TELKOMNIKA, Vol.15, No.3, September 2017, pp. 1022~1030
ISSN: 1693-6930, accredited **A** by DIKTI, Decree No: 58/DIKTI/Kep/2013
DOI: 10.12928/TELKOMNIKA.v15i3.5800

Weighted Least Squared Approach to Fault Detection and Isolation for GPS Integrity Monitoring

Ershen Wang*[1], Fuxia Yang[2], Pingping Qu[3], Tao Pang[4], Xiaoyu Lan[5]
[1,2,3,4,5]Shenyang Aerospace University, Shenyang 110136, China
[1]Liaoning General Aviation Key Laboratory, Shenyang 110136, China
*Corresponding author, e-mail: wes2016@126.com

Abstract

Reliability of a global navigation satellite system is one of great importance for global navigation purposes. Therefore, an integrity monitoring system is an inseparable part of aviation navigation system. Failures or faults due to malfunctions in the systems should be detected to keep the integrity of the system intact. In order to solve the problem that least squares method detects and isolates a satellite fault for GPS integrity monitoring, in this paper, a weighted least squares algorithm is proposed for satellite fault detection and isolation. The algorithm adopts the diagonal elements of the covariance matrix of GPS measurement equation as the weighted factor. Firstly, the weighted least squares approach for satellite fault detection establishes the test statistic by the sum of the squares of the pseudo-range residuals of each satellite for GPS. Then, the detection threshold is obtained by the false alarm rate of the fault detection, probability density function and visiable satellite number.The effectiveness of the proposed approach is illustrated in a problem of GPS (Global Positioning System) autonomous integrity monitoring system. Through the real raw measured GPS data, based on least squares RAIM method and the weighted least squares RAIM approach, the performance of the two algorithms is compared. The results show that the proposed RAIM approach is superior to the least squares RAIM algorithm in the sensitivity of fault detection and fault isolation performance for GPS integrity monitoring.

Keywords: *Global Navigation Satellite System (GNSS), Global Positioning System (GPS), Receiver Autonomous Integrity Monitoring (RAIM), Weighted Least squares method, Fault detection, BeiDou Navigation Satellite System (BDS)*

1. Introduction

The integrity includes the ability of navigation system to provide timely warnings to users when the system or some of its components are not trusted for navigation [1]. There are three kinds of integrity monitoring methods: satellite autonomous integrity monitoring, ground-based augmentation systems and receiver autonomous integrity monitoring (RAIM). The Receiver Autonomous Integrity Monitoring (RAIM) technology [2] is an algorithm running in the user receiver. It carries out the consistency check of the redundant observation information to achieve the purpose of integrity monitoring. The least squares (LS) algorithm is based on the consistency of the measured data to implement fault detection. The fault test statistic is computed by obtaining the residual error vector of the observed data error, and the fault test threshold is obtained by the probability distribution and the false alarm rate of the fault detection.

In the Global Navigation Satellite System (GNSS) positioning, it includes a variety of errors, such as satellite clock, satellite ephemeris error, atmospheric delay error (divided into the ionosphere and the troposphere), multi-path effects and electromagnetic interference. In the case of selective availability (SA) policy, SA is the main error effect; the ionosphere and troposphere are the main influencing factors [3] without SA policy. Pseudo-range measurement error is different because different elevation angles of the satellite signal come through the atmosphere at different satellite elevation [4]. When using least squares residuals detects and identifies fault, these errors are not fully considered. But these errors have different effects on satellites in the same system. To solve these problem, on the basis of the least squares algorithm, we consider the different effects of these errors and propose weighted least squares [5] (Weighted Least Squares, WLS) method, in which the reciprocal of the variance of these errors is used as a weighted ratio. In this paper, we first analyze the principle of weighted

Received April 26, 2017; Revised July 28, 2017; Accepted August 13, 2017

least squares for fault detection and identification. Then we analyze how to determine the weighting factor, and finally verify the performance of weighted least squares algorithm by measured data.

2. RAIM Algorithm
2.1. Fault Detection Model

The linearization equation of GNSS pseudorange observation equation can be shown as follows.

$$y = Hx + \varepsilon \qquad W \tag{1}$$

Where, x is an unknown 4×1 column vector, including receiver position and clock states. y is a n dimensional column vector of pseudo-range measurements minus the expected range for an all-in-view position solution. H is a $n \times 4$ geometry matrix in East North Up (ENU) coordinates with a clock component. ε is a n dimensional column vector of measurement noise. W is the weighted matrix.

$$W = \begin{bmatrix} 1/\sigma_1^2 & 0 & \cdots & 0 \\ 0 & 1/\sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/\sigma_n^2 \end{bmatrix} \tag{2}$$

To solve Equation 1, WLS approach is used. The solution is described as the Equation 3.

$$x_{wls} = \left(H^T W H\right)^{-1} H^T W y = Ay \tag{3}$$

WLS pseudo-range residuals (w) are as follows Equation 4.

$$w = y - H x_{wls} = \left(I_n - H\left(H^T W H\right)^{-1} H^T W\right) y = Sy \tag{4}$$

The covariance matrix (Q) of the matrix (S) is given by:

$$Q = W^{-1} - H(H^T W H)^{-1} H^T \tag{5}$$

The sum of the squared errors can be obtained from the residuals by:

$$SSE = w^T w \tag{6}$$

In the process of fault test, the following equation is used as test statistics (Td).

$$Td = \sqrt{\left(w^T w\right) / (n-4)}$$

When the system is running normally, pseudo-range residuals is smaller and Td is also smaller; When the deviation of a satellite measurement pseudo-range is larger, Td is also lager and it will detect the pseudo-range fault. If the noise errors (w) is normal distribution of zero normal and (σ_0^2) variance, SSE is a $\chi^2(n-4)$ distribution with (n-4) freedom degrees; If the noise errors (w) is not zero normal, SSE is a $\chi^2(n-4,\lambda)$ distribution with (n-4) freedom degrees [6]. Given the probability of false alarms (P_{fa}), the test threshold can be found by threshold is found by the following formula [7].

$$P(SSE < t) = \int_0^t f_{\chi^2(n-4)}(x)dx = 1 - P_{fa} \tag{7}$$

Solving Equation 7 will get the value (t). Then, $T = \sqrt{t/(n-4)}$ is used as test threshold (T).

The method of detecting a fault in the navigation solution is to compare test statistic (Td) with test threshold (T). If test statistic is less than test threshold, there is no failure; if not, there is failure and then we need to identify failure.

2.2. Fault Identifition Model

After detecting the fault, there will be two cases: one is the existence of a fault, the other is no fault. If there is a fault and the number of visible satellites is more than 6, receiver would identify which satellite is fault. If there is no fault, receiver would solve the position. The most commonly used error detection method is the Balda data detection method [8]. The steps of the method are: firstly, using the least squares residual structure the detection statistic obeyed a certain distribution; giving the confidence level obtains the detection threshold; finally, comparing the detection statistic with the detection threshold determines which satellite is faulty. So the test statistic of fault identification is defined as:

$$d_i = |w_i| / \sqrt{Q_{ii}} \tag{8}$$

Where, d_i is test statistics of the i^{th} satellite; w_i is the i^{th} element of the pseudo-range residual; Q_{ii} is the i^{th} diagonal elements of matrix (Q). Make a binary hypothesis for d_i. If there is no fault, d_i is $d_i \sim N(0,1)$; If there is a fault, d_i is $d_i \sim N(\delta_i,1)$.

Given the total false alarm rate (P_{fa}), the false alarm rate for each test statistic is P_{fa}/n. The equation for determining the identification threshold is given by:

$$P(d_i > T_2) = 2\int_{T_2}^{\infty} f_{N(0,1)}(x)dx = P_{fa}/n \tag{9}$$

By solving the Equation 9, we can obtain the identification threshold (T_2).

The method of identifying a fault in the navigation solution is to compare test statistic (d_i) with identification threshold (T_2), if test statistic is less than identification threshold, the satellite is no failure; if not, the satellite is a failure satellite and need to exclude the failure satellite in positioning solution.

2.3. RAIM Algorithm Availability

RAIM is affected by the number and geometrical structure of satellites when detecting and identifying satellite faults. In some areas, the satellite geometrical structure can not meet all the integrity performance requirements so that the integrity monitoring results at this time will not be credible [9]. Hence it is very necessary to judge whether the current satellite geometrical structure can meet the needs of fault detection before RAIM algorithm detects fault, which is called availability judgement under integrity requirement. Now determining the RAIM algorithm availability method includes: $\delta H_{\max}$, approximated radial error protected (ARP), horizontal protect limit (HPL). The theoretical analysis and comparison of these three availability judgments methods show that $\delta H_{\max}$ method and HPL method is equivalent in theory [10]. Then, we introduce the weighted HPL method to determine the the RAIM algorithm availability.

If i th satellite is faulty and the pseudo-range measurement error is b_i, then the horizontal positioning errors (E_i) is written as:

$$E_i = b_i \sqrt{A_{1i}^2 + A_{2i}^2} \tag{10}$$

Where, A_{1i} and A_{2i} denote the first and the second element of the i th column of the $m \times n$ matrix $A = \left(H^T W H\right)^{-1} H^T W$.

Ignoring the observation noise, the corresponding test statistic (r_i) is written as:

$$r_i = b_i \sqrt{S_{ii} W_{ii} / (n-4)} \tag{11}$$

Where, S_{ii} is the element of i th row and i th column of the matrix (S); W_{ii} is the element of i th row and i th column of the matrix (W).

So, the characteristic slope (K_i) of i th satellite is given by:

$$K_i = E_i / r_i = \sqrt{(A_{1i}^2 + A_{2i}^2)(n-4)/(S_{ii} W_{ii})} \tag{12}$$

At the same time, the test threshold (R) that satisfies the probability of missed alarm is given by:

$$R = \sqrt{\lambda / (n-4)} \tag{13}$$

Where, λ is the non-central parameter that satisfies the non-central χ^2 distribution of the missed detection probability. Thus, λ is calculated by the following formula.

$$P(SSE < t) = \int_0^t f_{\chi^2(n-4,\lambda)}(x) dx = 1 - P_{MD} \tag{14}$$

In Equation 14, P_{MD} is known in advance; t is obtained by the Equation 7. By solving the Equation 14, we can attain the non-central parameter (λ).

$$K_{\max} = \max_i (K_i) \tag{15}$$

So, the HPL is written as:

$$HPL = K_{\max} \times R = \max_i \left(\sqrt{(A_{1i}^2 + A_{2i}^2)/(S_{ii} W_{ii})} \right) \times \sqrt{\lambda} \tag{16}$$

The method of judging the RAIM algorithm availability is to compare the value of HPL with the value of HAL (which is horizontal alert limit), if the value of HPL is less than the value of HAL, the RAIM algorithm is available; if not, the RAIM algorithm is not available.

2.4. Calculation of the Weighted Factor

It is assumed that the observation noise variance of the satellite (i) is σ_i^2 and the observed noise of each satellite is independent. So the covariance matrix (C) of the observed n satellite noise is:

$$C=\begin{bmatrix}\sigma_1^2 & 0 & \cdots & 0\\ 0 & \sigma_2^2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \sigma_n^2\end{bmatrix} \quad (17)$$

In addition, the weighted matrix (W) is equal to the inverse (C^{-1}) of the covariance matrix [11]. Where, if selecting the observed noise variance is closer to the actual situation, the weighted least squares residual method would be more effective. Therefore, the choice of observation noise variance is essential. GPS satellite observation noise variance is expressed as [12]:

$$\sigma_i^2=\sigma_{i,URA}^2+\sigma_{i,iono}^2+\sigma_{i,tropo}^2+\sigma_{i,mp}^2+\sigma_{i,tcvr}^2 \quad (18)$$

Where, the variance($\sigma_{i,URA}^2$) correction parameters included satellite clock error and satellite ephemeris error is existed in the satellite broadcast ephemeris file; the value of ionosphere delay error variance ($\sigma_{i,iono}^2$) is $\max\left\{\left(cT_{iono}/5\right)^2,\left(F_{pp}\cdot\tau_{vert}\right)^2\right\}$, it has $c=2.99792458\times10^8$, the ionosphere corrections value(T_{iono})are calculated from the ionosphere Klobuchar model, the mapping function (F_{pp}) is:

$$F_{pp}=\left(1-\left(\frac{6378.1363\times\cos(E_i)}{6378.1363+350}\right)^2\right)^{-0.5}$$

E_i is the elevation angle of the satellite, φ_m is the magnetic latitude of the ionosphere penetration point on the ground, and τ_{vert} is described as follows:

$$\tau_{vert}=\begin{cases}9m & 0\le|\varphi_m|\le 20\\ 4.5m & 20<|\varphi_m|\le 55\\ 6m & 55<|\varphi_m|\end{cases}$$

The tropospheric delay error variance ($\sigma_{i,tropo}^2$) is specifically calculated from the troposphere UNB3 model [13], and the value is shown as follows.

$$\sigma_{i,tropo}^2=\left(0.12\cdot1.001\big/\sqrt{0.002001+\sin^2(E_i)}\right)^2$$

E_i is the elevation angle of i th satellite; the value of multipath error variance ($\sigma_{i,mp}^2$) is $\left(0.13+0.53e^{-E_i/10}\right)^2$; the value of receiver thermal noise variance($\sigma_{i,tcvr}^2$) is 0.1^2. Due to the variability and instability of the ionosphere and the troposphere, a specific empirical model is used for the description. Next we will simply introduce ionosphere model and tropospheric model.

2.4.1. Ionosphere Model

In order to reduce the impact of ionosphere, we use the ionosphere model to modify the observed data. In this paper, Klobuchar model is selected and introduced briefly [14].

The ionosphere correction model is as follows:

$$T_{iono} = \begin{cases} F \times \left[5.0\times10^{-9} + (AMP)(1-\frac{X^2}{2}+\frac{X^4}{2}) \right], & |X| < 1.57 \\ F \times 5.0\times10^{-9}, & |X| \geq 1.57 \end{cases} \quad (19)$$

Where, T_{iono} is the ionosphere correction value; the value of tilt ratio (F)is $F = 1.0 + 16.0 \times (0.53 - E)^3$; AMP is shown as follows.

$$AMP = \begin{cases} \sum_{n=0}^{3} \alpha_n \phi_m^n, & AMP \geq 0 \\ 0, & AMP < 0 \end{cases} (\text{sec})$$

And the phase(X) is $X = 2\pi(t - 50400)/PER \ (\text{radians})$.

Where,

$$PER = \begin{cases} \sum_{n=0}^{3} \beta_n \phi_m^n, & PER \geq 72000 \\ 0, & PER < 72000 \end{cases} (\text{sec})$$

$\phi_m = \phi_i + 0.064\cos(\lambda_i - 1.617) \ (\text{semi-circles})$,

$\lambda_i = \lambda_u + \psi \times \sin A / \cos\phi_i \ (\text{semi-circles})$

$\psi = 0.0137/(E + 0.11) \text{-} 0.022 \ (\text{semi-circles})$

and $t = 4.32\times10^4 \times \lambda_i + GPStime \ (\text{sec})$.

2.4.2. Tropospheric Model

The tropospheric correction model has Hopfiled model, Saastamoinen model and UNB3 model. This paper simply introduces the UNB3 model [13-14].

The UNB3 model is divided into dry delay and wet delay. The GPS tropospheric delay correction in either direction is expressed as:

$$d_{trop} = d_{hyd}^z \times m_{hyd} + d_{wet}^z \times m_{wet} \quad (20)$$

Where, d_{trop} is the total delay for the troposphere; d_{hyd}^z is the dry delay of the tropospheric in zenith direction; d_{wet}^z is the wet delay of the tropospheric in zenith direction; m_{hyd} is the mapping function for the tropospheric dry delay; m_{wet} is the mapping function for tropospheric wet delay.

The dry and wet delay model of the troposphere zenith direction in UNB3 is:

$$d_{hyd}^z = \left(10^{-6} k_1 R_d / g_m\right)\left(1 - \beta H/T_0\right)^{g/(R_d\beta)} P_0 \quad (21)$$

$$d_{wet}^z = \left(10^{-6} k_3' R_d \Big/ \left(g_m \lambda' - \beta R_d\right)\right)\left(1 - \beta H/T_0\right)^{\lambda' g/(R_d\beta) - 1} \left(e_0/T_0\right) \quad (22)$$

Where, $k_1 = 77.604K/mbar$, $k_3^{'} = 382000K^2/mbar$, $g = 9.80665m/s^2$, $R_d = 287.054J/(kg \cdot k^{-1})$, $g_m = 9.784[1 - 2.66 \times 10^{-3}\cos(2\Phi) - 2.8 \times 10^{-7} H]m/s^2$, $\lambda^{'} = \lambda + 1$. However, the meteorological parameters atmospheric pressure (P_0), temperature (T_0), water pressure (e_0), temperature change rate (β), and water pressure change rate (λ) are calculated by the following formula (23).

$$X_{\phi,doy} = Avg_{\phi} - Amp_{\phi} \cdot \cos(2\pi(doy - 28)/365.25) \tag{23}$$

Mapping function (m_{hyd} and m_{wet}) are obtained by using Niell mapping model.

3. Experimental Results and Analysis

In order to verify the correctness and superiority of the weighted least squares algorithm proposed in the fault detection process, the performance of weighted least squares algorithm is evaluated by two performance measures: detection rate and identification rate. And this performance is compared with the performance of the traditional least squares algorithm. We use the Matlab software to compare the detection and identification performance of the two algorithms. Data collection time is at 00:00 on May 29, 2014. We collect 24 hours data and collect a data per second. The receiver coordinates are [-2965385.050, -972576.616, 5543892.887].

In the progress of verifying the detection performance of the weighted least squares RAIM algorithm, we add different biases to the pseudo-ranges of 1 to 18000 epochs on the 12th satellite, and setting the false alarm rate is 0.002/h. The comparison results between least squares and weighted least squares approach are shown in Figure 1.

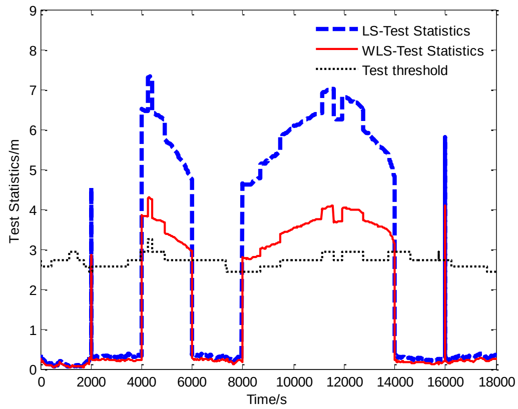

Figure 1. Comparison of test statistics between least squares and weighted least squares algorithms

In Figure 1, in the 2000 epoch and 16000 epochs, pulse errors with 50m and 65m deviation are added respectively. In the range of 4000 to 6000 epochs and 8000 to 14000 epochs, a step error of 55m and 60m was added respectively. In the same deviation, the detection statistic calculated by the weighted least squares method is greater than the detection statistic calculated by the least squares method. The both detection statistic are greater than the detection threshold at the time of adding the fault. Therefore, simulation results show that the both algorithms can detect faults and the two algorithms are valid.

To verify the detection and identification performance of the weighted least squares RAIM algorithm in the simulation, the deviation is added to the pseudo-range of the 12th satellite from 1 to 18000 epochs. The deviation is increased from 0m to 100m and the deviation step length is 5m. The least squares method and the weighted least squares method were used every 10 epoch for fault detection and identification.

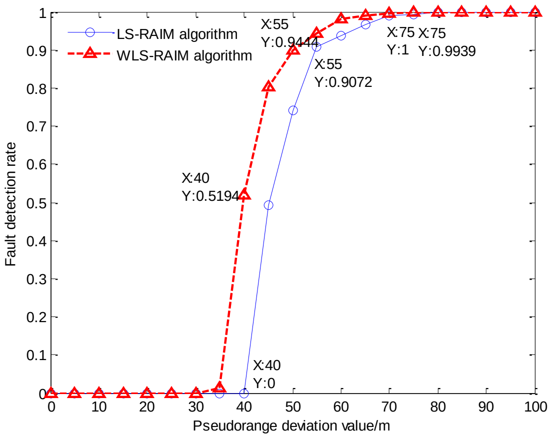

Figure 2. Performance comparison of fault detection rate between least squares and weighted least squares algorithms

In Figure 2, when the pseudo-range deviation is 50m, the fault detection rate of the weighted least-squares RAIM algorithm is 90%; when the pseudo-range deviation is 54m, the fault identification rate of the least-squares RAIM algorithm is 90%. So, we can get a conclusion that the weighted least squares RAIM algorithm for fault is more sensitive and has stronger detection performance than the least squares RAIM algorithm.

Figure 3 shows the fault identification rate comparison between the weighted least squares RAIM algorithm and the least squares RAIM algorithm when we add different biases to the pseudo-ranges of 1 to 18000 epochs on the 12th satellite.

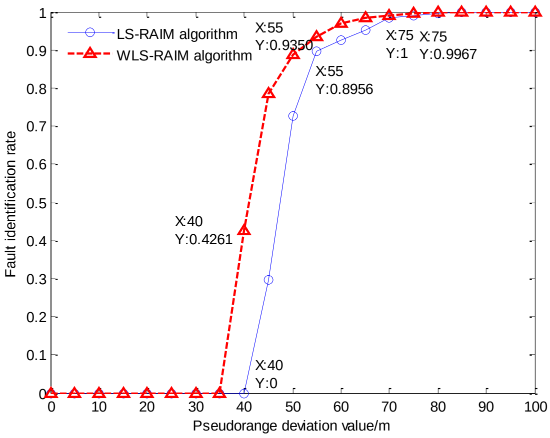

Figure 3. Performance comparison of fault identification rate between least squares and weighted least squares approach

In Figure 3, when the pseudo-range deviation is 50m, the fault identification rate of the weighted least-squares RAIM algorithm is 90%; when the pseudo-range deviation is 56m, the fault identification rate of the least-squares RAIM algorithm is 90%. So, we can get a conclusion that the weighted least squares RAIM algorithm for fault is more sensitive and has stronger identification performance than the least squares RAIM algorithm.

4. Conclusion

This paper proposes a new approach of fault detection and isolation (FDI) for GPS integrity monitoring by using weighted least squares approach. The proposed approach detects failure by establishing the test statistic with the weighted factor. And the detection threshold is discussed by the false alarm rate of the fault detection, probability density function and visiable satellite number. Based on the least squares method and the proposed appraoch, the effectiveness of the proposed approach has been demonstrated on GPS integrity monitoring to detect faults on navigation signal from GPS satellite. Experimental results demonstrate that the GPS satellite failures are successfully detected and isolated. Moreover, the performance of the proposed RAIM approach is better than that based on least squares method. Meanwhile, the results are instructive for the study of the autonomous integrity monitoring of BeiDou navigation satellite system (BDS).

Acknowledgements

This study is supported by the National Natural Science Foundation of China (Nos. 61571309 and 61101161)，the Liaoning BaiQianWan Talents Program (No.04021407).

References

[1] Yuanxi Yang, Junyi Xu. GNSS receiver autonomous integrity monitoring (RAIM) algorithm based on robust estimation. *Geodesy and Geodynamics*. 2016; 7(2): 117-123.

[2] Yang Kai, Guo Chengjun. Calculation of Optimal Vertical Protection Level Based on Integrity Risk and Continuity Risk. *Computer Engineering*. 2017; 43(1): 50-54.

[3] GA Mcgraw. *Tropospheric Error Modeling for High Integrity Airborne GNSS Navigation*. Proceedings of IEEE/ION PLANS 2012. Myrtle Beach, South Carolina. 2012: 158-166.

[4] Zhu Yanbo, Zhang Miaoyan, Zhang Jun. Research on Weighted Algorithm to Predict RAIM Availability. *Journal of Telemetry, Tracking and Command.* 2009; 30(1): 1-4.

[5] Walter T, Enge P. *Weighted RAIM for Precision Approach*. ION GPS. Palm Springs, CA. 1995.

[6] BW Parkinson, P Axelrad. Autonomous GPS Integrity Monitoring Using the Pseudorange Residual. *Navigation.* 1988; 35(2): 255-274.

[7] Wang, Ershen, Ming Cai, Tao Pang. *A Simple and Effective GPS Receiver Autonomous Integrity Monitoring and Fault Isolation Approach.* In Control Engineering and Communication Technology (ICCECT), 2012 International Conference on, IEEE. 2012: 657-660.

[8] Liu Dajie, Tao Benzao. Practical Methods for Surveying Data Processing. Beijing: Surveying and Mapping Press. 2000: 66- 71.

[9] Li Jingyu, Jiang Bin, Liu Jian wei. The Software System Design of Receiver Autonomous Integrity Monitoring. Aero Weaponry. 2008; 6: 52-55.

[10] Sun Minghan, Sun Guoliang, Zheng Yugui. Research on RAIM Availability Algorithm. *Journal of Telemetry, Tracking, and Command.* 2006; 27(2): 24-27.

[11] Xie Gang. Principles of GPS and Receiver Design. Beijing: Electronic Industry Press. 2014.

[12] RTCA DO-229C. Minimal Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment. U.S.A: RTCA, Inc. 2001.

[13] Leandro RF, Santos MC, Langley RB. *UNB neutral atmosphere models: development and performance*. Proceedings of ION NTM. 2006; 1: 564-573.

[14] Jiao Weidong, Tang Zhihu, Shen Xiaoyun. Analysis of influence of ionosphere and troposphere model on RAIM availability of COMPASS. *Journal of Communications*. 2015; 36(3): 74-81.