which results to

$$\sigma_u^2 = 2 \cdot \int_0^\infty \frac{\sigma_n^2}{\pi} \left(\frac{u}{\sigma_n^2}\right)^2 \cdot K_0\left(\frac{u}{\sigma_n^2}\right) du \quad (26)$$

$$t \square \frac{u}{\sigma_n^2} \Rightarrow du = \sigma_n^2 dt \quad (27)$$

$$\begin{aligned} \sigma_u^2 &= \frac{2\sigma_n^2}{\pi} \cdot \int_0^\infty t^2 \cdot K_0(t) \sigma_n^2 dt \\ &= \frac{4\sigma_n^4}{\pi} \Gamma^2\left(\frac{3}{2}\right) \\ &= \frac{4\sigma_n^4}{\pi} \left(\frac{\sqrt{\pi}}{2}\right)^2 \\ &= \sigma_n^4 \end{aligned} \quad (28)$$

The final criterion is

$$S(\tau) = \sum_{k=0}^{K-1} (s_{BB}^k s_{CC}^k + s_{BC}^k s_{CB}^k) \quad (29)$$

In the GNSS case, there are two conditions of signal presence and absence correspond to the two hypotheses:

The null hypothesis  $H_0$ : the signal is not present or not correctly aligned with the local replica;

The alternative hypothesis  $H_1$ : the signal is present and correctly aligned.

In particular, the detection and the false alarm probabilities are defined as:

$$P_{fa}(\gamma) = P(S(\tau) > \gamma | H_0) = P(S(\tau) > \gamma | \tau \neq \tau_0) \tag{30}$$

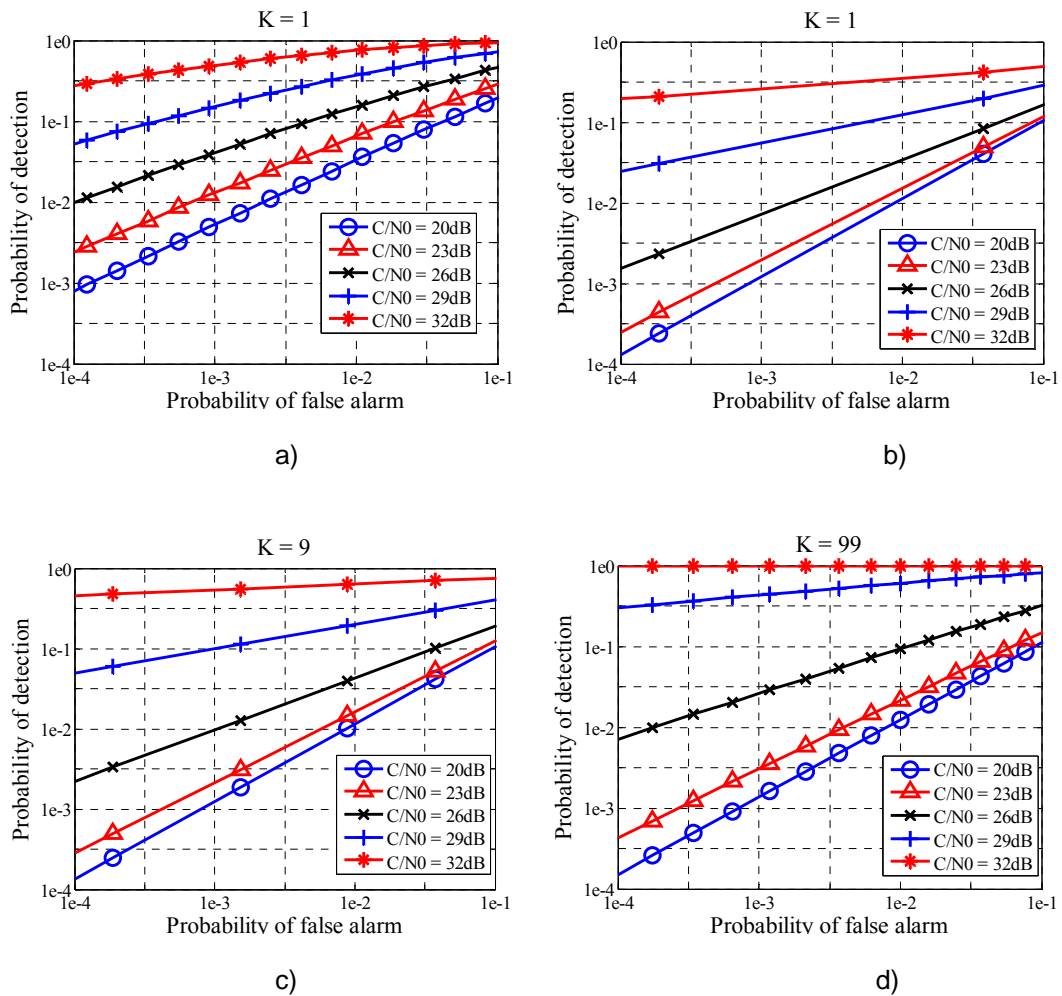
$$P_d(\gamma) = P(S(\tau) > \gamma | H_1) = P(S(\tau) > \gamma | \tau = \tau_0) \tag{31}$$

By applying the central limit theorem, for sufficiently large  $K$ , all noise term will converge to uncorrelated, zero-mean, Gaussian distributed variates. Therefore, the detection and the false alarm probabilities are:

$$P_{fa}(\gamma) = \frac{1}{2} \operatorname{erfc}\left(\frac{\gamma}{\sqrt{2K(4\sigma_n^4 + 4\sigma_n^6 + \sigma_n^8)}}\right) \tag{32}$$

$$P_d(\gamma) = \frac{1}{2} \operatorname{erfc}\left(\frac{\gamma - \frac{KC^2}{16}}{\sqrt{2K\left(\frac{C^2}{4}\sigma_n^2 + \left(\frac{C^2}{8} + 4\right)\sigma_n^4 + 4\sigma_n^6 + \sigma_n^8\right)}}\right) \tag{33}$$

where  $K$  is periods of coherent integration. Figure 11 shows Receiver Operating Characteristic (ROC) curve with different  $K$ .



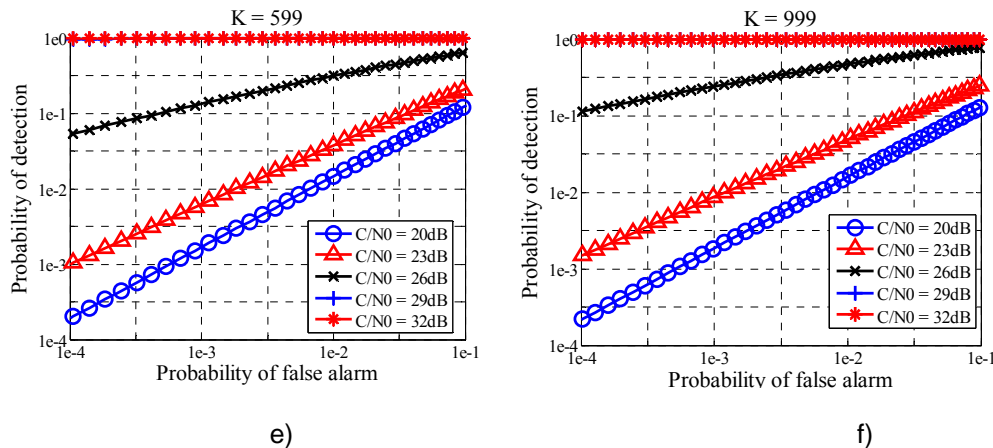


Figure 11. Receiver Operating Characteristic (ROC) curve with different  $K$

It is shown that when  $K=1$ , the performance of traditional scheme is better than that of proposed method. However, based on the discussion above, the proposed method can improve its performance by increasing the periods of coherent integration. With the increase of  $K$ , the performance of proposed method is obviously improved. It is also noted that it is very difficult to improve the performance when C/N0 of signal below 23dB by only increasing the periods of coherent integration.

## 6. Conclusions

In this paper, a new method is proposed to accomplish the unambiguous acquisition for Galileo E1 OS signal. At first, the problems existing in the process of acquisition are analyzed. Then, the new method is proposed and how the proposed method will solve the problem is discussed. Finally, Receiver Operating Characteristic (ROC) curves with different  $K$  are given. It is proved that the new method can performance as well as the traditional scheme in terms of probability of detection and false alarm.

## References

- [1] Jose Vicente Perello Gisbert, Gustavo Lopez Risueño, et al. *Galileo takes first flight: IOV aeronautical campaign results*. Proceedings of European Navigation Conference. 2014.
- [2] Javier Benedicto Ruiz. *Europe takes up its role*. Proceedings of European Navigation Conference. 2014.
- [3] Heiries V, Avila-Rodriguez JA, Irsigler M, et al. *Acquisition performance analysis of composite signals for the L1 OS optimized signal*. ION GNSS 18th ITM. 2005: 13-16.
- [4] Vincent Heiries, J'A Avila-Rodriguez, M Irsigler, GW Hein, E Rebeyrol, D Roviras. *Acquisition performance analysis of candidate designs for the l1 os optimized signal*. In Proceedings of the 16<sup>th</sup> Annual International Technical Meeting of the Satellite Division of the Institute of Navigation (ION)–Global Satellite Navigation System (GNSS). 2005.
- [5] Olivier Julien, Christophe Macabiau, M Elizabeth Cannon, Gerard Lachapelle. Aspect: Unambiguous sine-boc (n, n) acquisition/tracking technique for navigation applications. *Aerospace and Electronic Systems, IEEE Transactions on*. 2007; 43(1): 150–162.
- [6] Foucras M, Julien O, Macabiau C, et al. *A novel computationally efficient Galileo E1 OS acquisition method for GNSS software receiver*. ION GNSS 2012, 25th International Technical Meeting of The Satellite Division of the Institute of Navigation. 2012.
- [7] Borio D. *A statistical theory for GNSS signal acquisition*. Polytecnico di Torino NavSAS Group, Torino. 2008.
- [8] Avila-Rodriguez JA, Wallner S, Hein GW, et al. *CBOC-An implementation of MBOC*. First CNES Workshop on Galileo signals and signal processing. 2006: 12-13.
- [9] Weisstein EW. *CRC concise encyclopedia of mathematics*. CRC press. 2010.
- [10] Abramowitz M. *Handbook of mathematical functions*. New York: Dover. 1972.